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A Liquidity Approach to the Relationship between the Central Bank and the State

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Abstract

The Eurozone sovereign debts crisis(2010-2012?) raises a "sovereign rate puzzle". We show that no reasonable econometric model of sovereign interest rates can explain their sharp increase in 2011 and 2012. Our explanation is the following : due to differences in the preference for inflation, Central Banks (CB) don't show the same degree of commitment to intervene on the sovereign bond market - what we label the degree of liquidity-passivity.

While the literature until now has concentrated mostly on the relations between the State and the CB from the angle of solvency, we would want to underline the idea that the liquidity dimension of the relation is as important. We provide a clear framework to conceptualize the two dimensions of the relationship between the CB and the State (liquidity and solvency) and to understand how they interact.

Our theoretical contribution is to show how the monetary policy influences the determination of the equilibrium of a simple roll-over game where the State has to refinance its debt, the investors to decide upon reinvesting or not and the monetary upon whether to commit and possibly intervene as a lender of last resort of the State (LLRS). We study and show how the impact of the commitment to act as a LLRS depends on the informational setting - homogeneous signals as in Diamond-Dybvig ; heterogeneous signals as in Morris-Shin. We show that the CB faces a trade-off between stability of the financing of the State and inflation when the information is not complete, that the solution to this trade-off - the degree of liquidity-passivity - is decreasing with the preference for inflation and decreasing with the degree of solvability-activity. When the interest rate is allowed to be endogeneously determined, we show that it decreases with the degree of liquidity-passivity and increases with the preference for price stability of the CB. We also build an extension that shows why trying to influence the sovereign debt market through the refinancing of banks is not a sufficient solution when the troubles are caused by run for liquidity behaviors. This might explain why the ECB policies before the summer 2012 failed : as far as the sovereign debt markets were concerned, the liquidity hoarding behavior of banks was not the main issue.

We then propose a historical perspective to test our model and to introduce renewed - but tentative - interpretations of some key financial events, in the light of our model. Finally the econometric contribution based on two distinct event-analyses run on daily and quaterly data is to show that the policy announcements of the CB have decreased the sovereign yields mainly through the decrease in default risk, and not trough a rebalancing effect nor through the expected decrease in future policy rates - although we find evidence of a small decline in forwards rates on July 26. The expected inflation slightly increased in the three announcements. Our results support the "run for liquidity" interpretation. On the contrary, the fundamentalist and the liquidity hoarding interpretations are rejected by the data.

JEL Classification Codes: C230, E520, E580, E630, E650, G010, G140, H630

Keywords: sovereign bond spreads, interest rate, sovereign debt crisis, euro area, global games, monetary policy, lender of last resort, panel data, event studies

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Contents

1 Introduction

The "incestuous" relations between the State and the Central Bank were often envisaged in the economic literature from the angle of the " monetary dominance " or on the contrary from the one of the " budgetary dominance ". In the regime of " monetary dominance ", the Central Bank independently decides upon the path of inflation - and thus upon the path of incomes from seigniorage. In such a regime, the State has to choose its flows of income so as to meet its intertemporal budget constraint, by taking into account the monetary policy and by considering as given the income of seigniorage.

Nevertheless this literature envisages the relationships between the State and the Central Bank only from the angle of solvency. Yet the current events as well as the historic analysis suggest that the first - conceptually and historically - relationship between the State and the Central Bank is marked by liquidity. The tensions in 2011 and 2012 on the European sovereign debts markets and the peace on the markets of the Anglo-Saxon debts - a contrast that cannot be justified by fundamentals - indeed reveals the importance of liquidity. More exactly, the current events remind us of the role of lender of last resort of Central Banks. We rediscover that Central Banks are not only the lender of last resort to the banks but also the lender of last resort to the States. The history of the last three centuries is full of examples of situations in which the Central Bank had to play the role of lender of last resort to the sovereign. To tell the truth, the Central Bank was at first the lender of last resort of the State before being the one of the banking system, and the ban on the European Central Bank to lend directly to the members of the Eurozone is, in this perspective, an accident of history.

The study of liquidity made the object of in-depth studies in other domains of the economic research: since the works of Diamond and Dybvig (Diamond and Dybvig, 1983), the phenomena of "runs" and the sudden drying up of liquidity have been modelled and applied to various domains. More recently, the global games literature with Morris and Shin's works tried to restore the uniqueness of the equilibrium by introducing noises into the private signals (Morris and Shin, 1998, 2001, 2003, 2009). These models have been extensively used in the financial intermediaries literature: Allen and Gale show that run may be contagious depending on the linkages between financial institutions (Allen and Gale, 2000), Acharya and al. show that modern funding markets like the repo markets may be subject to run (Acharya, Gale and Yorulmazer, 2010) and Vives and Rochet show why an LLR is still needed in a global game framework with modern interbank markets (Rochet and Vives, 2004). Our dissertation is also related to the international crises literature, for example Chang and Velasco apply the Diamond and Dybvig framework in an international context (Chang and Velasco, 1999 ; Sachs, 1995 ; Flood and Mario, 1998 ; Goldfajn and Valdes, 2007, ; Chamon, 2007 ; Cohen and Villemot, 2008). In this international crises literature, a more specific stream studies the role of a International Lender of Last Resort (Jeanne and Wyplosz, 2001) ; Jeanne and Zettelmeyer, 2002) or the role of an International Lender of First Resort (Cohen and Portes, 2006). A set of older papers present models of "confidence crisis" on government debt that have a similar structure as in Diamond and Dybvig (1983) (Calvo 1988 ; Giavazzi and Pagano, 1990 ;

Alesina et al., 1990).

Our work is also related to the stream of the literature that studies the relationship between the State and the CB. Sargent and Wallace model two polar cases of the relationship between the monetary authority and the State. In the monetary dominance regime, the path of inflation is set exogeneously, the the State has to adjust the path of taxes to meet its budget constraint. In the fiscal dominance regime, the path of taxes is exogeneously given, and the path of inflation / seigneuriage revenus adjust so as for the State to meet its budget constraint (Sargent and Wallace, 1981). In the literature on DSGE models, there is monetary dominance when the monetary rule follows the Taylor principle and the fiscal rule is locally ricardian (Woodford, 2001). Our terminology - liquidity-activity and solvability-activity - refers to the one of Leeper : in his papers, a passive fiscal policy and an active monetary policy correspond to the monetary dominance of Sargent and Wallace (Leeper, 1991). Sims conceptualizes the fiscal theory of the price level where the price level becomes an asset price endogeneously determined to guarantee the solvency of the State (Sims, 1994).

Our model is historically very much related - although conceptually different as we will suggest - to the one of Jeanne (Jeanne, 2012). We differ from the latter to the extent that the role of *our* lender of last resort is not to provide the State in need to close its deficit with seigneuriage income but to simply provide it with liquidity when a crisis hits. Actually the model of Jeanne doesn't really model a LLR, since the CB doesn't lend liquidity but give the State real revenus. This criticism concerns all the literature that has modeled the relationship between the State and the CB only from the angle of solvency (Davig, Leeper and Walker, 2010 ; Uribe, 2002). There is consequently a major conceptual difference between the literature that have modeled the relationships between the State and the CB so far and our paper since the CB provides the State above all with liquidity, before possibly providing it with real revenus.

The main objective of this dissertation is to show that the primary - conceptually and historically - dimension of the relationship between the State and the Central Bank is the one of liquidity. It therefore aims at providing a clearer understanding of the role of Lender of Last Resort of the CB to the State and at showing how the liquidity dimension interacts with the solvability dimension. More concretely, we model a situation in which the State has to refinance its debt and the investors to choose whether to roll or not depending on signals that they receive, the behavior of other investors and the policy of the CB. In the benchmark model, we show how the monetary policy of the Central Bank influences the equilibrium of the game, and in particular the probability of defaults and inflation. When information is incomplete, the CB faces a trade-off between stability of the financing of the State and monetary stability.

We often hear of and talk about the independence of the CB without knowing exactly the precise meaning of the word. There is indeed no precise definition of it. The degree of independence is usually approached by the laws that rule the relationship between the CB and the State. In economics, the conceptual work remains to be done. The only conceptual content is given by Sargent and Wallace : an independent policy is a policy that freely defines the optimal path of future inflation without taking into account the budget constraint of the State. But this definition is only about solvability ; and therefore neglects the liquidity dimension. In these frameworks, liquidity is actually completely absent. In our dissertation, we provide a framework that allows to distinguish two dimensions in the relationship between the State and the CB. We define a degree of liquidity-passivity (activity) as the degree to which the CB is (not) ready to lend to the State and a degree of solvability-activity (passivity) - what Sargent and Wallace label monetary dominance - as the degree to which the CB has (doesn't have) to provide income from seigneuriage to meet the budget constraint of the State. Two main results stem from the analysis. Firstly we show that the degree of liquidity-passivity is all the higher as the preference for price stability is low, which is intuitive and can rationalize the difference in the behaviors of the ECB and the Fed. Secondly, and more surprisingly, in our investigation on the interaction between the two dimensions, it is shown that the degree of liquidity-passivity is all the higher as the solvability-activity is also high: it is optimal for the Central Bank to guarantee to lend to the State when the degree of solvability-activity is high, which is the case in the Eurozone.

We then propose some extensions to the basic model. At first, we seek to endogenize the interest rate considered as exogenous in most of the models of "run". This is indeed all the more annoying as financial markets are characterized by flexible interest rates and as modern crises take the form of sharp increase in interest rate. Following a simple method and by means of simulations, we determine the equilibrium rates and draw general conclusions on the effect of the monetary policy on this equilibrium rate. In particular we show that the more liquidity-passive the CB policy the lower the equilibrium interest rate. In a second extention, we build a model that aims at showing why the underlying principle of the policies of the ECB before September 6^{th} which consisted in trying to influence the sovereign debt markets through the channel of the refinancing of banks could not be as efficient as a direct intervention on the market ; although it may have been legitimate.

The following part consists in a historical path through key events of the financial history of the Treasuries and of the relationship between the State and the CB in order to test our toy model and to show the interesting opportunities of reinterpretation it offers. The last part is an econometric study of the recent events in the Eurozone. We argue that the "sovereign debt puzzle" - the facts that no reasonable econometric model is able to replicate the recent sharp increase in sovereign yields in Europe and that the Anglo-Saxon countries benefit from the lowest rate while their fundamentals are similar or worse than those of Southern States - can be understood provided we take into account the liquidity dimension of the relationship between the ECB and the members of the Eurozone. We run two kinds of event-analysis on the three announcements dates in the summer 2012 that have led to a greater commitment on the part of the ECB to support sovereign debt markets. One focuses on the short term reaction of financial markets (1 to 4 days) and the other on the cumulative effect over the quarter. This allows us to shed light on the channels through which these announcements have been effective in decreasing sovereign yields. Although these results are interesting by themselves, we use them to test our toy model and more precisely to distinguish between different interpretations of the eurozone crises. Indeed, these three announcements offer an quasi-natural experiment of a shift of a monetary policy : since this shift has different implications depending on the interpretations one has in mind, it allows us to reject the fundamentalist and the liquidity-hoarding - described in the second extension - interpretations.

2 The model

2.1 General setup

CB takes action and announces it
θ_{BC} realizes
$t = 0$

Uncertainty resolved
θ_2 realizes
$t = 2$

$t = 1$
Θ_I realizes
Investors take action

There are three periods $t = 0, 1, 2$. At $t = 0$, the Central Bank (CB) receives a private signal θ_{BC} in Θ_{BC} and chooses an action according to its strategy $\mu(\theta_{BC}) \in [0; 1]$ chosen *a priori*. $\mu = \frac{L_{CB}}{D}$ is defined as the amount of money the CB is ready to issue per unit of maturing debt - for simplicity all the stock of debt will be maturing in period 1 - ; it is also the amount of public debt that it is ready to buy. μ measures the degree of financial support by the CB to the State. It is the key variable in the model. The CB plays first, so that its action is known by investors in the following period.

In period 1, the State has to roll-over the stock of maturing debt that we normalize $D = 1$. Each private investors $i \in I$ - where I is of mass 1 - receives a, possibly different, signal θ_i. It may be public - and common knowledge - or private. The investor also learns about the action of the CB in the previous period. It can also possibly learn about θ_{CB} if the CB announces it. During this period, they choose their best action according to their information set (θ_i,μ) or (θ_i,μ,θ_{CB}) if she also learns the signal of the CB. The set of actions is very standard, an investor can either rollover -R- its debt contract with the State or liquidate -L- its position.

In period $t = 2$, θ_2, the fundamental value, realized and uncertainty about the solvency of the State is resolved. We do as if the infinity of future periods could be summarized into one single period. The solvability of the State depends on the comparision between the discounted flow of future tax revenus that we summarize into

$$\sum_{t=0}^{\infty} \frac{T_t}{(1+r)^t} = \theta_2$$

and the amount of debt D. We make the parallel between the financial situation of the State and the one of the financial institution and implicitly consider the State from an accounting point of view: on the asset side of the balance sheet, the State has the discounted sum of revenus from taxation and on the liability side its debts. As in accounting, we call net value the difference between the asset value and the face value of debt. The State's balance sheet at period i can therefore be represented as:

Asset	Liability
Cash M_i	Net Value NV_i
Taxes $\sum_{t=0}^{\infty} \frac{T_t}{(1+r)^t}$	Debt D_i

We could distinguish between short-term and long-term debt, but this would complexify the analysis without bringing new crucial insights.

The State is solvent if $NV_2 \geq 0$ i.e. if $M + \theta_2 - D_2 \geq 0 \iff \theta_2 \geq \theta^{**} = D_2 - M$. θ^{**} represents the threshold below which the State becomes insolvent. In our model, the decision to default is not strategic contrary to a lot of models in the sovereign default literature. Here the State is completely passive, and defaults when it is forced to.

The total amount of liquidity the State can have access to is $L = M + A = M + \mu D$. For simplicity and without loss of generality, we make the assumption that $M = 0$; the main source source of liquidity is therefore the CB : $L = L_{CB} = \mu D$. μ is the crucial parameter because it governs the abailibity of liquidity for the State in case of a liquidity crisis. If $\mu(\theta_{BC}) = 0$ for every θ_{BC} then the State has never access to any CB liquidity and any refusal to rollover the debt contracts on the part of private investors could lead to default. Conversely, if $\mu(\theta_{BC}) = 1$ for every θ_{BC}, this means that in all circumstances, the CB is ready to lend to the State any amount necessary to reimburse the private investors. The parameter μ is important because it conditions the success or the failure of a run. Indeed a run is successful if the proportion of runners, δ is superior to the available liquidity by unit of debt μ. In particular, a run will always fail if $\mu = 1$.

A monetary strategy for the CB is, in general, a function that associates to any signal θ_{BC}, a probability distribution over the possible $\mu \in [0; 1]$. For simplicity and clarity, we restrict the analysis to trigger strategies :

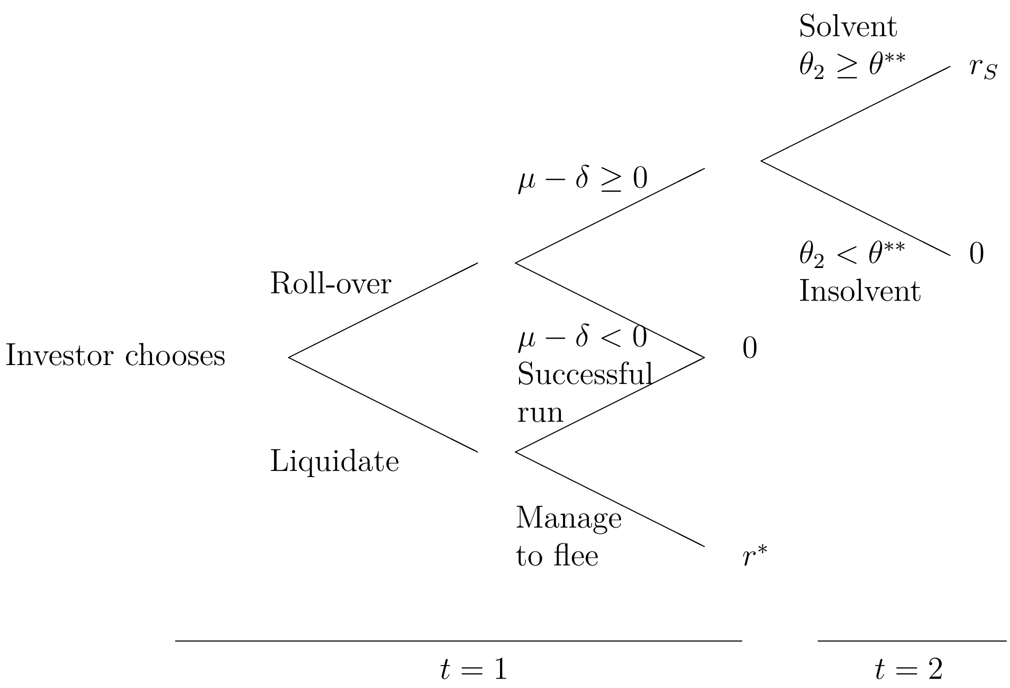
$$\begin{cases} \mu = 1 \quad if \quad \theta_{BC} \geq \theta^{'} \\ \mu = \mu_{min} \quad if \quad \theta_{BC} < \theta^{'} \end{cases}$$

According to this strategy, The CB intervenes without any quantitative limit if it receives a signal higher than a threshold and does not intervene at all otherwise[1]. The monetary strategy is summarized by the threshold $\theta^{'}$. It captures a given regime of relationship between the State and the Central Bank. Since the game is dynamic, the CB will choose $\theta^{'}$ optimally taking into account the best response of private investors in the following period. The goal of the CB will be to minimize a loss function that will take into account expected inflation and expected output loss from crisis - which allows to capture the trade-off between monetary and financial stability. The key parameter in this function will be the rate of preference of price stability over financial stability.

The investors can either invest in public debt or in an international safe asset with exogeneous real return r^*. The return on public debt, conditional on "no default", is

[1] We will see that in equilibrium there will sometimes be some restrictions on the lower bound of μ that's why we don't assume *a priori* any value for the minimum μ.

not random and equal to r_S. There are two causes of default which correspond to the two periods of the game: either the State experienced a successful run in $t = 1$ which forces it to default, would the State have been solvent in period $t = 2$ or not ; or it did not experience any successful run but it turns out to be insolvent when θ_2 realizes in $t = 2$. If there is no default neither in period 1 nor in period 2, an investor receives r_S. Now it remains to specify the gains in case of default in period 1 and in period 2. For clarity, we make the most simple assumtion that in case of default either in period 1 or 2, an investor loses the whole value of its investment and the payoffs are simply zero. We discuss this assumption in section 3.3. It must also be noticed that contrary to a lot of models of run, for example in Diamond and Dybvig (1983) - where there is a liquidation cost -, we don't assume that the realized θ_2 depends on whether a run occurred or not. However, from the point of view of the investors, the mere fact that the gains are zero in the case of a self-fulfilling run in period 1, is a more than sufficient condition to create the strategic complementarities that will be at the root of the liquidity crisis. The following diagram summarizes the expected payoffs for an individual investor.



We organize the solving of the model according to the information distribution among the investors and the CB. In the first section, the information is complete for all so that $\theta_{BC} = \theta_i = \theta_2$ and there is no uncertainty regarding the solvency of the State. In the second part, the information will become incomplete because the signal is not perfectly informative about the solvency of the state. But the information will remain symmetric since every investor will receive the same signal. Notice that it is possible for the information to be perfect for the Central Bank and incomplete for the investors. Finally incomplete information can be symmetric or asymmetric depending whether all investors receive the same signal or not. The last section solves the asymmetric information case. We summarize the different configurations in the following table:

Investors CB	Complete	Incomplete Sym	Incomplete Asym
Complete	(1)	(2)	(3)
Incomplete Sym	(4)	(5)	(6)
Incomplete Asym	(7)	(8)	(9)

The nature of the information setup - complete/incomplete, asymmetric/symmetric, superiority of information received by CB or not - plays a great role for the equilibrium determination. We will focus on the setups for which the information of the CB is at least as good as investors'. We thus won't analyse cases (2), (3) and (6). Notice that the more general setup is (9): (5) is a subcase of (9) where all agents receive the same noisy signal ; (1) is a subcase of (5) where the noise tends to zero. For clarity, we go from the simplest setup (1) to the more general (9).

Liquidity is a commonly used word in the economic literature. It generally refers to the ease to access monetary ressources. In the microeconomic literature, it is used to describe the liquidity of an asset, the liquidity of a market or the liquidity of a financial intermediaries - or the degree of maturity mismatch. In macrofinance it is usually less clear : it is sometimes related to the monetary policy and the base money - say the monetary aggregates -, sometimes it refers to the degree to which the financial system is ready to bear maturity transformation risk and the ease with which sleeping liquidity is transformed into circulating liquidity - are banks ready to lend ; what is the interest rate required by banks and markets ? In our model, as in the literature on runs, liquidity refers to funding liquidty - i.e. the possibility for an institution, here the State, to finance or refinance its debt in a context of maturity mismatch of the balance sheet. In order to give more conceptual structure to our reasoning, we define a financing equilibrium as an equilibrium in which the State can roll-over the entire stock of maturing debt, whatever the interest rate. By contrast, a non-financing equilibrium refers to an equilibrium in which the State doesn't have access to enough liquidity to roll-over the stock of maturing debt.

Finally we assume that inflation takes place only in period $t = 2$ so that prices remain constant from $t = 0$ to $t = 2$. There is different ways to specify the sources and dynamics of inflation. The first one would be a pure quantitative view according to which, for a given level of real transactions and velocity of money, inflation is the immediate consequence of an increase in the supply of money (Quantitative Theory of the Price Level, QTPL). A second one would be to make the price level adjust in period $t = 2$ so that public debt in real terms is at most equal to the discounted flow of taxes (Fiscal Theory of the Price Level, FTPL). A third one makes the price level adjust in period $t = 2$ so as to make the liability side of the CB balance sheet at most equal to the asset side : we call this assumption the Balance Sheet Theory of the Price Level (BSTPL). The latter is an adaptation of the FTPL to the CB. Since the liability side (base money) is denominated by definition in nominal terms, inflation is the means by which the real value of CB's debt is reduced. In this interpretation, notes are therefore viewed by investors mainly as real assets and not only as means of transactions. In the remaining of the dissertation, we consider the third assumption. As we discuss in section 3.4, this assumption is important, although not crucial for the results. Indeed, according to the QTPL, which sees the notes as means of transactions, any increase in the quantity of money must be reflected in the level of prices ; be the counterpart of this money creation valuable or worthless. According to the BSTPL, any increase in the quantity of money is not necessarily reflected in the price level if the created notes are the counterpart of real bills. This distinction is crucial in the current debate on the inflationary implications of quantitative easing.

An (pure strategy Bayesian Nash-)equilibrium requires the following statements to be satisfied :

- the CB chooses θ' the cutoff value of its trigger strategy in order to minimize the loss function L(.), governed by the rate of preference for price stability.
- the investors choose their best strategy - in the asymmetric information case, the cutoff value κ of their trigger strategy - that maximizes their payoffs taking the strategy of the CB, the interest rate and the strategy/actions of others investors as given.

We solve this dynamic game backward : in period $t = 1$, investors make forecasts over the distribution of θ_2 and over the distribution of δ, given their information set $\Omega_i = (\Theta_I(\theta_i); \Theta_{BC}; \mu)$ where Θ_I is the information set about the vector of signals received by the continuous set of private agents and Θ_{BC} the information set about the CB's signal. Investors are risk-neutral and seek to maximize their expected payoff at $t = 2$. In period $t = 1$, investors thus choose their best response -either to roll-over or to liquidate-associated with the realization of signals received and the behavior of others taking the policy announcement as given and credible. In $t = 0$, the CB chooses its action μ after having received a signal θ_{CB} and according to its own trigger strategy which is a best response to the trade-off between inflation and financial instability governed by the preference for monetary stability, a.

2.2 Setup (1) Complete Information

The Southern countries of the Eurozone - including France - claimed that the CB should credibly commit to buy any amount of public debt and that will by itself - without being necessary to effectively intervene - bring back financial markets to a financing, "good", equilibrium. This position is supported by models of multiple equilibria where a financing equilibrium can coexist with a non-financing or run equilibrium. This is what this section -as well as the following two, with some refinements - models.

The CB and private investors receive the same perfectly informative signal $\theta_{BC} = \theta_i = \theta_2$ which allows each of them to conclude about the solvency of the State in $t = 2$. Since information is symmetric, we can restrict the analysis to symmetric equilibria where all investors act in the same way. We first focus on $t = 1$ to show how the model works and how the equilibrium depends on the choice of θ' made by the CB.

First, we study the case when the CB never intervenes : $\mu \to 0$ for all θ_2.

If the signal received reveals that the State is insolvent - i.e. $\theta_2 < \theta^{**}$ - then the expected gain from rolling-over the debt contract is 0 since the State will default for sure in period 2, independently of the occurence of a run in period 1. In this situation, it is optimal for the investors not to rollover and to leave. Indeed, if she stays she will get 0 for sure. If nobody leaves, but she leaves, she can hope to get the alternative r^* -hence the assumption that μ is arbitrarily small but still positive. If everybody leaves, she will gain 0 for sure. Therefore leaving is strictly prefered to staying. It is a dominant strategy to choose the alternative r^*. But since everybody does the same and since the CB doesn't intervene, the only possible equilibrium is the liquidity crisis. Notice that there is undoubtedly a liquidity crisis in period 1 *because* everybody knows that the State is insolvent in period 2.

If the signal received reveals that the State is solvent - i.e. $\theta_2 \geq \theta^{**}$ then the expected gain from rolling-over the debt contract now depends on the behavior of others investors. If everybody rolls-over, then an investor rolling-over can get r_S which is superior to the alternative r^* by assumption. In this case, she will choose to roll-over. But if others investors don't roll-over, the State will be forced to default for sure since it has no access to CB liquidity and an investor rolling-over would for sure lose the whole value of her investment. Notice that the preference for leaving is this case is not strict since both action gives 0 profits. But by construction of a symmetric equilibrium, everybody will run in such an equilbrium.

Denoting E(NP / δ ; r_S ; r^* ; μ) the expected gain given a monetary strategy, interest rates, when a proportion δ of investors don't rollover, the matrix of gains is the following:

$$\text{E(NP} \ / \ \delta \ ; \ r_S \ ; \ r^* \ ; \ \mu \to 0)$$

$$\left\{ \begin{array}{ll} r_S - r^* & if \quad \delta \leq \mu \\ -r^* & if \quad \delta > \mu \end{array} \right. \Longleftrightarrow \left\{ \begin{array}{ll} r_S - r^* & if \quad \delta \leq 0 \\ -r^* & if \quad \delta > 0 \end{array} \right.$$

This implies the existence of two equilibria: a "good" equilibrium where $\delta = 0$ and where it is optimal for everybody to stay because $r_S - r^* > 0$; and a "bad" equilibrium where $\delta = 1$ and where it is optimal for everyone to run since $r^* > 0$. This is the the traditional view of the literature on liquidity crisis: because of strategic complementarities there are multiple equilibria.

This result, of course, is conditional upon $\mu = 0$ - ie conditional upon a given monetary strategy on the part of the CB. This strategy can be seen as the optimal response of a CB that sacrifices financial stability on the altar of monetary stability. While the monetary stability is guaranteed, it let completely opened the possibility of a self-fulfilling liquidity crisis that lead to the default of the State.

We now turn to the opposite case where the CB is always ready to intervene, be the State solvent or insolvent: $\mu = 1$ for all θ_2.

When the State is solvent, the expected gain of rolling-over doesn't depend on the behavior of others investors since $\mu = 1$:

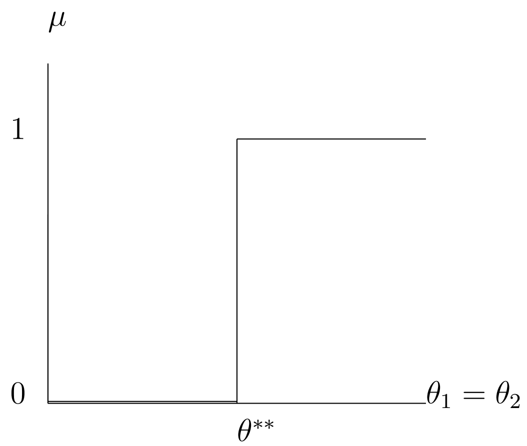
$$\text{E(NP} \ / \ \delta \ ; r_S \ ; r^* \ ; \mu = 1) =$$

$$\begin{cases} r_S - r^* & if \quad \delta \leq \mu \\ -r^* & if \quad \delta > \mu \end{cases} \iff \begin{cases} r_S - r^* & if \quad \delta \leq 1 \\ -r^* & if \quad \delta > 1 \end{cases} \iff r_S - r^* \quad for\ all \quad \delta$$

Gains are independent of the behavior of others investors. International investors are guaranteed they would get their capital plus interests even if all others investors run, because the CB is ready to buy any amount of public debt in $t = 1$. Consequently all investors strictly prefer to stay when the State is solvent. There is only one equilibrium: $\delta = 0$.

When the State is insolvent, the expected gain from rolling-over are zero. They therefore liquidate their positions and don't roll-over the debt contracts, which forces the CB to monetize the entire stock of public debt. In period $t = 2$, according to our BSTPL, this translates into inflation. This strategy kills financial instability but at the expense of monetary stability because there is inflation each time the State is insolvent.

Finally, we turn to a specific intermediary case where the CB conditions its commitment to intervene upon the realization of a high enough signal. More precisely, the CB commits to intervene as soon as the State is solvent: $\mu = 1$ if and only if $\theta_2 \geq \theta^{**}$.



With this strategy, the CB minimizes financial instability and perfectly maintains monetary stability in all circumstances. In perfect information, it is the "best" strategy in the trade-off between financial stability and monetary stability.

Indeed if the State is solvent, $\theta_2 \geq \theta^{**}$, the CB commits to acquire any amount of public debt needed to support the sovereign debt market. Expected gains for private investors are independent of the proportion of runners, δ :

$$E(NP \ / \ \delta \ ; r_S \ ; r^* \ ; \mu = 1) =$$

$$\begin{cases} r_S - r^* & if \quad \delta \leq \mu \\ -r^* & if \quad \delta > \mu \end{cases} \iff \begin{cases} r_S - r^* & if \quad \delta \leq 1 \\ -r^* & if \quad \delta > 1 \end{cases} \iff r_S - r^* \quad for \ all \quad \delta$$

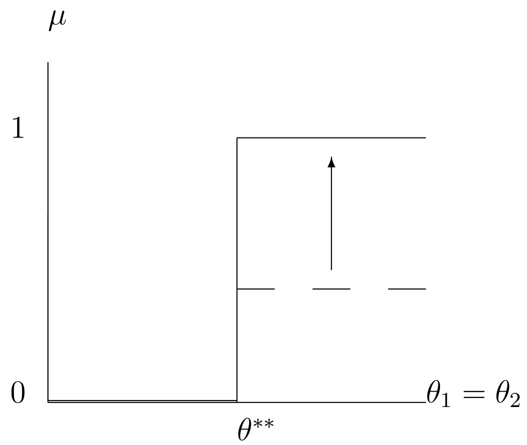
Since by assumption $r_S - r^* > 0$, all investors prefer to roll-over their debt contract with the State. There is only one equilibrium $\delta = 0$. The policy of the CB avoids liquidity crisis when the State is solvent. Notice that this commitment to act as a Lender of Last Resort to the State is a free lunch since the CB never has to intervene.

However, when the State is insolvent, $\theta_2 \geq \theta^{**}$, the CB never intervenes $\mu \to 0$. Expected gains of private investors are for sure 0 if they roll-over their contract since the State will default whatever the behavior of others market participants. In this situation, it is optimal to leave and to choose the alternative r^* (notice that, in equilibrium, all investors will lose their capital in this situation). The only possible equilibrium is the liquidity crisis because of expected insolvency.

This monetary strategy is optimal in the sense that it maintains the stability of prices for all realization of θ_2 since the CB never has to intervene in equilibrium and also minimizes the occurence of financial instability event. It avoids self-fulfilling liquidity crisis that are unrelated to fundamentals. But it doesn't avoid liquidity crisis caused by insolvency. This strategy is not far from the doctrine of Bagehot (1873) adapted to the States : the CB stands ready to bail-out solvent States ; but lets insolvent state default and let private investors bear the losses.

To summarize this first section, when the information is complete, there can be multiple equilibria when the State is solvent if and only the CB refuses to act as a lender of last resort as long as the State remains in the solvency zone. Acting as a LLR kills the strategic complementarities and is a free lunch as long as the CB provides liquidity only when the State is solvent. To a certain extent, this is the interpretation of the Southern countries of the Eurozone : we - the Southern countries - are still in the solvency zone, we are hit by a pure liquidity crisis.

This first section allows us to draw a first interpretation of the sudden rise and then decrease in sovereign spreads in 2011 and 2012. While the fundamentals have not changed so much between 2007 and 2011, the sudden shift in yields may be interpreted as the shift from a "good" equilibrium - or financing equilibrium - to a bad equilibrium - non financing equilibrium. This interpretation in terms of multiple equilibria is very common in the press for instance. According to this view, the subsequent decrease in yields can be thought as the consequence of a policy shift from μ strictly inferior to unity $\mu(\theta_{CB} \geq \theta^{**}) < 1$ to the "optimal" policy $\mu(\theta_{CB} \geq \theta^{**}) = 1$. In the situation before the policy shift, a solvent State could be subject to a run on its debt if $\delta > \mu$ which was always possible since $\mu < 1$. The shift to an optimal monetary policy kills the strategic complementarities that are at the root of the inefficient run when the State is solvent.



2.3 Setup (4) and the informational role of the policy announcement

In this section we suggest why the only policy announcement of the CB suffices to recoordinate investors on a good equilibrium when it is better informed and when investors knows it.

Suppose now that, the CB still receives the perfectly informative signal $\theta_{BC} = \theta_2$ but that investors receive the same noisy signal $\theta_I = \theta_2 + \sigma_2 \epsilon_2$ with $\epsilon_2 \sim \mathcal{N}(0, 1)$. If all investors did not observe the policy announcement, they would all have the same belief over the distribution of θ_2 : $\theta_2 \sim \mathcal{N}(\theta_I, \sigma_2)$. However the sequential structure of the game is important because the policy announcement reveals information about the true θ_2 to private investors. Indeed, if the strategy of the CB is dichotomic around θ' :

$$\mu = \begin{cases} 1 & if \quad \theta_{BC} \geq \theta' \\ 0 & if \quad \theta_{BC} < \theta' \end{cases}$$

then any policy announcement $\mu = 1$ or $\mu = 0$ reveals to the investors the sign of $\theta_2 - \theta'$. Investors will consequently update their belief over the distribution of θ_2. We analyse the simplest case where $\theta' = \theta^{**}$, which the strategy that minimizes financial instability under the constraint that prices remain stable. The updated probability that the State be solvent given θ_I and $\mu = 1$ is :

$$P(\theta_2 \geq \theta^{**} \setminus \theta_I ; \mu = 1) = P(\theta_2 \geq \theta^{**} \setminus \theta_I ; \theta_2 \geq \theta^{**}) = 1$$

and the probability that it is insolvent given θ_I and $\mu = 1$ is :

$$P(\theta_2 < \theta^{**} \setminus \theta_I ; \mu = 1) = P(\theta_2 \geq \theta^{**} \setminus \theta_I ; \theta_2 \geq \theta^{**}) = 0$$

When the CB's threshold is $\theta' = \theta^{**}$, the policy announcement perfectly reveals the private information of the Bank. Investors update their information set and deduce from the observation of the announcement the solvency of the State. When the CB follows this strategy, we are back to the perfect information case analysed in the previous section.

We saw that the equilibrium was unique : liquidity crisis in period 1 when the State is insolvent and rolling-over of the entire public debt when the State is solvent.

This informational role of the announcement of CB is a potentially powerful argument and we will deal with it all along the dissertation : the LLR not only interacts with market participants on the liquidity dimension and the underlying dynamics of strategic complementaries, it also influences investor beliefs about fundamentals because announcements reveal information. In this section, we have illustrated the extreme case when the CB is perfectly informed and the announcement perfectly revealing. In general, the CB is not perfectly informed and the announcement not perfectly revealing. However, it captures the idea that policy announcements contribute to shape market expectations because they convey information and that the pure announcement of committing can convey enough information to reassure market participants.

This section allows us to draw a potential second interpretation about the recent events. In 2011 and 2012, markets became suddenly uncertain about the fundamentals of some countries. This in turn led to large swing and increase in the yields of those countries. By committing to intervene on sovereign debt markets, the ECB reveals that its own signal was higher that its threshold : if the threshold of the CB is θ^{**} - which is not necessarily the case if the CB also takes care of financial instability as discussed later -, and if the markets believe that the signal of the CB is perfect, then the mere fact that the CB commits suffices to reshape market expectation about fondamentals. The announcement in the Summer 2012 may therefore have contributed to reshape the private expectations about the solvency of some southern States.

2.4 Setup (5) and the role of fundamental uncertainty

In this section, we simply introduce uncertainty in the common signal, the model will have exactly the same structure as in the first setup, except that nobody knows *ex ante* what is the real value of bonds *ex post*. This model is therefore a refinement of the first one. It also models the position of Southern countries in the sense that there are multiple equilibria and that the credible commitment of the CB to intervene can recoordinate investors on the financing equilibrium without any intervention. However because of the uncertainty, a trade-off between price and financial stability arises: the CB may want to buy the entire stock of debt even when all investors want to sell because it cares about financial stability.

In the setup (5), investors and the BC receive the same noisy signal $\theta_1 = \theta_2 + \sigma_2\epsilon_2$ with $\epsilon_2 \sim \mathcal{N}(0,1)$. Since everybody gets the same message, we focus on symmetric equilibria $\delta = 0$ or $\delta = 1$.

If the realized signal is $\theta_1 \geq \theta'$ then everyone knows that $\mu = 1$. Consequently, expected gains don't depend on the proportion of runners that don't roll-over their debt contract. The expected gains is r_S multiplied by the probability that the State won't default in period 2.

$$P(\theta_2 \geq \theta^{**} \setminus \theta_1) = P(\frac{\theta_2 - \theta_1}{\sigma_2} \geq \frac{\theta^{**} - \theta_1}{\sigma_2} \setminus \theta_1) = 1 - G(\frac{\theta^{**} - \theta_1}{\sigma_2})$$

where G is the cumulative distribution of ϵ_2.

Nets Expected gains are therefore :

$$\text{E(NP / } \delta \text{ ; } r_S \text{ ; } r^* \text{ ; } \mu = 1) = \begin{cases} r_S(1 - G(\frac{\theta^{**}-\theta_1}{\sigma_2})) - r^* & if \quad \delta = 0 \\ r_S(1 - G(\frac{\theta^{**}-\theta_1}{\sigma_2})) - r^* & if \quad \delta = 1 \end{cases}$$

Gains are independent from δ. It exists a dominant strategy for all players depending on the realization of θ_1. Denoting $BR(\theta_1; \delta; r^*; r_S)$ the best response of an investor to the signal received and to the behavior of others investors, we have :

$$BR(\theta_1; \delta; r^*; r_S) = \begin{cases} \text{Roll-over if} \quad \theta_1 \geq \kappa = \theta^{**} - \sigma_2 G^{-1}(1 - \frac{r^*}{r_S}) \\ \text{Liquidate if} \quad \theta_1 < \kappa = \theta^{**} - \sigma_2 G^{-1}(1 - \frac{r^*}{r_S}) \end{cases}$$

There is therefore an unique equilibrium when $\theta_1 \geq \theta'$, but the nature of it depends on the value of θ_1. If the signal is too low, the probability that the State default is too big, and all investors prefer to leave.

If the signal received by the CB is inferior to the threshold of intervention $\theta_1 < \theta'$ then it is common knowledge that $\mu = 0$. In this case, strategic complementarities are back and expected gains depend, once again, on the behavior of others investors.

$$\text{E(NP / } \delta \text{ ; } r_S \text{ ; } r^* \text{ ; } \mu = 0) = \begin{cases} r_S(1 - G(\frac{\theta^{**}-\theta_1}{\sigma_2})) - r^* & if \quad \delta = 0 \\ -r^* \quad if \quad \delta = 1 \end{cases}$$

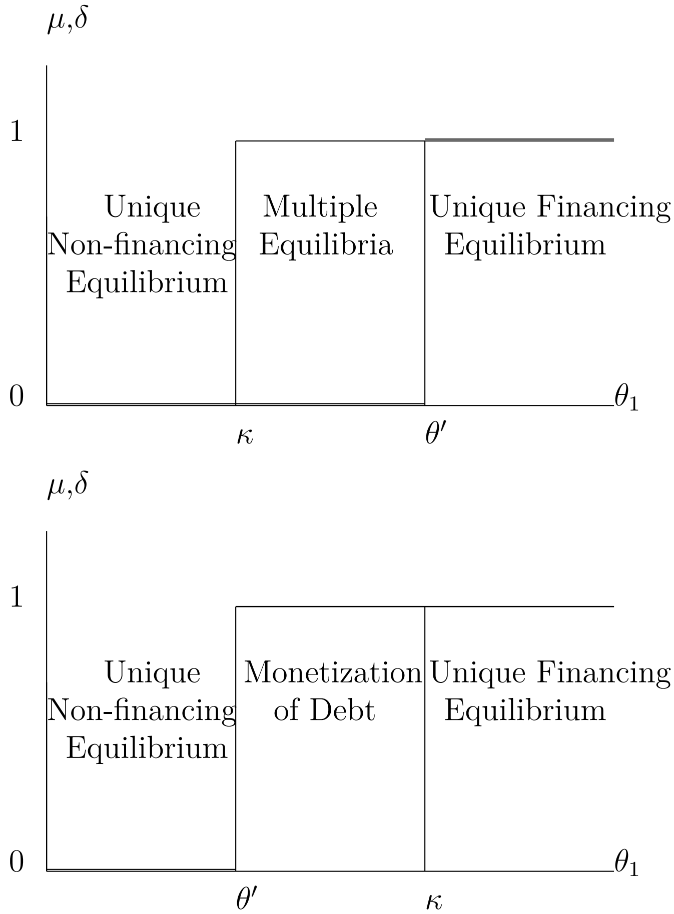
In this situation, the best response depends not only on the signal θ_1 but also on the behavior of others investors. Denoting $\kappa = \theta^{**} - \sigma_2 G^{-1}(1 - \frac{r^*}{r_S})$, we have :

$$BR(\theta_1; \delta; r^*; r_S) = \begin{cases} \text{Roll-over if} \quad \theta_1 \geq \kappa \quad \text{and} \quad \delta = 0 \\ \text{Liquidate if} \quad \theta_1 \geq \kappa \quad \text{and} \quad \delta = 1 \\ \text{Liquidate if} \quad \theta_1 < \kappa \end{cases}$$

In this situation, where the CB doesn't commit to intervene there is a unique equilibrium when the probability that the State is insolvent is large - ie when θ_1 is small. But there are multiple equilibria when the probability of default is low.

We summarize the setup (5) labelled "with fundamental uncertainty" with the following two diagrams. The first one illustrates the outcomes when the CB threshold θ' is higher than the private investor threshold ; and the second one when it is lower than the private investor threshold.

Figure 1: Equilibria as a function of θ' and κ



Notice that the monetary policy can eliminate multiple equilibria by changing its threshold θ'. Indeed, if it fixes $\theta' = \kappa = \theta^{**} - \sigma_2 G^{-1}(1 - \frac{r^*}{r_S})$, the multiple equilibria area disappears from the picture. Moreover this commitment to intervene is a free lunch since the CB never has to buy public debt.

In the specific case where $\sigma_2 = 0$, we are back to the setup (1) and multiple equilibria are eliminated for $\theta' = \theta^{**}$. As mentioned above, the setup (5) is a generalization of the setup (1). The introduction of fundamental uncertainty doesn't modify our conclusions. : the only difference is the replacement of θ^{**} by $\theta^{**} - \sigma_2 G^{-1}(1 - \frac{r^*}{r_S})$ which can be interpreted as the "default risk" premium -people require a higher signal to compensate for the risk.

To summarize this second section, fundamental uncertainty almost doesn't change the key results from the previous setup. The multiple equilibria can arise when the CB doesn't commit to intervene and even when the insolvency risk of the State is not too high - compared to the interest rate spread compensation. As previously, it is a free lunch for the CB to commit to intervene as long as its threshold of intervention is not lower than the one at which investors sell. In particular, it leads to interpret the recent events as a situation of multiple equilibria and the following decrease as the consequence of a policy shift of $\theta' > \kappa$ towards the left and closer to κ. However, it must be noticed - and this is the only difference to the previous case - that nobody knows *ex ante* whether the State will be solvent *ex post* so that if the CB decides to buy all the debt - which happens if and only if $\theta' < \kappa$, it takes a inflation risk.

Introducing the Trade-off between Financial Stability and Price Stability

This stochastic framework allows to introduce explicitely into the analysis the trade-off between financial stability and monetary stability. Indeed, *ex ante*, neither the CB nor the investors know the true value of θ_2 which will be realized only at $t = 2$. *Ex post* the CB might own public debt whose value is inferior to the face value of notes issued in $t = 1$ to buy it. According to the BSTPL assumption, this leads to inflation. It is obvious from the previous analysis that there will never be inflation as long as $\theta' \geq \kappa$. But the CB may want to minimize the occurence of financial instability events. As seen before, it can eliminate multiple equilibria by setting $\theta' \leq \kappa$. As in the setup (1), the only monetary strategy that keeps prices stable and that minimize the occurence of liquidity crisis consists in deleting the multiple equilibria area by setting $\theta' = \kappa$.

However -and this is a crucial point - in such a situation, the CB doesn't eliminate all situations where the State is forced to default because of a liquidity run despite the fact that it turns out to be solvent in $t = 2$. Indeed, when the signal is too low, neither the Bank nor the private investors are ready to roll-over the debt contracts. With positive probability, the State is forced to default while solvent *ex post*. The CB may consequently be tempted to decrease its threshold to avoid such liquidity crisis caused by uncertainty about the solvency of the State. This will have a cost, though : this will increase the probability that the Bank holds assets whose value had been over appreciated which would create inflation.

As above mentioned, the recent events have often been interprated as examples of multiple equilibria. But another interpretation that is not understandable in the previous framework can also be made. In the previous models, the intervention of the CB is a free lunch as long as its threshold is not lower than the private threshold κ. When it is effectively lower, the CB must buy all the stock of debt. During the recent events, the CB of the developed world have bought - more or less - large quantities of public debt, but only a small proportion of it. The real world world seems therefore to be in a intermediate situation. We need a model that allows to capture the possibility that the CB buys only a fraction of the debt.

2.5 Setup (9) and the interaction between fundamental and stategic uncertainty

The previous settings allowed for multiple equilibria, therefore leaving a role for sunspots. In the orthodox view of the Eurozone crisis - that we naively label the 'German view" -

however, the financing of the Southern States would be firstly useless because the problem is about solvability, not liquidity and run on public debt, and secondly detrimental to price stability. This section proposes a model that formalizes this idea in the same framework as the first three sections.

More related to the theoretical literature, these types of models have been critized for their lack of tractability and the associated difficulty to make comparative static analysis. More importantly, there are clear evidence that self-fulfilling runs most of the time occur when fundamentals deteriorate. On the contrary in the previous setups, as long as the CB doesn't commit to intervene, there can be multiple equilibria and successful run even if the State is solvent. And the fundamentals played no role in the triggering of a pure liquidity crisis - by definition not related to fundamental concerns. Either everybody was selling because they expect the State to be insolvent - we label such a situation a liquidity crisis caused by insolvency, or the fundamentals were not so bad -the State was solvent in expectation-, but everybody was selling because all investors expected the other to sell. As economist, we would like to have a model that predicts that roll-over crisis are more likely to occur when fundamentals enter a certain region.

The Global Game literature with the work of Carlsson and van Damme (1993) and Morris and Shin (1998, 2003) deals with this issue by introducing heterogeneity in the private signals received by investors which may allow under certain conditions to find a unique equilibrium. This section is inspired from the recent paper of Morris "The Illiquidity Component of Credit Risk" (2009) which deals explicitely with the following question : how coordination failure depends on future fundamental uncertainty and "how illiquidity risk depends on future insolvency risk"[2]?

2.5.1 Solution with heterogeneous beliefs and trigger strategy

In the case of asymmetric private information, a unobserved underlying signal θ_1 realizes at $t = 0$ - $\theta_1 = \theta_2 + \sigma_2\epsilon_2$. Nobody is able to observe it, neither the Bank nor the investors. Each investor i receives a private signal : $\theta_i = \theta_1 + \sigma_1\epsilon_{1,i}$. The CB also receives a private signal $\theta_{BC} = \theta_1 + \sigma_1\epsilon_{1,BC}$. Since the latter plays first, its action reveals, like in section 2.3, some information about its private signal. Private investors will therefore update their beliefs according to the observation of the policy announcement.

Recall that a monetary strategy is of the form :

$$\mu = \begin{cases} \mu_{max} = 1 \quad if \quad \theta_{CB} \geq \theta' \\ \mu_{min} \quad if \quad \theta_{CB} < \theta' \end{cases}$$

The policy announcement reveals to investor the position of the private signal of the Bank θ_{BC} with respect to the threshold θ'. To simplify the analysis, without loss of

[2]Morris, S. and S. Song Shin. (2009). "The Illiquidity Component of Credit Risk", pp. 4

interesting insights, we consider the specific case where the CB makes its private signal public. Empirically modern Central Banks release a lot of reports and bulletins and do press conferences to make their view public. People know not only the position of θ_{BC} with respect to θ' but they also learn the exact value of θ_{BC}. Consequently all investors observe a private signal and a public signal θ_{BC}. Investor will thus have the following beliefs about the distribution of θ_1. If

$$\epsilon_{1,CB} \sim \mathcal{N}\Big(0; \frac{1}{\alpha}\Big)$$

$$\epsilon_{1,i} \sim \mathcal{N}\Big(0; \frac{1}{\beta}\Big)$$

then

$$\theta_1 \sim \mathcal{N}\Big(\frac{\alpha\theta_{BC} + \beta\theta_{1,i}}{\alpha+\beta}; \frac{1}{\alpha+\beta}\Big)$$

with α and β the precision - which is defined as the inverse of the variance -, respectively, of the signal of the CB and of the private one.

As traditional in the Global Games literature, a strategy for an investor is a "trigger strategy" that associates an action to an updated belief $\bar{\theta_i} = \frac{\alpha\theta_{BC}+\beta\theta_{1,i}}{\alpha+\beta}$:

$$\begin{cases} \text{Roll-over} \quad if \quad \bar{\theta_i} \geq \kappa \\ \text{Liquidate} \quad if \quad \bar{\theta_i} < \kappa \end{cases}$$

An equilibrium is a value of κ such that if every investors follow this trigger strategy, then it is optimal for investor i to follow this strategy as well. We therefore need to solve for this κ.

Investor i's beliefs about the distribution of the private signal of another investor j, θ_j, knowing $\bar{\theta_i}$ is :

$$\theta_j \sim \mathcal{N}\Big(\bar{\theta_i}; \frac{2\beta+\alpha}{(\alpha+\beta)\beta}\Big)$$

Consequently investor i's belief about the distribution of the updated belief $\bar{\theta_j}$ of another investor j is :

$$\bar{\theta_j} \sim \mathcal{N}\Big(\frac{\alpha\theta_{BC} + \beta\bar{\theta_i}}{\alpha+\beta}; \frac{\alpha+2\beta}{\alpha^2+\beta^2+3\alpha\beta}\Big)$$

Finally her beliefs about the distribution of θ_2 is given by :

$$\theta_2 \sim \mathcal{N}\Big(\frac{\alpha\theta_{BC} + \beta\theta_{1,i}}{\alpha+\beta}; \frac{\alpha+\beta+\gamma}{(\alpha+\beta)\gamma}\Big)$$

where γ is defined as follows :

$$\theta_2 \sim \mathcal{N}\Big(\theta_1; \frac{1}{\gamma}\Big)$$

We denote $p_2 = \frac{(\alpha+\beta)\gamma}{\alpha+\beta+\gamma}$ the precision of θ_2 knowing $\bar{\theta_i}$.

Assume there is no run at $t = 1$. The probability that there is no default in $t = 2$ is thus the probability that θ_2 is greater than θ^{**}.

$$P(\theta_2 \geq \theta^{**} \setminus \bar{\theta_i}) = P((\theta_2 - \bar{\theta_i})\sqrt{p_2} \geq (\theta^{**} - \bar{\theta_i})\sqrt{p_2} \setminus \bar{\theta_i}) = 1 - \Phi\Big[(\theta^{**} - \bar{\theta_i})\sqrt{p_2}\Big]$$

with $\Phi(.)$ the cumulative function of a standard normal.

Conditional on the fact that there is no run, and since we have assumed that the payoff for an investor is 0 when the State default at $t = 2$, the expected payoff from rolling-over the debt contract is :

$$r_S\Big(1 - \Phi\Big[(\theta^{**} - \bar{\theta_i})\sqrt{p_2}\Big]\Big)$$

Because the payoff when the State is forced to default at $t = 1$ by a run on its debt is 0 by assumption, the expected -unconditional- payoff for an investor when she rolls-over the debt contract is :

$$P(\delta \leq \mu) r_S\Big(1 - \Phi\Big[(\theta^{**} - \bar{\theta_i})\sqrt{p_2}\Big]\Big)$$

The crucial parameter that remains to be explicited is $P(\delta \leq \mu)$.

Given that the number of investor is uniformly distributed over $[0; 1]$, the proportion of individuals that don't roll over their contract is also the probability that any agent j doesn't roll it over. But since all agents follow the same strategy :

$$\begin{cases} \text{Roll-over} & if \quad \bar{\theta_j} \geq \kappa \\ \text{Liquidate} & if \quad \bar{\theta_j} < \kappa \end{cases}$$

$$\delta = P(\bar{\theta_j} < \kappa \setminus \bar{\theta_i}) = P\Big((\bar{\theta_j} - \theta_1)\sqrt{\beta + \alpha} < (\kappa - \theta_1)\sqrt{\beta + \alpha} \setminus \bar{\theta_i}\Big) = \Phi((\kappa - \theta_1)\sqrt{\beta + \alpha})$$

Hence $\delta \leq \mu \iff \Phi((\kappa - \theta_1)\sqrt{\beta + \alpha}) \leq \mu \iff \theta_1 \geq \kappa - \frac{1}{\sqrt{\beta+\alpha}}\Phi^{-1}(\mu)$

The probability of the event $\delta \leq \mu$ is therefore also the probability of the event $\theta_1 \geq \kappa - \frac{1}{\sqrt{\beta+\alpha}}\Phi^{-1}(\mu)$. But investor i has a belief about the distribution of θ_1 conditionnal on her updated signal, hence :

$$P(\delta \leq \mu \setminus \bar{\theta_i}) = P(\theta_1 \geq \kappa - \frac{1}{\sqrt{\beta + \alpha}}\Phi^{-1}(\mu) \setminus \bar{\theta_i}) = 1 - \Phi\Big(\Big(\kappa - \frac{1}{\sqrt{\beta + \alpha}}\Phi^{-1}(\mu) - \bar{\theta_i}\Big)\sqrt{\alpha + \beta}\Big)$$

Finally the expected payoff -unconditional, of rolling-over the contract is :

$$\Big[1 - \Phi\Big(\big(\kappa - \frac{1}{\sqrt{\beta+\alpha}}\Phi^{-1}(\mu) - \bar{\theta_i}\big)\sqrt{\alpha+\beta}\Big)\Big] \times r_S \times \Big(1 - \Phi\Big[(\theta^{**} - \bar{\theta_i})\sqrt{p_2}\Big]\Big)$$

A necessary condition for κ to be an equilibrium threshold is that an investor that receives an updated signal exactly equal to κ must be indifferent between rolling-over and liquidating. This necessary condition writes :

$$\Big[1 - \Phi\Big(\big(\kappa - \frac{1}{\sqrt{\beta+\alpha}}\Phi^{-1}(\mu) - \kappa\big)\sqrt{\alpha+\beta}\Big)\Big] \times r_S \times \Big(1 - \Phi\Big[(\theta^{**} - \kappa)\sqrt{p_2}\Big]\Big) = r^*$$

$$\iff \Big[1 - \Phi\Big(-\Phi^{-1}(\mu)\Big)\Big] \times r_S \times \Big(1 - \Phi\Big[(\theta^{**} - \kappa)\sqrt{p_2}\Big]\Big) = r^*$$

$$\iff \Big[1 - (1-\mu)\Big] \times r_S \times \Big(1 - \Phi\Big[(\theta^{**} - \kappa)\sqrt{p_2}\Big]\Big) = r^*$$

$$\iff \kappa = \theta^{**} - \Phi^{-1}\Big[1 - \frac{r^*}{r_S}\frac{1}{\mu}\Big]\frac{1}{\sqrt{p_2}}$$

$$\iff \kappa = \theta^{**} - \Phi^{-1}\Big[1 - \frac{r^*}{r_S}\frac{1}{\mu}\Big]\sqrt{\frac{\alpha+\beta+\gamma}{(\alpha+\beta)\gamma}}$$

There is a unique solution to this condition. This is only a necessary condition that caracterizes the equilibrium, but not a sufficient condition. The existence may therefore be an issue. For κ to exist, one needs also to have

$$0 < 1 - \frac{r^*}{r_S}\frac{1}{\mu} < 1$$

$$\iff 0 < \frac{r^*}{\mu} < r_S$$

This implies $r^* > 0$ and $\mu > \frac{r^*}{r_S}$. This last condition imposes some restrictions on the values that μ can take. For instance $\mu_{min} = 0$ is not possible. Consequently we redefine the monetary strategy as follows :

$$\mu = \begin{cases} \mu_{max} = 1 \quad if \quad \theta_2 \geq \theta' \\ \mu_{min} = \frac{r^*}{r_S}\nu \quad if \quad \theta_2 < \theta' \end{cases}$$

with $\nu > 1$ arbitrarily close to 1.

If we try to put numbers on r^* and r_S, we find a high minimum value of μ. For $r^* = 1.00$ and $r_S = 1.1$, we find $\mu_{min} = 0.909$. This strange result is heavily dependent on the simplifying assumption that when a liquidity crisis occurs, the payoff for an investor is 0. Indeed, since the loss when a liquidity crisis hits is enormous, investors ask for an enormous premium - here informational premium. That's why no equilibrium threshold is sustainable when μ is less than 0.909.

Interestingly, the optimal trigger threshold, κ, is very close to the threshold found in the symmetric information section with an additional term μ that takes into account the risk of a liquidity crisis in $t = 1$. It is as if the investors were asking a premium $1/\mu$ above the rate of return to compensate for the additional risk.

Finally μ itself depends on the signal received by the CB, θ_{BC}. There are two different values for κ depending on the realization of θ_{BC}.

$$\kappa = \begin{cases} \theta^{**} - \Phi^{-1}\left[1 - \frac{r^*}{r_S}\frac{1}{\mu_{max}}\right]\sqrt{\frac{\alpha+\beta+\gamma}{(\alpha+\beta)\gamma}} & si \quad \theta_{BC} \geq \theta' \\ \theta^{**} - \Phi^{-1}\left[1 - \frac{r^*}{r_S}\frac{1}{\mu_{min}}\right]\sqrt{\frac{\alpha+\beta+\gamma}{(\alpha+\beta)\gamma}} & si \quad \theta_{BC} < \theta' \end{cases}$$

After replacing μ by their values, it becomes

$$\kappa = \begin{cases} \kappa_L = \theta^{**} - \Phi^{-1}\left[1 - \frac{r^*}{r_S}\right]\sqrt{\frac{\alpha+\beta+\gamma}{(\alpha+\beta)\gamma}} & si \quad \theta_{BC} \geq \theta' \\ \kappa_H = \theta^{**} - \Phi^{-1}\left[1 - \frac{1}{\nu}\right]\sqrt{\frac{\alpha+\beta+\gamma}{(\alpha+\beta)\gamma}} & si \quad \theta_{BC} < \theta' \end{cases}$$

Clearly $\kappa_L < \kappa_H$ since $\frac{1}{\nu} > \frac{r^*}{r_S}$. Here, the signal received by the CB has a great impact on the outcome of the game since it governs the value of the threshold of the private investors' trigger strategy. The threshold is higher when the signal received by the CB is low (bad) and lower when the signal received by the CB is high. Before the private and asymmetric signals realize at $t = 1$, the policy announcement of the CB has influenced the outcome of the game : if the signal received by the CB is low, it will not commit to intervene in the following period, this tend to make the investors nervous because they fear that the others market participants will run and force the State to default since there will be almost no Central Bank liquidity in the market. On the contrary, a high enough signal lets the CB commit which tends to calm market participants. In this case, private investors just have to care about expected solvency, and not at all what others investors do. Notice that κ_{CB} is the same threshold as in the symmetric information setup studied in the previous section.

2.5.2 Crisis Zone

Contrary to the previous section, it is not an easy task to draw a diagram showing the equilibrium outcomes as a function of the signals, the different thresholds and the policy of the CB. The condition for a crisis to hit is that the total amount of liquidity -either public or private- available to refinance the debt is less than the stock of maturing debt. This writes $\mu - \delta < 0$.

It is possible to give a simple expression of the proportion of runners at $t = 1$, δ. Notice that contrary to the investors who consider θ_{CB} as a public signal and a additional source of information and who try to make their best guest about θ_1 and θ_2, as economist we do as if we knew θ_1 and sees θ_{CB} as a key determinant of the equilibrium since it influences the updated signal of all investors althouth it is itself noisy. The equilibrium number of runners will heavily depend on the public signal received by the CB. In $t = 1$, θ_{BC} has already been realized and will shape market expectations sometimes positively and sometimes negatively. More formally, the "real" distribution - as theoretician we know the true underlying θ_1 - of $\bar{\theta}_i$ is not centered around θ_1 contrary to what an individual investor would think but around $\frac{\alpha\theta_{CB}+\beta\theta_1}{\alpha+\beta}$[3].

$$\bar{\theta}_i \sim \mathcal{N}\Big(\frac{\alpha\theta_{CB}+\beta\theta_1}{\alpha+\beta}; \frac{1}{\alpha+\beta}\Big)$$

Therefore the proportion of runners is:

$$\begin{aligned}
\delta &= P(\bar{\theta}_i < \kappa) \\
&= P\Big((\bar{\theta}_i - \frac{\alpha\theta_{CB}+\beta\theta_1}{\alpha+\beta})\sqrt{\alpha+\beta} < (\kappa - \frac{\alpha\theta_{CB}+\beta\theta_1}{\alpha+\beta})\sqrt{\alpha+\beta}\Big) \\
&= \Phi\Big((\kappa - \frac{\alpha\theta_{CB}+\beta\theta_1}{\alpha+\beta})\sqrt{\alpha+\beta}\Big) \\
&= \Phi\Big((\theta^{**} - \Phi^{-1}\Big[1 - \frac{r^*}{r_S}\frac{1}{\mu}\Big]\sqrt{\frac{\alpha+\beta+\gamma}{(\alpha+\beta)\gamma}} - \frac{\alpha\theta_{CB}+\beta\theta_1}{\alpha+\beta})\sqrt{\alpha+\beta}\Big)
\end{aligned}$$

From this expression, we draw the following comparative static lessons:

- δ is an increasing function of the alternative asset returns r^* and decreasing of public bond yields r_S
- δ is an increasing function of the difference $\theta^{**} - \frac{\alpha\theta_{CB}+\beta\theta_1}{\alpha+\beta}$. In particular, the higher the signal of the CB, the lower the proportion of runners.
- δ is an decreasing function of μ the amount of liquidity per unit of debt that the CB is ready to provide to the market.
- δ is a decreasing function (resp. increasing) of α and β the precision of signals, if the hidden signal θ_1 is superior (resp. inferior) to the threshold κ. The more precise the signals, the more sensitive the proportion of investors to the gap between θ_1 and κ.

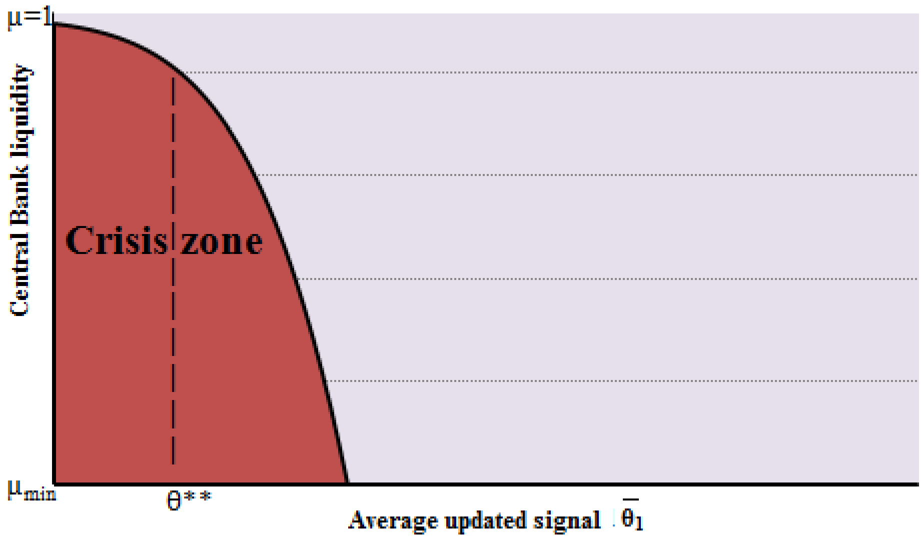
[3] This goes back to the point we made in section 2.3 on the informational role of the policy announcement.

Denoting, $\bar{\theta}_1 = \frac{\alpha\theta_{CB}+\beta\theta_1}{\alpha+\beta}$, the crisis condition therefore writes

$$\mu - \delta < 0 \iff \mu - \Phi((\theta^{**} - \Phi^{-1}\Big[1 - \frac{r^*}{r_S}\frac{1}{\mu}\Big]\sqrt{\frac{\alpha+\beta+\gamma}{(\alpha+\beta)\gamma}} - \bar{\theta}_1)\sqrt{\alpha+\beta}) < 0$$

As mentioned in the introduction to this Global Game section, introducing noisy signal allows us to get a unique equilibrium and to get clearer prediction regarding the role of different key variables on the outcome. On figure 2, we have represented the zone of liquidity crisis as a function of "fundamentals" - namely the underlying *updated* signal $\bar{\theta}_1$ - and central bank liquidity availability per unit of maturing public debt μ. Obviously for $\mu = 1$ liquidity crisis cannot occur in equilibrium even in case of very bad average signal since the Central Bank is always ready to intervene ; for $\bar{\theta}_1 - \theta^{**}$ large enough a liquidity crisis cannot occur because fundamentals *and* the CB signal are good enough ; as updated average signal about fundamentals deteriorates - $\bar{\theta}_1$ is getting close to θ^{**} - the State can rapidly fall into the crisis zone even if the expected gap from insolvency remains positive and large because private investors fear that the others investors got a worse signal than themselves and consequently run which could force the State to default. The crisis zone rapidly expands as the liquidity parameters decreases. We can also see on this figure that the increase in μ is all the more efficient to avoid liquidity crisis as the country is close to the "cliff edge". CB liquidity commitment is all the more important as the fundamentals are close to θ^{**}. The increase in μ reduces the shape of the crisis zone by two channels : it decreases δ because it reassures the investors ; and it clear the market by making the condition $\mu > \delta$ more likely for a given δ.

Figure 2: Crisis Zone

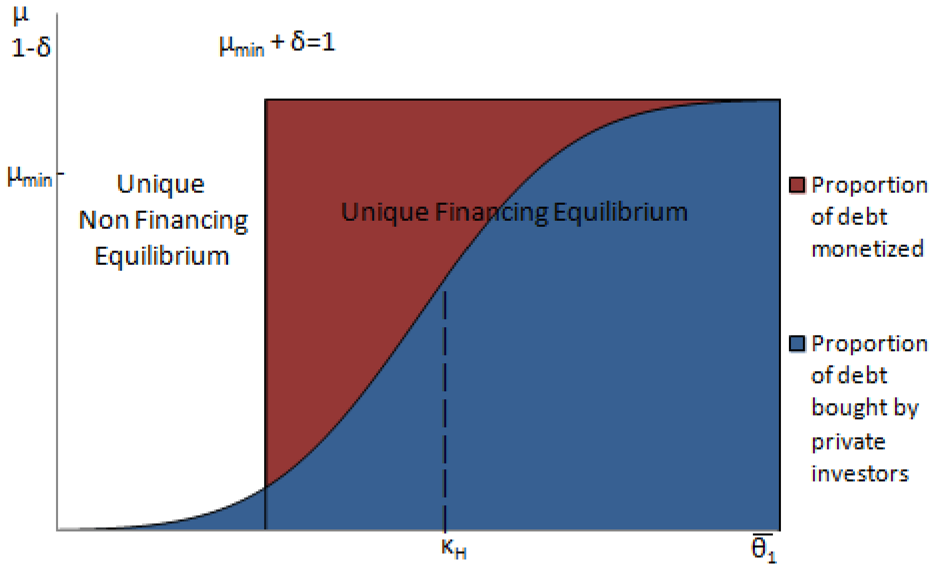


Depending on the realization of θ_{CB} with respect to θ', the CB decides to commit or not :

Figure 3: Policy of the CB

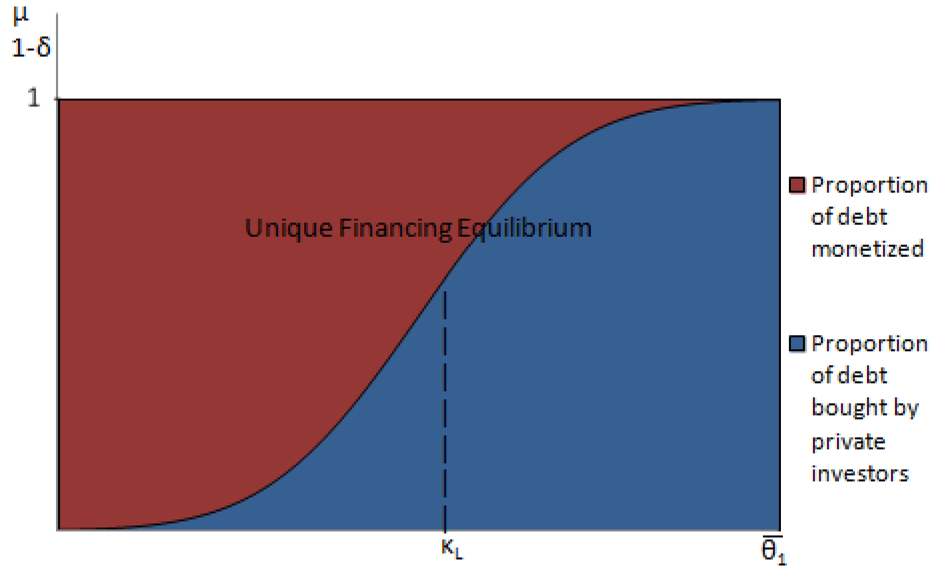
The game and the potential outcome as a function of $\bar{\theta}_1$, the true updated underlying signal, is either represented by the following diagram when $\theta_{CB} < \theta'$ and $\mu = \mu_{min}$:

Figure 4: Equilibria when $\theta_{CB} < \theta'$ and $\mu = \mu_{min}$



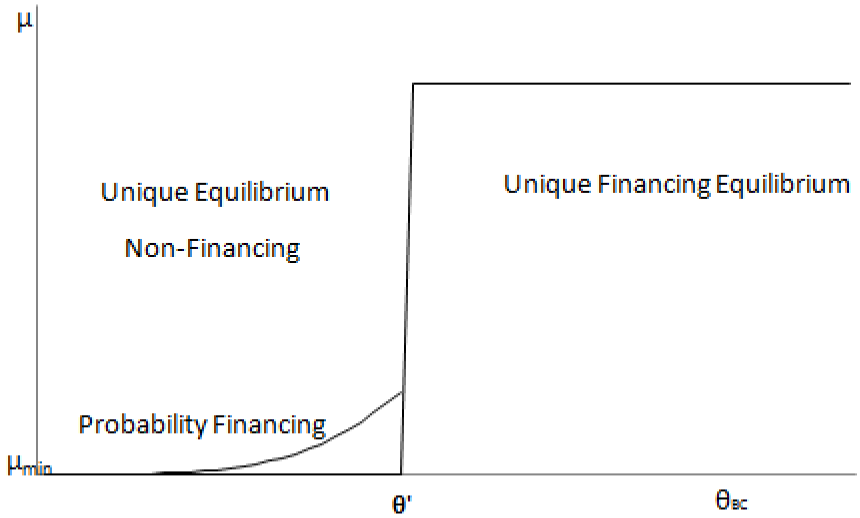
or it is represented by the following diagram when $\theta_{CB} \geq \theta'$ and $\mu = 1$:

Figure 5: Equilibria when $\theta_{CB} \geq \theta'$ and $\mu = 1$



The previous diagrams can be summarized to get a closed form representation of the determination of equilibria as in the previous sections :

Figure 6: Equilibria under Assymetric Information and Normal Distribution



To summarize this fourth section, when the information is heterogeneous among investors, solvency concerns and liquidity concerns are no longer easy to separate and are intertwined. As fundamentals deteriorate a country may face a liquidity shortage as pessimistic investors are afraid not only of insolvency but also of run of others investors. However fundamentals need to be very bad for investors unanimously to run. For the CB any commitment to intervene becomes no longer a free lunch, and as fundamentals deteriorates it will have to intervene effectively and buy part of the total stock of bonds in order to clear the market, at the potential expense of price stability.

2.6 Optimal Monetary Policy and Relationships with the State

2.6.1 Optimal Monetary Policy : solving for θ'

In this section, we endogenize the choice of θ', the optimal threshold above which the CB buys bond if needed. We provide a simple positive explanation why the ECB was more reluctant to intervene than the Fed: its only objective is price stability and financial stability doesn't have the same weight in its loss function as for the Fed. This could lead to suboptimal situation where there is too low inflation and too much financial instability. In addition, when the solvability-activity of the CB is guaranteed it is optimal for it to provide as much liquidity as needed since it would never give rise to inflation. We argue that this condition is satisfied in the Eurozone, which leads to the conclusion that the CB has long behaved in a suboptimal way according to our results.

In all previous sections, θ' was given. We now solve for the first period of the game and draw general conclusions about the impact of the rate of preference for price stability on the equilibrium outcome and the financial stability. We assume that the CB chooses the optimal trigger strategy threshold θ' in order to minimize a loss function made of two terms. On the one hand, the CB wants to minimize the expected inflation, denoted $E(\pi; \theta')$ which depends on θ'. On the other, it seeks to minimize the expected output loss Δy stemming from crisis that happens with probability $P(Crisis; \theta')$ which is a function of θ'. a denotes the rate of preference for price stability over financial stability.

$$Loss = L\Big(E(\pi; \theta'); \Delta y \times P(Crisis); a\Big)$$

We impose the traditional restriction that it must be quadratic. Therefore, we have that :

$$\theta'_{opt} = argmin_{\theta'}\Big[\Big(\Delta y \times P(Crisis; \theta')\Big)^2 + a\Big(E(\pi; \theta')\Big)^2\Big]$$

Proposition n°1

If $P(Crisis; \theta')$ and $E(\pi; \theta')$ are twice differentiable in $\theta' = \theta'_{opt}$, then θ'_{opt} is increasing in a.

Proof in Appendix 1.

Corollary

If the social rate of preference for price stability, a^{social}, is lower than the rate of preference of the CB, then $\theta'_{opt}(a) \geq \theta'_{opt}(a^{social})$, the expected inflation rate is too low and the probability of crisis too high compared to the Pareto optimum.

Proof

This is a direct consequence of proposition n°1 : since θ'_{opt} is increasing in a, it follows that if $a > a^{social}$, then $\theta'_{opt}(a) \geq \theta'_{opt}(a^{social})$. The others two statements follows from the fact that the probability of crisis is an increasing function and the expected inflation is a decreasing function of θ' as shown previously.

2.6.2 Liquidity and solvability : a two-dimensional relationship with the State

As already stressed, the relationship between the State and the CB has long been studied in the perspective of solvability, neglecting the liquidity dimension. In this section, we want to show how the two dimensions interact in the framework of the model and how it shapes the outcome of the game.

We say that the the monetary policy is solvability-active when it freely sets the path of prices compatible with its objective - e.g. maintaining price stability - and the State has to adjust to meet its budget constraint, potentially by defaulting. This solvability-activity implies that if the CB holds public debt while the State turns out to be insolvent, an institution - the State by borrowing or taxing its citizen, or international public actors like, for example, the EFSF - must recapitalize the CB so as to make sure that its net value will always remain positive. Indeed, this is a direct implication of our assumption that inflation is determined according to the Balance-Sheet Theory of the Price Level (BSTPL). A necessary and sufficient condition for prices to remain stable is that the net value of the CB remains positive. On the contrary, a solvability-passive monetary policy means that the State doesn't have to make sure that the net value of the CB becomes negative. Notice that in the traditional literature, the monetary policy is said to be solvability-passive when it takes the budget contraint of the State as given and provides him with the necessary seigneuriage revenus to meet its solvability constraint. However, in our toy model, the CB cannot create inflation by simply printing money since the QTPL doesn't hold. That's why we slightly modify the definition of a solvability-passive monetary policy.

In order to put more structure and to think more formally about it, we denote $\theta_2 + z$ the maximum amount of ressources made available by the State - or any other institution - to recapitalize the CB. If the State defaults, we keep assuming that the investor loses its entire capital and gets 0. In case of default, the State could possibly use the realized θ_2 to recapitalize the CB. A solvability-active monetary policy implies that the State will use all the ressources it can mobilize to recapitalize the CB when needed, $\theta_2 + z$ is the upper limit. z can be greater than zero, for example in the Eurozone since the others States can recapitalize the CB even when one of the States is insolvent. When z is greater than zero, it captures the extent to which capital is made available by external actors. When z is smaller than zero, it captures the degree of solvability-activity of the monetary policy. Indeed in the extreme situation where $z = -\theta_2$, the State never recapitalizes the CB and the whole adjustment will have to be born by an increase in the level of prices when the State defaults and the CB holds public debt.

Figure 7: Interaction between liquidity-activity and solvability-activity

	The State must recapitalize CB Solvability-active policy $z = +\infty$	The State doesn't recapitalize CB Solvability-passive policy $z = \min(0; -\theta_2)$
The CB must close the financing gap Liquidity-passive policy $\mu = 1$	No Liquidity Crisis *Potentially* No Inflation *Potentially* Best solution Problem : Is recapitalization compatible with unsolvency?	No Liquidity Crisis Often Inflation Most inflationary configuration of relations
The CB doesn't have to close the financing gap Liquidity-active policy $\mu = 0$	Very Often Crisis Never Inflation Eurozone situation ? CB could still kill multiple equilibria	Very Often Crisis Never Inflation even if the State never recapitalizes since the CB never buys public debt

We say that the monetary policy is liquidity-active when the CB is not constraint to meet the liquidity needs of the State. On the contrary, a liquidity-passive monetary policy means that the CB must make sure that the State will always be financed. The degree of liquidity-activity is captured by the function $\mu(\theta_{BC})$ previously defined. In figure 7 we report the four possible combinations that are also the four polar cases of a more general analysis.

The case in the north-west quarter is *a priori* the best case from a welfare point of view since it allows to eliminate the crises while maintaining the stability of prices. However, it raises the following question: how could an insolvent State recapitalize its insolvent CB that had bought public debt ? An easy solution would be to assume that an international institution always stands ready to recapitalize the CB, like this is implicitely the case in the Eurozone. Indeed, in the Eurozone, even if the CB has a negative capital because one of the States to whom it would have lent turns out to be insolvent, the others States would recapitalize the CB. However, in general, for stand-alone countries, it is hard to think of an international institution that would accept to recapitalize the CB. The second solution would be to assume that the State can recapitalize the CB either by borrowing on the market. But why should investors lend to a State that just defaulted? Or by raising taxes. But then, why the State did not raise taxes to meet its own budget constraint in the first place ? We let this tricky but key question opened.

Interestingly, it emerges from the table that the two dimensions are not independent from each other. A first approach to think of this interaction is to assume that the solvability passivity or activity is exogeneously set by law but that the CB is still free to choose the degree of activity in the liquidity-dimension which is captured by θ'. It appears that for a given rate of preference for price stability, a, the CB will choose an optimal threshold of intervention, θ'_{opt} all the lower as the monetary policy is solvability-active. Notice that the solvability-activity is guaranteed through the recapitalization of the CB ; but, in the real world, also through the conditionality of loans. The intuition is the following : a solvability-active monetary policy implies that the risk of inflation is basically zero. So that in the loss function, the expected inflation is always zero. Then, the CB only cares about financial stability, even when the rate of preference for price stability, a, is very high. This is summarized in the following proposition :

Proposition n°2

θ'_{opt} is a decreasing function of z. In words, the optimal degree of liquidity-passivity of the CB is increasing with the degree of solvability-activity.

Proof in Appendix 1.

A shift from a solvability-passive monetary policy to a solvability-active monetary policy decreases the optimal threshold of intervention by the CB. A solvability-active monetary policy implies that a liquidity-passive monetary policy is optimal. Conversely, a solvability-passive monetary policy implies that a higher degree of liquidity-activity is optimal.

This theoretical result is consistent with the behavior of CB since their creation. For example, during the Napoleonic War, the Bank of England suspended convertibility and lent huge amount to the State at war. This apparent but real liquidity-passivity must not be misinterpreted, it was not the monetization of the public deficit. The Bank of England followed a liquidity-passive monetary policy because the solvability-activity was, arguably, guaranteed. The Bank of England accepted to do that because the suspension was temporary, i-e it was clearly said that the pre-war parity will be restored as soon as possible. The State committed to reimburse at the end of the war in order to guarantee the solvavility-activity of the Bank. This was effectively done : the State reimbursed the Bank, and the parity was restored.

Another interesting example is the behavior of the Bank of France after the First World War. As long as the governing council of the Bank believed the State would maintain the solvability-activity of the Bank, reimburse the advances made during the war and return to the pre-war parity, they accepted to lend and fill the financing gap in the beginning of the 1920's. But when they suspected the governments to give up their commitments, and therefore encroach upon the solvability-activity of the monetary policy, in 1924, it seriously threatened the Treasury not to support it anymore, which may have fuelled the roll-over crisis as described later.

This analysis also suggests that to maintain the credibility of its policy and the inflation expectation anchored a CB can decide to behave completely passively in the dimension of liquidity as long as it makes sure and tells the people that it is active along the dimension of solvability. Of course, this remark relies on the assumption that the BSTPL is a good approximation of the determination of inflation. In the QTPL is a better approximation, then the result is potentially invalidated. It also relies on the assumption that the CB will always be recapitalized, potentially by international actors. Although reasonable for the Eurozone, such an assumption may be questionable for others countries.

Finally, this results shed light on the current debate about the intervention of the ECB on debt markets of Southern states. It indeed suggests that the problem - from the German viewpoint - is not only inflation, in a pure monetarist view, but more importantly the potential cost of recapitalizing the ECB in case the bonds it holds turn out to be worthless nad the implicit transfers. What is hidden behind the debate about the intervention of the ECB, is thus the issue of fiscal transfers between members of the Eurozone countries.

3 Extensions and discussions

3.1 Endogeneous Interest Rate

One of the main limits of these traditional models of run is the exogeneity of interest rate. In these models, the price of money is fixed, and quantities adjust. However, the functioning of modern financial markets requires to think in terms of price adjustment. As economists, we would like to have an idea of how interest rates react in such a situation. Interest rates must become a property of the equilibrium, not an exogeneous parameter.

The transformations of the world financial system has modified the definition and the nature of crises. Although it was possible to define a sovereign crisis as an episode of default during the 1970s and 1980s, it seems to be no longer the case and the definition must be revised to include sharp and brutal increase in sovereign interest rates (Pescatori and Sy, 2004).

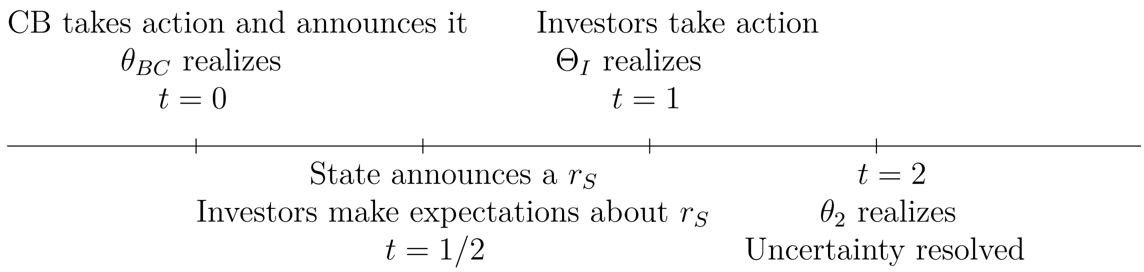
Another reason that pushes towards the need to endogenize the interest rate is the role that it plays in the determination of the solvency of the State. Indeed, the interest rate is not only a measure and the mirror of an exogeneous "risk of credit", but it is one of the main determinants of this risk. From now on, the cutoff value θ^{**} below which the State is no longer solvent becomes a function of the interest rate charged on the debt : $\theta^{**} = D_2 - M = D_2 = r_S \times D_1$.

Neverthess endogenizing the interest rate is not without any cost. The difficulty to do it comes from the fact that interest rates are traditionally assumed to be exogeneously given at the beginning of the game. In this framework, the interest rate can't be at the same time an exogeneous parameter and an equilibrium outcome. To solve this issue, we solve for agents' best expectation about the interest rate *ex ante* that will prevail in equilibrium *ex post* given the available information. Under the rational expectation hypothesis (REH), agents don't make systematic mistakes so that on average their expectations are correct.

In general terms, the endogeneous "equilibrium" rate is defined as the rate that makes the sovereign bond market clear. Because the structure of the game requires that investors take the interest rate r_S as given before they choose their best strategy/action in $t = 1$, we will need actually to solve for the equilibrium expected rate before the realization of the investors' signals in $t = 1$. That's why we introduce a new period between $t = 1$ and $t = 2$ during which the State makes an announcement of the rate at which he will be willing to refinance its debt in $t = 1$ and during which investors make all the same expectation about r_S which will then play the same role as the exogeneous r_S of the previous section.

This is an "equilibrium" rate in the sense that for it to be credible and compatible with the Rational Expectation Hypothesis, is has to be the best prediction based on the available information in $t = 1/2$ - ie θ_{BC} and μ. However, notice that in the asymmetric information case, it could be that the announced and believed *ex ante* interest rate turns out to be wrong. For that reason, we define in the framework of our model, an *ex ante* equilibrium rate -on which everybody coordinates at the end of period $t = 1/2$- and an *ex post* equilibrium rate -that will clear the market in $t = 1$. The latter will be systematically equal to the former in the symmetric information case. However in the imperfect information case, they will be equal only on average and they will most of the time be different since the information set θ_{BC} on which the expectation are formed may be "biased" and not centered around the average signal θ_1, as already mentioned earlier.

Finally, as we will see, to endogenize the interest rate also requires to give the State a greater role in determining the equilibrium interest rate. The State will play a big role because it is the supply side of the financial markets - it is the bond seller - which will propose an interest rate to market participants. We will see that this new variable -the interest rate- will bring a new trade-off into the analysis : minimizing the service of the debt and thus the likelihood of default and minimizing the amount of debt that will have to be bought in equilibrium by the Central Bank. The State will choose the rate it proposes to market participants by minimizing a loss function that takes into account these two elements under the constraint that the markets should clear: $r_{eq} = argmin_r \frac{1}{2}[r^2 + b\delta(r)^2]$ such that $E(\delta(r)) - \mu \leq 0$.



A (pure-strategy Nash-)equilibrium with endogeneous interest rate requires the following statements to be satisfied :

- the CB chooses θ' the cutoff value of a trigger strategy in order to minimize the loss function L(.), governed by the parameter of preference for price stability.
- the State proposes the interest rate that minimizes its loss function taking into account the market clearing constraint based on the current information set, θ_{BC}, μ.
- the investors maximizes their payoffs taking the strategy of the CB as given, the credible interest rate and the strategy of others investors as given (see above for more details).

As in the previous section, we solve it backward. Notice that we have already solved for the third point when investors make their best choice taking all others parameters as given. In this section, we will mostly focus on the solving of the second point : what is the *ex ante* equilibrium rate ? We will pay a particular attention on and will draw general conclusions about the role of the CB policy in the determination of this equilibrium rate.

3.1.1 Perfect Information

In the perfect information case, private agents receive a signal θ_2 that perfectly informs them about the future flow of taxes. One must notice that contrary to the previous section, this doesn't say explicitely whether the State will be solvent or not since this will also depend on the interest rate that that the State will have to pay. The question that is raised at this stage is what is a credible interest rate ie one that is compatible with the best expectation that market participants can have?

We will show that r^* is always a credible interest rate when there exists an equilibrium with financing. First recall that in the new framework θ^{**} is now equal to $D_1 \times r^*$, so that the solvency condition is $\theta_2 > \theta^{**} \iff \theta_2 > D_1 \times r_S$. Therefore if the condition $\theta_2 > D_1 \times r^*$ is fulfilled, the State can credibly propose r^* since investors are sure that it will be solvent at this rate. Moreover it is compatible with its goal to minimize the debt service. It would be absurd for the State to increase the rate to supposedly attract investors since the elasticity of demand of bond to the interest rate is null. The only thing that matters is : is the State solvent at the minimum interest rate r^*.

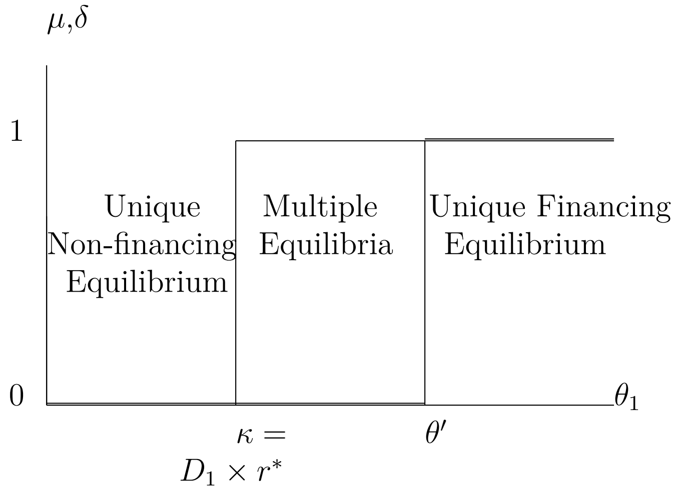
If the condition $\theta_2 > D_1 \times r^*$ is not fulfilled, then no equilibrium rate exists. We set the "equilibrium" value of r_S at $+\infty$ by convention and we label this situation liquidity crisis caused by certain insolvency.

It could also be that the condition $\theta_2 > D_1 \times r^*$ be fulfilled but that no equilibrium exists. Indeed, imagin all investors expect the others to run in $t = 1$ and expect the State to default, given r^* that would have set and agreed upon the previous period. Who can believe that r^* will be the market-clearing rate in $t = 1$? When everybody expects a run to occur next period, such a rate is not credible. No rate can be credible and therefore no equilibrium rate exists. We set the "equilibrium" value at $+\infty$ by convention and we label this situation pure liquidity crisis.

It is interesting to notice that, the flexibility of the interest rate alone doesn't guarantee that the State will not encounter a liquidity crisis. Even if the State is solvent at the minimum rate and the interest rate allowed to adjust to clear the market, a liquidity crisis can hit in the interim period at $t = 1$.

What can the CB do? As in the previous section, the commitment on the part of the CB to intervene will make self-fulfilling liquidity crisis less likely. More explicitely, if the CB strategy is :

Figure 8: Equilibria as a function of θ' and κ



$$\begin{cases} \mu = 1 \quad si \quad \theta_2 \leq \theta^{'} \\ \mu = 0 \quad si \quad \theta_2 > \theta^{'} \end{cases}$$

then, consistently with our findings in the previous section -where θ^{**} is now equal to $D_1 \times r^*$, we get the following diagram when $\theta^{'} > r * \times D_1$:

This can be rewritten in terms of equilibrium interest rates :

$$r_{eq} = \begin{cases} +\infty \quad if \quad \theta_2 < D_1 \times r^* \\ +\infty \quad if \quad \theta' < \theta_2 < D_1 \times r^* \quad \text{and liquidity crisis} \\ r^* \quad if \quad \theta' < \theta_2 < D_1 \times r^* \quad \text{and no liquidity crisis} \\ r^* \quad if \quad \theta_2 > \theta' \end{cases}$$

In particular, if the monetary strategy is to commit to intervene as soon as the State is solvent at the minimum interest rate (riskless rate r^*), ie to set $D_1 \times r^* = \theta'$ then the multiple equilibria area disappears and the equilibrium interest rate is :

$$r_{eq} = \begin{cases} +\infty \quad if \quad \theta_2 < D_1 \times r^* = \theta' \\ r^* \quad if \quad \theta_2 > D_1 \times r^* = \theta' \end{cases}$$

3.1.2 Imperfect Information

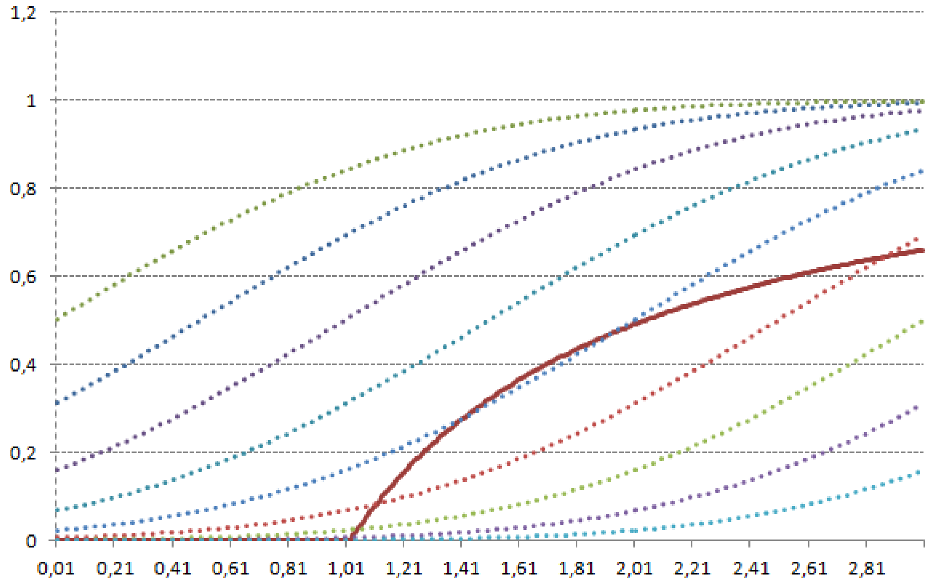
In incomplete information, the signal is no longer perfectly informative and the equilibrium interest rate will have to include a credit risk premium. This implies that the State, by setting the lowest rate for which investors accept to roll-over the public debt will seek the minimum rate r_S such that $r^* = r_S\Big(1 - G(\frac{\theta^{**}-\theta_1}{\sigma_2})\Big) = r_S\Big(1 - G(\frac{D_1 \times r_S - \theta_1}{\sigma_2})\Big)$. This is the minimum rate for which investors are ready to roll-over the debt since it includes a premium that offset the additional credit risk. It is credible because at this rate if all investors roll-over then the marginal investor will also roll-over. It is compatible with the goal of minimizing the service of public debt since the State seeks for the minimum solution to the equation. We solve this equation with Excel. On the following charts, the solid line is the spread term $1 - \frac{r^*}{r_S}$ and the dotted lines are the credit risk terms $G(\frac{D_1 \times r_S - \theta_1}{\sigma_2})$ for different realization of θ_1 ; the equilibrium rates are at the intersection of the two :

We should notice that the rate(s) r_S that verifies(y) this equality is (are) not the addition of the riskless rate and a credit risk premium because the credit risk is not exogeneous to the interest rate itself -we see on the graphs that the dotted lines are increasing functions. When the interest rate increases, the credit risk premium also increases. The solutions to the equation are thus the values of interest rates for which the yield spread is compatible with the level of risk stemming from the interest rate itself. As in the model *à la Calvo* there exist multiple "fundamental" equilibria : for a low interest rate, the risk is low and the risk premium is low ; for a high interest rate, the risk is larger, and the risk premium as well.

This multiple equilibria result is very different from the multiple equilibria that stem from liquidity stressed until now - financing *vs* non-financing equilibria. These two types of multiplicity are independent from each other. Conditional on the fact that no liquidity crisis hit, there are always multiplicity of fundamental equilibria, and this is unrelated to liquidity concerns. It has only to do with "fundamental" solvency.

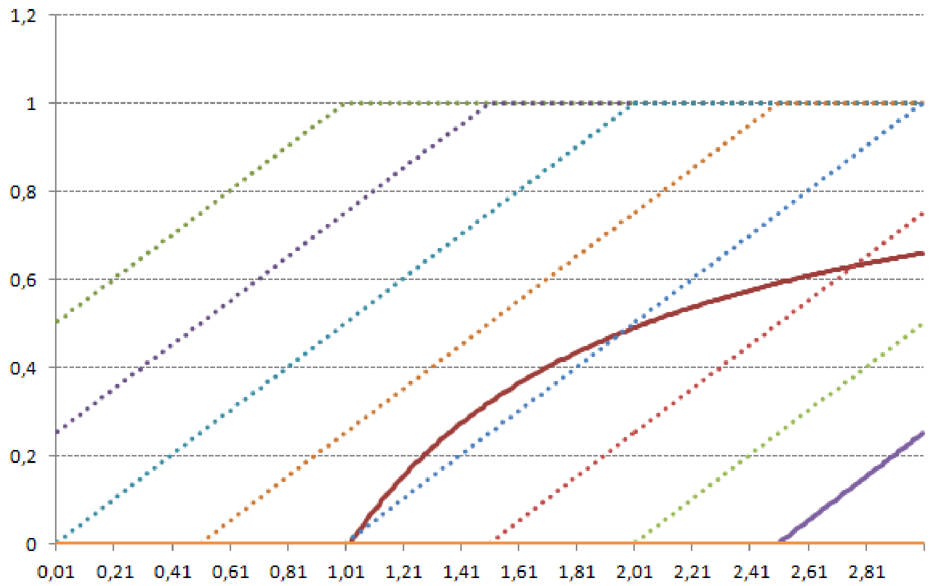
How to select between these possibly multiple fundamental equilibria? The selection of the equilibrium is a traditional question in the literature. Obviously the sunspot determination is always a possible solution. Chamon (2006) argues that there exist contracts that induce investors to select the good equilibrium. Cohen and Portès (2006) argue that a lender of *first* resort is needed to guide the investors. In the framework of our model we show that the selection is not a problem and that the investors will endogeneously converge to the good equilibrium. Indeed investors know that the State seeks to minimize the interest rate on sovereign bonds. The State will select the best equilibrium consistent with its goal to refinance its debt at the lowest rate. By definition, it is credible since this is an *ex ante* equilibrium rate. Therefore, all investors will coordinate on the low rate equilibrium in case there are more than one equilibrium. It stems from this reasoning that there is no need for the CB - or any other lender of *first* resort - to intervene in order to select the best equilibrium : the desire on the part of the State to refinance at the best rate suffices to coordinate expectations on the good

Figure 9: Equilibrium Interest Rate with Normal Distribution



Notes : The credit risk is the dotted line. The risk premium is the solid line. The equilibrium rate is the intersection of the two.

Figure 10: Equilibrium Interest Rate with Uniform Distribution



Notes : The credit risk is the dotted line. The risk premium is the solid line. The equilibrium rate is the intersection of the two.

equilibrium.

For low enough signal, the equation has no solution. We denote $\underline{\theta}$ the lowest signal for which $r_{eq}(\theta_1)$ exists -ie for which such an equilibrium rate exists. For all realization below $\underline{\theta}$, we set $r_{eq} = +\infty$ and label it liquidity crisis caused by probable insolvency.

Moreover it should be noticed that, like in the perfect information section, even if there exists an *ex ante* equilibrium rate conditional on the fact that investors don't expect a liquidity crisis to hit, all investors may expect others investors to run and to make the State default. In this case, no equilibrium exists. We set $r_{eq} = +\infty$ and label it pure liquidity crisis. Denoting $r_{eq}(\theta_1)$ the solution of the equation for a signal θ_1 when investors don't expect a liquidity crisis to hit, one has :

$$r_{eq} = \begin{cases} +\infty \quad si \quad \theta_1 < \underline{\theta} \\ +\infty \quad si \quad \underline{\theta} < \theta_1 < \theta' \quad \text{and liquidity crisis} \\ r_{eq}(\theta_1) \quad si \quad \underline{\theta} < \theta_1 < \theta' \quad \text{and no liquidity crisis} \\ r_{eq}(\theta_1) \quad si \quad \theta_1 > \theta' \end{cases}$$

Once again, notice that the flexibility of the interest rate doesn't eliminate liquidity crisis. Only the quarantee the the CB intervenes can eliminate multiple liquidity equilibria by setting the threshold of intervention at $\underline{\theta}$ below which there are no rate that can clear the market.

$$r_{eq} = \begin{cases} +\infty \quad if \quad \theta_1 < \underline{\theta} = \theta' \\ r_S(\theta_1) \quad if \quad \theta_1 > \underline{\theta} = \theta' \end{cases}$$

The two previous sections allow us to draw some general conclusions about the impact of the CB commitment on the equilibrium interest rate when information is symmetric -either perfect of imperfect.

Proposition n°3

By lowering its threshold, θ', the CB can make the sovereign yields decrease.

Proof

Decreasing the intervention threshold reduces the multiple liquidity equilibria area. Therefore it eliminates the occurence of pure liquidity crisis which makes the yield decrease from $+\infty$ to $r_{eq}(\theta_1)$. When θ' becomes inferior to $\bar{\theta}$, the CB is always the marginal investor on the market and can set any finite rate it wants.

Corollary n°1

A smaller preference for price stability implies a lower equilibrium interest rate. Symmetrically, a higher preference for price stability implies a higher equilibrium interest rate.

Proof

According to proposition n°1, θ' is increasing in a. According to proposition n°3, the equilibrium interest rate is increasing in θ'. Therefore the equilibrium interest rate is also increasing in a.

Corollary n°2

In the perfect (resp. imperfect) information case, lowering the intervention threshold lets the equilibrium rate unchanged or induces a decrease from $+\infty$ to a fundamental equilibrium rate r^* (resp. $r_S(\theta_1)$).

3.1.3 Asymmetric Imperfect Information

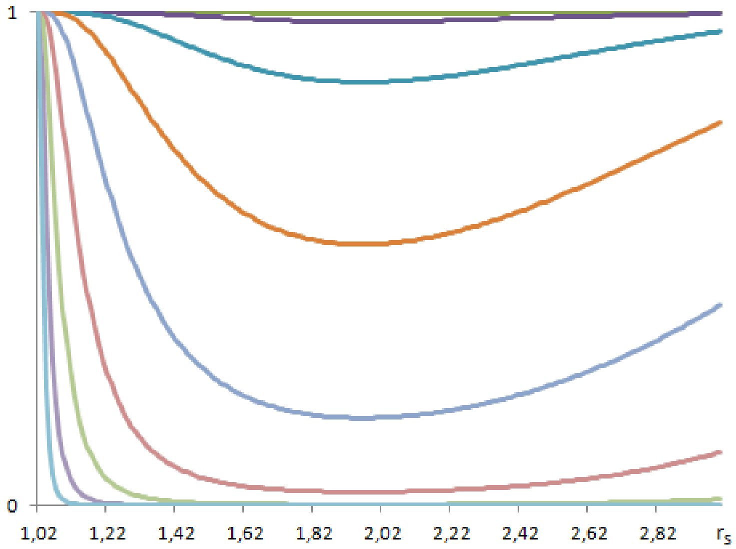
In the asymmetric imperfect information case : the information set in $t = 1$ is not the same for all investors. How could they form the same expectation about the equilibrium interest rate without any external and commun reference? This is the reason we have modified the game so that the State makes a credible announcement in $t = 1/2$ which is then taken as given during the following period when investors chooses their best strategy. This announcement will be the device -if credible- by which investors will have the same expectations.

As in the previous section, we first analyse the dynamics of the model when the distribution is normal and then when it is uniform. When the distribution of errors is normal, the proportion of investors that don't roll-over the debt contract, δ, is never zero. This is a direct consequence of the property of the cumulation function of a normal law. So that the State will, with a high probability, fall in a liquidity crisis and be forced to default is the CB doesn't intervene - ie when $\mu = \mu_{min}$ with $\mu_{min} > \frac{r^*}{r_S}$ as in section 2. This implies that the CB will have to buy bonds if it committed to intervene - when $\mu = 1$. It means that the CB is always the marginal investor when it committed, which is the one that *in fine* determines the interest rate to buy the bonds. Any rate is therefore credible if the CB is committed to buy any amount of public debt.

However, as schown in the graphs below where the different colours stands for different realizations of θ_1, δ is a function of r_S : when the announced rate is close to r^*, δ tends to 1. If the CB seeks to minimize the quantity of debt it buys, the credible rate must be strictly superior to r^*. Consequently there is a trade-off between choosing a very low interest rate, for which nobody wants to buy and thus having to buy a large amount of public debt ; a higher interest rate for which people are willing to buy, but at a higher spread. Notice that when the interest rate rises too much, the proportion of investors that is willing to buy will decrease since the increase in the rate deterioriates the solvency of the State. When the CB is the marginal investor, any rate is credible but there is a trade-off for public institutions.

The solution to the trade-off may depend on the cooperation between the two public entities. In case of no cooperation, like in the Eurozone, the State takes the CB policy as given and tries to reach its own goal which is to minimize the service of debt. The State

Figure 11: δ with Normal Distribution



would therefore be tempted to fix $r_S = r^*$ when the CB set $\mu = 1$ since it knows that a CB will always stand ready to intervene. Nevertheless, the CB expecting the State to do so, could decide not to intervene $\mu = \mu_{min}$ when the risk of inflation is too large given the size of public debt that would have to be bought. On the contrary, a cooperating CB and State would minimize the two loss functions simultaneously to find the optimal θ' and r_{eq}. We leave open the possibility of cooperation and denote r_S the rate chosen by the State.

Denoting r_{eq} the equilibrium rate and r_S the one announced by the State :

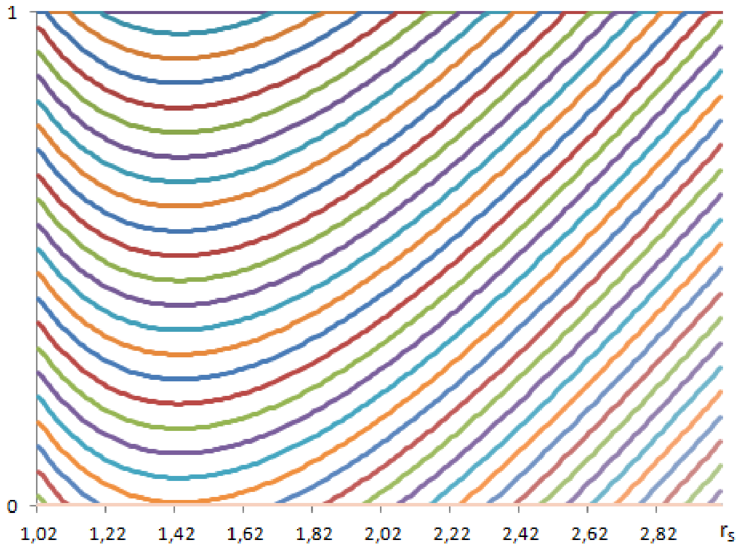
$$r_{eq} = \begin{cases} +\infty & if \quad \theta_{BC} < \theta' \\ r_S & if \quad \theta_{BC} > \theta' \end{cases}$$

In the case where there is no cooperation :

$$r_{eq} = \begin{cases} +\infty & if \quad \theta_{BC} < \theta' \\ r^* & if \quad \theta_{BC} > \theta' \end{cases}$$

When the distribution is uniform, the proportion of investors that don't roll-over the debt contract may be zero provided θ_1 is not too small. The smallest rate that makes δ equal to zero is the one that makes the marginal investor -the one who has received the worse signal- indifferent between leaving and staying. The rate must therefore include -at least- the marginal credit risk premium - the premium necessary to attract the marginal

Figure 12: δ with Uniform Distribution



investor.

When the distribution is uniform, the problem of the determination of the expectations about r_{eq} *ex ante* by investors is more problematic since we cannot use the argument that the CB is always the marginal investor. The only signal that everybody shares in $t = 1/2$ is θ_{BC}. It must therefore be on the ground on this public signal that the State will propose a rate which will have to be consistent with the investors' best expectation of its *ex post* value. Notice that expectations won't be necessary correct since nobody observes θ_1. More precisely r_{eq} is the smallest solution to the following equation :

$$E_{\theta_{BC}}\delta(\theta_1, r_{eq}) = \delta(E_{\theta_{BC}}(\theta_1), r_{eq}) = \delta(\theta_{BC}, r_{eq}) \leq 0$$

since δ is linear in θ_1. In the following graph, one can see δ as a function of r_{eq}. The different colours stand for different realizations of θ_{BC}. The solution to the equation are the values of r_{eq} for which the curves cross or are below the horizontal axis.

If this equation has no solution - if the curve associated with the realization of θ_{CB} never crosses the horizontal axis -, which is the case when the public signal is too low, then two cases arise :

- either there exists at least one r_{eq} so that $\delta(\theta_{BC}, r_{eq}) - \mu \leq 0$. In this case, the marginal investor is the CB and r_{eq} can therefore take any value - see the previous discussion with the normal distribution.

- or, for all r_S, $\delta(\theta_{BC}, r_S) - \mu > 0$. In this case no equilibrium rate exists. We set it at $r_S^e = +\infty$.

As in the symmetric section, we now draw some general conclusions about the impact of the CB policy and preference for inflation on the equilibrium rate.

Proposition n°3
r_{eq} is a non-increasing function of μ.

Proof in Appendix 1.

Proposition n°4
In the asymmetric imperfect information case, making the monetary policy more liquidity-passive - i.e. lowering the intervention threshold - makes the equilibrium interest rate decrease.

Proof. For a given θ_{BC}, lowering the threshold from θ' to θ'' modifies r_{eq} if and only if $\theta'' < \theta_{BC} < \theta'$. Otherwise, r_{eq} remains unchanged. In the case where $\theta'' < \theta_{BC} < \theta'$, μ increase from μ_{min} to 1 following the lowering of the threshold, by definition of the monetary policy. According to proposition n°3, r_{eq} is a decreasing function of μ, therefore the lowering of the threshold implies a decrease in r_{eq}.

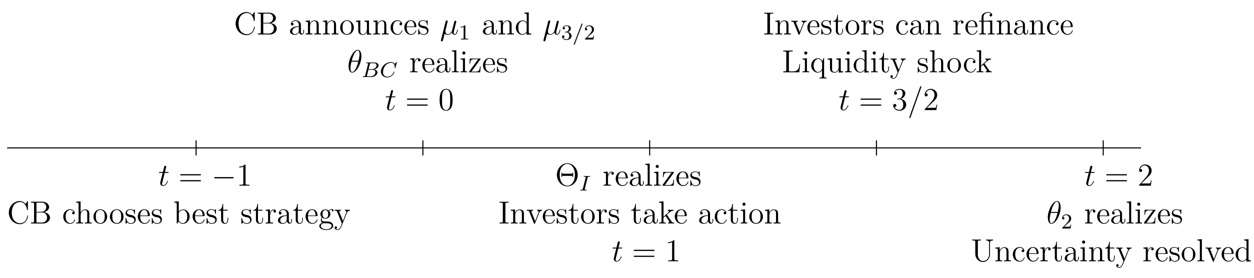
□

This result is key to our dissertation: it shows that the more liquidity-passive the CB, the lower the interest rate. On the contrary, the higher the degree of liquidity-activity the higher the "liquidity premium" and the higher the interest rate. This gives a theoretical justification to our central proposition according to which the sovereign rate puzzle can be explained by the different degrees of liquidity-activity of Central Banks, which themselves stem from different rates of preference for prices stability.

3.2 Interventions on open market vs Bank refinancing

Why did the previous policy of the ECB which consisted mainly in trying to influence the sovereign market conditions via the refinancing of banks fail? We show that this is because it relies on a partial analysis of the situation : the difficulty of sovereign states to sell their bonds was not only due to the hoarding behavior of financial institutions but also - and mainly - due to the run on public debt mechanism previously described. It is impossible to kill the latter vicious mechanism by solving the former.

Let's build a simple extension of the previous two periods model with exogeneous interest rate by adding one additional period between $t = 1$ and $t = 2$. This new period $t = 3/2$, before the information is revealed, is the "refinancing period" during which the investors can exchange their bonds against liquidity at the discount window. The degree to which the Central Bank is ready to make this repos agreement specifies the degree of the liquidity constraint faced by the Banks. We also add aggregate liquidity shocks randomly hitting the banks during this period to make the liquidity constraint relevant. We need "aggregate" shock because if shocks were idiosyncratic the financial institutions that have excess liquidity would lend to financial institutions in need for liquidity, and there would be no need for public liquidity (Holmstrom and Tirole, 1998). We don't need to solve the model in the general case to get the main idea of this part : when the CB leaves its discount window completely opened, there is no liquidity constraint so that the bank can get any amount of money during the third period in exchange for bonds. Clearly, if this was not the case, the banks could have an incentive to hoard liquidity and not to buy bonds. Letting the refinancing window open was a way for the Central Bank to ensure that the Bank would not hoard liquidity and would buy bonds. This was the diagnosis made by the European Central Bank in 2010 and 2011 and it may have justified and can explain the impact of the Long Term Refinancing Operations in the winter 2011-2012. However, this channel has not been efficient enough to permanently drive sovereign bond yield down.



Theoretically, this failure can be explained in the framework of our model. Call μ_2 the quantity of money that the CB is ready to issue in exchange for bonds in $t = 3/2$. This is the exact equivalent of μ for the previous period. Notice however that the CB doesn't buy the asset since the agreement between the Banks and the CB is a repo. μ_2 can be seen as a measure of the margin. When $\mu_2 = 0$, clearly there is an incentive for the Banks to hoard liquidity in order to self-insure against a aggregate liquidity shock.

The bank could thus be reluctant to roll-over their contracts in $t = 1$. It seems that this was the diagnosis of the the European Central Bank. This can justified to open the discount window. In terms of the model, this means increasing μ_2 to 1. Such a policy would completely remove this "liquidity" risk and therefore eliminate the precautionary and hoarding behavior of investors.

Now we will show that insuring the financial system against a aggregate liquidity shock in future period doesn't avoid a roll-over crisis occuring in $t = 1$. Assume $\mu_1 \to \mu_{min}$ and $\mu_2 = 1$: the CB never intervenes in $t = 1$ when the State has to roll-over its debt but is ready to refinance the banks against collateral without any discount. The core of our argument is the following : even if the banks know they will not be liquidity constraint in the future since $\mu_2 = 1$, and therefore have no incentive to hoard liquidity for that purpose ; they could still refuse to refinance the debt of the state. Adding an additional period where the Banks can be hit by a liquidity shock and adding $\mu_2 = 1$ does not change anything to the results we had before : since the Central Bank can provide liquidity only in the subsequent period $t = 3/2$ but not in $t = 1$, the refinancing channel cannot avoid a self-fulfulling liquidity crisis happening.

Explanations

Assume a aggregate liquidity shock γ may hit the whole financial system in $t = 3/2$. γ is the indicator function. If it hits - $\gamma = 1$ - the investor must liquidite its entire portfolio. It occurs with probability τ. With probability $1 - \tau$, no aggregate liquidity shock hits. We assume that θ_1 and γ are independent so that the signal received by the CB and investors is not informative at all about the realization of the liquidity shock in $t = 3/2$. We also assume that the market of the alternative asset is perfectly liquid contrary to the market for public debt which is partially illiquid : liquidating in $t = 3/2$ implies a loss of t - t may be a random variable depending on exogeneous financial conditions. We will think of t as being margins in repos agreement. We first solve in the symmetric information setup and then in the asymmetric one.

3.2.1 Symmetric information

With symmetric information, we can make the same reasoning as in section 2.4 where we simply take into account the possibility that the financial institutions may have to refinance - possibly with a loss - some of their asset to face a liquidity shock :

$$\text{E(NP / } \delta \text{ ; } r_S \text{ ; } r^* \text{ ; } \mu = 0 \text{ ; } \mu_2 = 0 \text{ ; t) =}$$

$$\begin{cases} r_S(1 - G(\frac{\theta^{**} - \theta_1}{\sigma_2}))(1 - \tau) + \tau r_S(1 - G(\frac{\theta^{**} - \theta_1}{\sigma_2}))(1 - t) - r^* \quad if \quad \delta = 0 \\ -r^* \quad if \quad \delta = 1 \end{cases}$$

Notice that we are back to section 2.4 when the shock occurs with probability 0 or when the public debt market is perfectly liquid $t = 0$.

Like in section 2.4, the best response depends not only on the signal θ_1, on the behavior of others investors and from now on on the probability of the shock and on the associated potential loss :

$$BR(\theta_1;\delta;r^*;r_S) = \begin{cases} \text{Roll-over if} \quad \theta_1 \geq \theta^{**} - \sigma_2 G^{-1}(1 - \frac{r^*}{r_S(1-t\times\tau)}) \quad \text{and} \quad \delta = 0 \\ \text{Liquidate if} \quad \theta_1 \geq \theta^{**} - \sigma_2 G^{-1}(1 - \frac{r^*}{r_S(1-t\times\tau)}) \quad \text{and} \quad \delta = 1 \\ \text{Liquidate if} \quad \theta_1 < \theta^{**} - \sigma_2 G^{-1}(1 - \frac{r^*}{r_S(1-t\times\tau)}) \end{cases}$$

The crucial result is that the new threshold $\theta^{**} - \sigma_2 G^{-1}(1 - \frac{r^*}{r_S(1-t\times\tau)})$ is higher than in the case without liquidity shock. Indeed, it is increasing in t and τ. When financial institutions fear to be liquidity constraint in future periods, they will require a higher signal to roll-over the debt. We call such a situation "liquidity hoarding".

3.2.2 Asymmetric information

In the same way, we show that κ is higher when there is a risk of uninsurable liquidity shock in future periods.

$$\kappa = \theta^{**} - \Phi^{-1}\left[1 - \frac{r^*}{r_S(1-t\tau)}\right]\sqrt{\frac{\alpha+\beta+\gamma}{(\alpha+\beta)\gamma}}$$

Notice that both in the symmetric and the asymmetric case, the higher the illiquidity of public bond markets t - which is generally related to global bad financial conditions - the higher the threshold.

3.2.3 Bank Refinancing : $\mu_2 = 1$

When the CB decides to lend freely - without any discount - against collateral to the financial institutions, it allows them to relax the liquidity constraint. If $\mu_2 = 1$, the institutions will refinance at the CB when they are hit by a liquidity shock. This implies that the loss t completely disappears and is now equal to 0. It stems from the preceding two subsections that when $t = 0$, we come back to section 2. Consequently allowing the Bank to borrow any amount of liquidity needed let the threshold of buying and selling lowersince investors have no incentives to hoard liquidity.

But we also know from section 2, that there is still multiple equilibria in the symmetric information case and that the threshold is higher - in the same way the crisis zone is larger - in the asymmetric information case when μ_1 is less than one. Independently of the value of μ_2, only μ_1 can eliminate the multiple equilibria - in the symmetric case - or decrease to its "fondamental" value the threshold of buying and selling -in the asymmetric case.

The goal of this extension was to show that even when the financial institutions don't fear anymore facing a liquidity constraint in the future due to aggregate liquidity shock and therefore have no incentives to hoard in order to self-insure since the CB stands ready to provide any amount of liquidity to those institutions, a roll-over crisis can still occur since this policy doesn't kill the strategic complementarities that may arise in period $t = 1$ when those actors have to decide whether to roll-over or not.

This could explain why the policy that consisted mainly in trying to influence the sovereign market conditions via the refinancing of banks was a short-term success but did not manage to completely eliminate the vicious circles at a longer horizon.

4 Public debt and liquidity : a historical study

The monetary and public finance history of the last four centuries offers the opportunity to test our toy model. What follows is therefore by no means the work of an historian, but rather a way for us to see whether our model can help to reinterpret some key events of the recent monetary history. In the light of past sovereign crises, we would like to convince the reader of two things. Firstly liquidity matters and that the relationship between Central Banks and States is above all marked by the issue of liquidity. Secondly our model is flexible enough to shed light on the running and outcome of different crises depending on the context. We isolate six main dimensions of the context which are related to parameters of the model : the organization of the financial system (mobility of capital), the monetary system (metallic money, convertible money, fiat money), the unit of account of the debt, the relationship between the State and the Central Bank (independence, main objectives of the Bank), the political and military context, the macroeconomic context.

As for the theoretical literature, in economic history, the role of liquidity in the relationship between Central Banks and the States has been neglected. Most papers have rather focused on the solvency issue. Bordo and White show that Great Britain, but not France, resorted partly to the inflation tax to finance the war (Bordo and White, 1990). Sargent and Velde explicitely apply the unpleasant fiscal arithmetic of the former to the Old Regime and to the hyperinflation caused by the assignats (Sargent and Velde, 1995). Hamilton who studies the role of wars in modern inflation shows that wars and revolutions without taxation have been the main causes of modern inflation (Hamilton, 1977).

4.1 Monetary system

The path from commodity currency to pure fiat money is one of the structural dimensions in the history of money. This transition from metallic money to convertible money in modern western european states is the history of the invention of Banks of Issue in the seventeenth and eighteenth century. These are the outcome of two convergent factors : the need for liquidity coming from the development of trade and the need for liquidity emanating from the nation-States in constitution. Regarding the implication for our model, we will show that this transition and the emergence of Banks of Issue have transformed the running and the outcome of sovereign debt crisis.

The nature of the monetary system influences the way liquidity can be made available when needed. In the case of a pure commodity money and when financial markets are not very developed, the State - or the institution dedicated to provide liquidity to the State - has no other possibility to get liquidity but to *ex ante* self-insure by accumulating coins. In the absence of a LLR which could have the ability to provide any amount of liquidity, the State must insure itself against future potential binding liquidity constraint.

The English State in the seventeenth century and the French State in the eighteenth century are in the same situation as a developing country of our modern times : it cannot issue the money it needs to settle its debts. As in the *original sin* story, the Old Regime French State cannot have immediate access to any amount of liquidity it would need. The possibility that the States had to modify the metallic content of coins could help to solve solvability problem by imposing a seigneuriage tax. But this could not solve the liquidity needs. Like the developing countries of our times, the only way for the States to increase M and prevent a liquidity crisis from happening in this monetary regime is to self-insure by accumulating coins.

4.1.1 Comparison between eighteenth century France and England

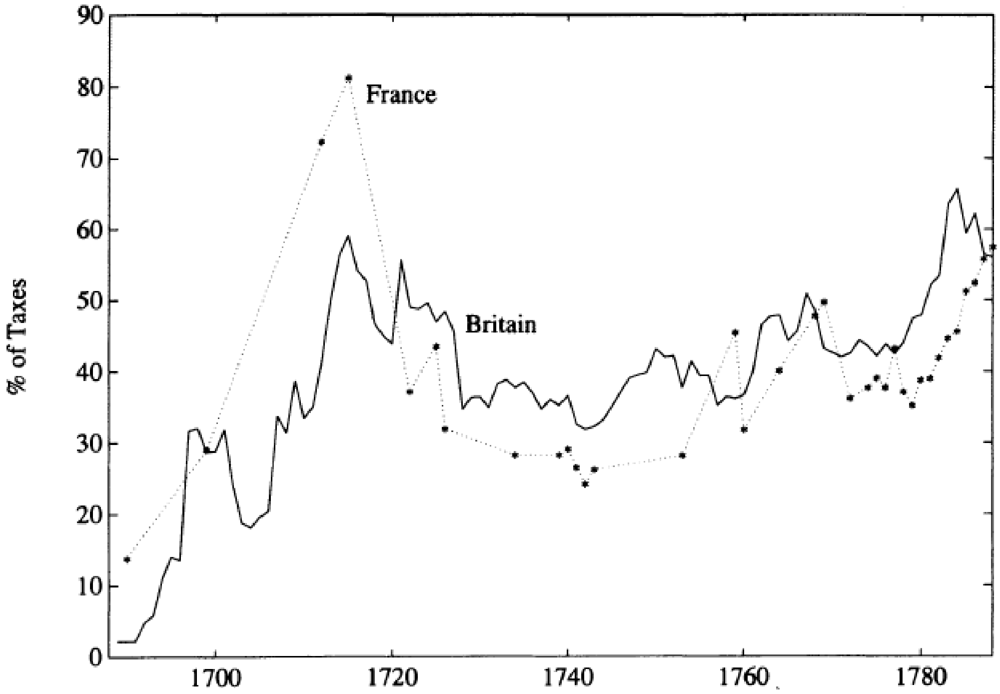
In the framework of our model, the absence of a provider of liquidity translates into $\mu = 0$. The prediction of our toy model is simple : $\mu = 0$ implies an increase in the probability that a liquidity crisis hits and makes the equilibrium interest rate rise. Is it what we observed? A good test of this prediction can be to compare France and England in the eighteenth century since the countries are quite similar in terms of development, macroeconomic perspective and institutions (with the major exception of the political system and its implication for the organization of public finance as stressed by North and Weingast, 1989).

Historically, this can help to explain the recurrence of liquidity crises and the numerous suspensions of payments which is the other theoretical means to deal with a liquidity crisis and to avoid default, in the eighteenth century France (Legay, 2011). By contrast, in England, whose traditional indicators of fiscal situation featured the same pattern as France, as suggested by the charts below (Sargent et al., 1995) and which had built since 1694 an institution specialized in providing liquidity to the States, the Bank of England, the State did not experience as many liquidity crisis and suspensions of payments as France. However, it must be noticed that the budget was probably more credible and better managed due to Parliament control (North and Weingast, 1989).

During the hundred years following the Glorious Revolution in 1688, Great Britain has never defaulted on its debt. On the contrary, France experienced several episodes of suspensions of payments, usually, but not always, accompanied by a reduction in interest rate. As written by Sargent and Velde (1995), France suspended payments and reduced interest payments after the Spanish War in 1713, after the system of John Law in 1720-1723, in 1759 during the Seven Years' War, then in 1770 - when Louis XV appointed Joseph Marie l'abbée Terray as controller-general which suspended payments of some short-term debt, lowered interest rate on rentes, reduced pensions by 80 percent, raised new revenus and cut expenditures (White, 1989) - and finally in 1783 and from 1787 until the constitution of the Estates General in May 1789.

Although the service of debt as a percent of taxes which is a traditional indicator of sustainability was very similar as revealed by figure 1, the French monarchy had to

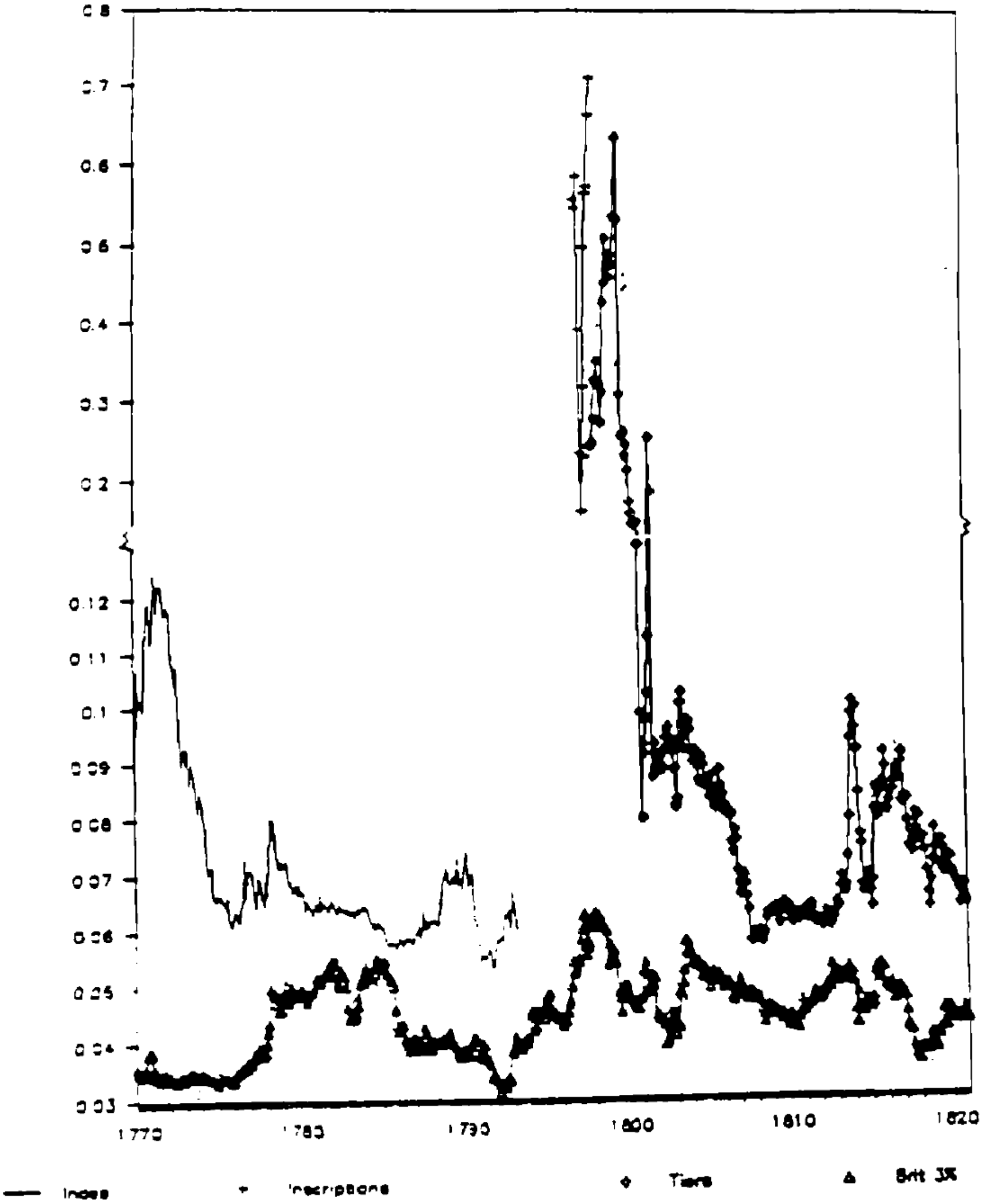
Figure 13: Debt Service as a % of Taxes



Source : Sargent et al.

pay a higher interest rate on its debt than the English crown, all along the eighteenth century century (Crouzet, 1992). This is also suggested by figure 2 on which Bordo and White (1990) reports the monthly yields on the three percent consols (Brit3%) and the monthly yield on the stock of the Compagnie des Indes (Indes), the French East Compagny whose series begins in 1770 when the Crown took over the Compagnie des Indes and converted its stock into a five percent consols (Bordo and White, 1990). This positive differential between the French and the English interest rates could also be interpreted, at least to a certain extent, as the consequence of the absence of a provider of liquidity of last resort in France. Consistently with the second prediction of our model, this could be the symptom of the fact that France had to pay a premium to attract the marginal investor and to give the latter a compensation for the default risk coming for the absence of a provider of liquidity in last resort, while this risk was absent in Great Britain. Consequently, in our view, recurrent liquidity crises and higher interest rates were two sides of the same coin.

Figure 14: Yields of British and French Securities



Source : Bordo and White (1990)

However this argument based on the absence of a lender of last resort to explain both the repetition of liquidity crisis and the higher interest rate has been debated and is now quite controversial (Crouzet, 1993). The literature has rather stressed the differences in the institutional features between the French absolute monarchy and the English Parliamentary regime. According to North and Weingast (1989), Great Britain, since the Glorious Revolution, had the institutions that made its State creditworthy. In particular, the Parliament had the power to vote the budget. According to Sargent and Velde, the Bank of England also played a role in improving the commitment of the State not to default. However according to these authors, this is not by acting as a Lender of Last Resort, but by making default more costly for the State. On the contrary, the absence of institutions designed to implement a full commitment on the part of the State and the absence of an efficient tax system which goes with modern comptability and tax collection, as stressed by Legay (2011), might contribute to explain the high number of crises, suspensions of payments and higher interest rates in the eighteenth century France. Crouzet argues that there are two modes of regulation of the sustainability of the public debt. On the one hand, the English way is consistent with the "tax-smoothing" theory of government budget (Barro, 1969) : when a war hit, the tax rates were raised in order to make sure that the present value of the additional fiscal revenus matched the deficit in wartime. Moreover, after every war, the floating debt was consolidated into perpetual annuities. On the other hand, the French State, because of difficulties to raise taxe rates and to improve the collection of taxes partly due to its political system, could not apply the same scheme and regulation was more chaotic: increase in taxes when possible, liquidity crises, and default. All this literature agrees, although there are a lot of internal debates, on the fact that the differential is due to the difference in the organization of the fiscal system and the subsequent credibility.

Is the liquidity interpretation credible? We will argue here that even if it is impossible to measure the contribution of each explanation -fiscal system vs LLR-, the liquidity interpretation must not be neglected. Firstly, the view according to which there would have been an English "virtuous" way and a French chaotic way is too simplistic to be true. Indeed, the taxes were permanently increased in France during war time, as in Great Britain ; and the reduction in interest made after the wars or the conversion into perpetuals of floating debt were also commonly used by the English Crown - they should not be viewed as episodes of default as stressed by Chamley (2011).

Secondly and more importantly, the recurrence of liquidity crises in France is the *immediate* symptom and consequence of the absence of a LLR. Should the French monarchy have the support of a LLR, it would not have experienced so many liquidity crises, although could have maybe encountered sustainability problems. Indeed a LLR would have lent to the State the liquidity it would have needed. Therefore the only fact that liquidity crises happened is the direct consequence of the absence of a Bank of Issue devoted to the stability of the financing of the public debt. As a consequence, it is hard to believe that this very fact that a liquidity may hit was not reflected in the interest rates required on French bonds. One may argue that the suspensions of payments were not the consequence of roll-over crises but, most of the time, a decision of the State. This serious counter-argument does not totally invalidate our point since the decision and the cost associated to suspend payments themselves depend on whether the State has access to large amount of emergency liquidity from a CB or not. Moreover, as we will argue later, the role of the CB in providing liquidity should not be restricted to roll-over crises period. These two questions would obviously deserve a rigorous modeling.

Finally, there is evidence that during events of high uncertainty and stress, the Bank of England played a stabilizing role by lending to the State allowing the rate to remain at reasonable levels. For instance Dickson argues that the intervention of the Bank of England during the War of the Spanish Succession allowed the state to borrow at low interest rate (Dickson, 1969). From figure 2, even if the yields on 3% british consols increased during the war of American Independence and the revolutionary period wars, they remained under the threshold of 6%.

However, for the liquidity view to remain reasonable, it must not be restricted to the interpretation of the LLR as sketched in the first part - i.e. in the context of a roll-over self-fulfilling crisis. Indeed, as already mentioned, the financial crises of the French State were not only and probably not mainly caused by self-fulfilling run crises. Nevertheless the lack of available liquidity and the need to refinance a huge short-term debt were often keys to understand these crises. The presence of a provider of liquidity may also play a role in the - optimal - decision to suspend payments which could also support our argument. For the liquidity interpretation to remain reasonable, one needs to enlarge the stabilizing contribution of the Bank of Issue to a wider domain than simply intervening during textbook-like roll-over crises. Clearly we need more research on these points to understand the channels by which the Bank of Issue could have enhanced the stability of the State financing.

4.1.2 Emergence of Banks of Issue and liquidity needs of the State

The invention of issuing banks in developed countries is, most of the time, intrinsically linked to the strenghtening of the modern nation-state and its need to relax the credit constraint in wartime, to secure a flow of financing and refinancing of its debt. The implicit - or sometimes explicit - aggreement between the State and the Bank is usually the following : these banks get a privilege - or in modern words, a monopoly - usually the right to issue notes within a certain area, which means that the State delegates to the bank the right to issue money - seigneuriage - ; in exchange the Banks must make sure that the notes remain convertible into the reference metal and must help the government in case of liquidity needs. We would like to show that the first three Banks of Issue have been created partly to relax the credit constraint of a strenghtening modern nation-state at war with its neighbours.

The Bank of Sweden is usually viewed as the first Central Bank, although the word is anachronistic[4]. The Bank of Sweden has been created in 1688 at a time of large political, social and economic transformations. In the 17^{th}, Sweden was becoming a major Empire in the Baltic region and the state had to finance repeated wars against its neighbours. As E. Heckscher [5] showed in 1936, the rapid development of trade and industry during the 17^{th} century also required new ressources to finance larger investments. Exports of iron and the metalworking industry concentrated most of the stock of credit. It must be recalled that at that time, the operations of credit were still informal and personnal ; there was no such things as banks or centralized market to issue debt. The need for a better organization of financial transactions to sustain the economic expansion, the problem raised by the copper standard [6] and the need to relax the credit constraint on the part of a militarized state led to the creation of the Stockholm Banco by a rich merchants Johan Palmstruch and then by its replacement by the Bank of the Estates of the Realm after a successful run on the bank-notes that had been issued in very large amounts compared to the metal in reserves in order to finance mostly public debt occurred. This latter bank was under the direct control of the Rikstag of the Estates to regain the public confidence, to prevent the interference with the King and to avoid the massive use of new credit lines to finance the wars - this could be understood in the framework of our model as a way to guarantee the solvability-activity of the bank's policy. However, rapidly, the political pressures and the financial necessities made this control more and more laxist ; and the deposits of the Bank were massively used to finance the war expenditures. Rita Bredefledt has shown that the state remained the main debtor of the Bank, by far, at the end of the century. Why is the intermediation of the Bank so beneficial? For the state this intermediation is a guarantee that it could have an easy access to a source of liquidity when needed.

The Bank of England has been set up in 1694. It is with Sweden (1668) and France (1800) one of the oldest Banks of Issue. As stressed by Crouzet, this is not by mere chance than France and Great Britain are also the first nations-states in Europe[7]. Like the Bank of Sweden, the Bank of England has been created at a time of major economic development and financial needs on the part of not only private merchants but also the states at war. Indeed, after the Great Revolution of 1688, Great Britain entered a new episode of conflicts against France from 1689 to 1697 - the Nine Years' War. The deterioration of the public finance, the very high interest rates and the difficulty for the Londonian goldsmiths to provide the necessary financial support to the state were the key and short-term determinants of the creation of the Bank of England in 1694 by W. Patterson (Plessis, in

[4] What follows is mostly inspired by "La genêse de la Banque de Suède dans le contexte économique et social de la fin du $XVII^e$ siècle", Rita Bredefeldt, in *Politiques et pratiques des Banques d'émission en Europe ($XVII^e$-XX^e siècle)*

[5] E. Heckscher, *Sveriges ekonomiska historia fran Gustav Vasa*, 1, 2, Stockholm, Bonniers, 1936, p. 573.

[6] In 1624, the government decided to mint copper in order to sustain its market value (Sweden was a major exporter of copper) ; therefore entering a period of bimetallism (silver, copper). Given the market value of copper in terms of silver and its weight, the coins in copper were extremely heavy and inconvenient to carry (it could weigh between 2.72 kg and 19.7 kg)

[7] F. Crouzet, *De la supériorité de l'Angleterre sur la France. L'économique et l'imaginaire, $XVII^e - XX^e$ siècle*, Librairie académique Perrin, Paris, 1985, p. 452.

Feiertag et al.(dir.), 2003). In exchange for receiving the right to issue banknotes and to discount bills, Patterson lent to the Crown £1 200 000. First conceived as an emergency support, the Bank then became a key institution in the economic and financial development of the country and did bring liquidity support to the state in the management of a high debt -in particular during the War of the Spanish succession, as stressed by Dickson (Dickson, 1989). As for the Bank of Sweden, the creation of the Bank of England allowed the state to transfer wealth from the future to the present and therefore to relax the liquidity constraint which was all the more costly as the country was engaged in a war.

The first experience of an issuing bank in France is the Royal Bank of Law in 1718 (Plessis, 2003). A mass of short-term debt had been accumulated during the War of Spanish Succession (1706-1714) and despite the reduction in the face value of the debt from 596m to 180m in the Visa of 1716, the stock of debt amounted to 2 billions - around 167% of national wealth according to rough estimations (Chavagneux, 2011)-, the debt service remained thus high and the payment of interest uncertain (Velde, 2006). This can explain why the Regent allowed in May 1716 John Law to establish a privately owned bank issuing notes payable to the bearer : this is the creation of the first french issuing bank. The Bank and the Company -the Company of the West, which will soon become the Indies Company- bought billets d'État which became the compagnies' main assets, in exchange for shares. Progressively the Company bought out all the compagnies in existence, in particular the South Sea Company, the French Indies, and it got the monopoly on trade with Louisiana. One of the aim of Law was to buy the entire public debt and refinance it at a lower rate (Velde, 2006). These operations were conversions of government debt into equilities. At the same time Law got the management of the collection of all taxes in France. We don't detail all the devices that Law invented to attract investors, like for instance rising share prices partly fuelled by active monetary creation by the Bank. This first experience which ended up in a complete chaos might be seen as the temporary and tentative response to two problems. First, as it has been emphasized by the literature, John Law brought some proposals to solve the structural solvency problems of the State : for the first time, the taxation system was centralized, all officers of the receivers general had been bought, almost all taxes were collected by the Compagny and the purchase of others assets (Louisiana, French Indies etc..) were supposed to bring additional revenus. Secondly, and this point has been less emphasized by the literature, the existence of the Bank implied that the State could avoid roll-over crisis and could better transfers funds intertemporarily.

The invention of the Bank of France comes after two other experiences of notes issuance in the context of the Revolution. In 1776, the "Caisse d'escompte" a private institutions that issues notes is created. With the rise in public debt in the folowing decades, it becomes rapidly the Bank of the State (Plessis, 2003). Most of its asset is made of public debt, and because of runs, it has to suspend payments from 1787 on. It is liquidated by the Convention in 1793. The second experience is the issuance directly by the State of paper money : the "assignats". From december 1789 on, the State becomes its own bankers and its own lender of last resort. There is no longer any intermediation by a financial institution. In the monetary dimension also, the Revolution gave birth to radically new ideas. As Galbraith writes it, the paper moneys are the instrument of the revolutions. Before it the Americans also used the paper money to finance their Revolution ; and the Soviets will do the same later. The "assignats" are issued to finance the deficit that stems from the absence of a new taxes to replace the old that have been removed and from 1792 from the military expenditures - the Revolutionary State has many ennemies outside and on the French territories. In 1793 and 1794, under the management of Cambon, the value of the assignats is stabilized at 50% of its face value (Galbraith, 1975). In 1796, before being replaced by the "mandats territoriaux" and before the "banqueroute of the two-thirds", it has lost more than 90% of its face value. *Ex post* - even if it would be teleological to give the actors such a preview - it is clear that the French Revolutionary State needed to have not only a liquidity-passive monetary policy and the subsequent possibility to directly issue notes but also a solvability-passive policy which enabled it to finance its enormous expenditures. Clearly, the Revolution has been financed - in the two dimensions of the word - through the assignats and the associated "Great Inflation" (Crouzet, 1993).

The fear of notes of French people inherited from the Law or Mississippi bubble and the subsequent high inflation and reactivated by the experience of the assignats did not resist the financial needs - and the willingness to give the French economic system a stable and efficient monetary system - of the Revolutionary State led by Bonaparte. The Bank of France is created in January 1800 by a group of bankers close to Bonaparte who obtained from the latter the right to issue notes. The Bank of France will rapidly become the Bank of the State at war. It obtained the monopoly over notes issuance in 1802. From the starts, it lends to the State either directly through advances or though the refinancing of Treasury bills. This led to a run on the Bank in 1803 and in Autumn 1805, which doesn't turn out badly thanks to the victory of Austerlitz in december 1805 (Plessis, 2003). April 22 1806, the monopoly of the Bank is guaranteed until 1843, and will be managed by a governor appointed by the Emperor and revocable at any time. Finally in 1808, the new status of the Bank create new commissions that will rule the Bank together with the "conseil de régence". This leads Plessis to write that the independence of the Bank has only been short-term and apparent. However the term independent - as usual - is used in a rather vague manner.

It must be noticed that contrary to the 1790-1796 periods, the degree of solvability-activity of the monetary policy was a lot stronger. Indeed, the head of the State, Napoleon Bonaparte itself, wanted the Bank to maintain the value of the notes stable over time, and he knew that this depended on the relationship between the Bank and the State (Gabillard, 1953). Consistently with its will to give France a modern Bank of Issue, and despite the huge expenditures required by the campaigns in Europe, Napoleon maintained the solvability-activity of the Bank. As noted by Bordo and White, the Napoleonic War in France were mosltly financed through taxes and taxation of conquered countries (Bordo and White, 1990). Consequently, as far as solvability is concerned, we cannot say that "the independence of the Bank of France was short-term and apparent". One could still argue that the monetary policy was liquidity-passive although we have shown previously that this may not be necessarily the sign of a lack of independence since it may be optimal for the monetary policy to be liquidity-passive when the solvability-activity is guaranteed.

As for the Bank of England, the birth of the Bank of France is not separable from the financial needs of a State at war and the need from the private merchants for currency stability and credit expansion. More generally the invention of the first issuing banks are not separable from the financial needs of the state and the necessity for it to relax the liquidity constraint. From the perspective of the incomplete market theory, the invention of a Public Bank is a way to find a solution to a credit constraint problem, as far as the State is concerned, which may come from asymetries of information and more generally from imperfections of markets (Holmstrom and Tirole, 1998) or from the asymetries of power between the creditors and the State. The first issuing banks are *de facto* public banks even if they are usually held and owned by private merchants - the private ownership is also a way to reinforce the credibility in the institution, and in particular in its notes.

One of the key questions raised by the creation of this Banks of Issue is why did the credit constraint bind ; or why did the creation of the Banks relax it? The traditional explanation of credit constraint is based on information asymmetry (Stiglitz and Weiss, 1992). The creation of these financial institutions would have solved the informational asymmetry between creditors and debtors since the Banks could more easily gather information about the states. Without neglecting this interpretation, our model pushes us towards another types of interpretations : the creation of these Banks solves different kinds of coordination/liquidity problems that prevented the liquidity to be intermediated. The first one modeled in the second extension is the hoarding problem. In a world caracterized by credit constraint, people are very reluctant to lend their monetary asset and buy public debt because they fear to be hit by a liquidity shock in future periods. The creation of a Bank of Issue, by allowing the discount of Treasury bills, may have partly solve this problem. The second one modeled in the first part is the coordination problem that emerges from the very large number of creditors and that makes a large debt hard to roll-over. The Bank of Issue by providing liquidity makes sure that a liquidity crisis won't force the State to suspend payments and potentially to default.

Since the Jamaica agreement, the monetary system is no longer based on a convertility principle into a metal. Our contemporeneous system is characterized by "fiat money" (Keynes, 1930). Of course, confidence, in particular in the convertibility of notes into metal, was already at the core of the convertibility system since the liquidity coverage of the Bank of Issue was never 100%. But the end of the convertilibty system implies that the value of the currency is no longer tied to a reference commodity. The "value of money" has no meaning anymore ; the only definition of it is by essence tautological : it is the inverse of the nominal value of a representative basket of goods (Orléan, 2011) . The key question is the following : how is the price level and inflation determined in such a system characterized by self-referentiality ? Is the assumption of a "Central Bank balance sheet theory of the price level" relevant ? As mentioned in the theoretical part, this question is one of the most important that must be answered before analysing the consequence of the interventions of CB on public debt markets.

4.2 Unit of account of debt and original sin

Most states in developing countries cannot borrow in their own currency. This phenomenon - called the "original sin" - means that the more the state is indebted the higher the currency mismatch of the balance sheet since the liability side is denominated in a foreign currency - typically the US dollar, the euro, the sterling, the swiss franc or the yen. As it has been well-documented, this inability to borrow in national currency has a procyclical effect in bad times since the depreciation of the national currency has a negative wealth effect and because the monetary policy is also likely to be procyclical - in order to avoid further depreciation - (Eichengreen et al, 2003, 2003a, 2003b). Empirically, it has been shown that countries hit by the "original sin" are also characterized by a higher output and capital flows volatility and a greater reluctance to let their exchange rate float.

The other important point that the literature has stressed is the absence of a domestic Lender of Last Resort and consequently the likelihood of a liquidity crises. Because those countries have no access to the liquidity they would need - apart from accumulated foreign exchange reserves -, a sudden stop can rapidly lead to a liquidity crisis (Goldfajn and Valdes (2007), Chang and Velasco (1999) who apply the Diamond and Dybvig framework in an international context, Sachs 1995 or Flood and Mario (1998)). Different solutions have been proposed to solve the liquidity need problem in those cases.

Firstly those countries could accumulate large foreign exchange reserves in order to insure against self-fulfilling run as stressed by Aizenman et al. (2004), Flood and Mario (2002), Aizenman and Mario (2004). This precautionary saving can be justified based on consumption-smoothing models with risk aversion and non-insurable risk (Weil,1993). However, this solution is costly compared to a first best situation with complete markets. Indeed, if the increase in the foreign exchange reserves is exactly equal to the increase in foreign debt, the net foreign debt remains equal to zero, the operation is completely useless - there is no net capital flow to finance new investment within the country - and costly - the opportunity cost of holding foreign reserves is potentially high - ; if the increase in the foreign exchange reserves is less than the increase in foreign debt, then the country can benefit from the opportunity of the international trade of capital - consumption smoothing - but there is still a opportunity cost of the proportion of foreign liquidity held and the risk of a successful liquidity crisis is not eliminated since the reserves do not fully cover the foreign debt. Hence some authors have formalized this trade-off and presented formulas for an optimal level of reserves (Jeanne and Rancière, 2006).

A second solution would be to create an Internation Lender of Last Resort who would stand ready to intervene without *ex ante* limit in case of international liquidity crisis that involve those countries unable to issue the currency they are indebted in (Jeanne and Wyplosz (2001), Jeanne and Zettelmeyer (2002)). These institutions would completely cut the roots for a self-fulfilling run on countries' debt. It would also avoid the social cost imposed by the accumulation of foreign reserves : an ILLR would allow the world economy to reach the first best situation. However the moral hazard problem and the political difficulty to implement such a strong supranational institutions have until now prevented its setting up.

The original sin and the impossibility to issue the currency in which the state is indebted means, in the framework of our toy model, that $\mu = 0$. Nevertheless it is not easy to test our model in this international and historical context by simply regressing the probability of crisis on a dummy capturing the "original sin" and a set of controls. Indeed, the original sin is not an exogeneous parameter, and even if there is still no consensus about the deep roots of the phenomenon (Eichengreen et al. (2003)), we can at least presume that the likelihood of a crisis and the original sin are jointly determined. A better way to test our model here is to take all developing countries and regress the probability of a crisis on the level of international reserves and a set of controls. The foreign reserves variables is a good proxy of the parameter μ in our model ; even if we would have to take also into account the maturity of the debt - since μ is the amount that the Central Bank is ready to provide to the State by unit of maturing debt. The econometric papers have found a negative and significant relationship between the level of international reserves and the likelihood of a crisis (Detragiache and Spilimbergo, 2001). There is also a positive relationship between the proportion of short-term and maturing debt and the likelihood of a crisis (Pescatori and Sy, 2004). Without surprise - the literature on developing country sovereign crisis is one of the first one to have modeled the self-fulfilling nature of liquidity crisis- these evidence of the recent period is clearly in favor of our theory.

4.3 Relationship between the Central Bank and the State

The nature of the relationship between the Central Bank and the State matters to understand the outcome of a liquidity crisis. A "liquidity-passive" Central Bank will prevent a successful run to occur. On the contrary, if a Central Bank is reluctant to intervene as a provider of last resort, everything else being constant, a loss of confidence - a possibly bad signal - may degenerate into a liquidity crisis.

As we will argue, the model is particularly well-fitted to understand the financial stress in France after the First World War and after the Cartel des Gauches took over in May 1924. We will suggest that there exists a lot of interesting parallels between the financial dynamics of the after-war period in France and the ones of the post-2008-crisis period in Europe.

WWI in France has been financed trough taxes (15%), long-term debt (22%), short-term debt called "Bons de la Défense Nationale" (38%), external debt (17%) and Bank of France credit (8%)[8]. Since a large part of the short term has been bought by financial institutions - among which the Bank of France - the claims on the Treasury represented 56% of the counterpart of M2 in 1919 - to be compared with 6% in 1913. The debt as a percent of GDP amounts to 150% of GDP and M2 has been multiplied by 3 since 1913. The solvency of the French State is not guaranteed and a huge stock of short term threatens the financial stability. The question of the reparations, the reimbursement of the French debt to the Allies and the financial and monetary stress due to the large short-term debt will dominate the event from 1919 to 1926. We will divide the analyse into 3 periods : from 1919 to 1924, a potentially stress but stable situation, financial tensions are not far but the government managed to roll-over the debt easily. From 1924 to 1926, the Cartel des Gauches rules the State and the fragile situation degenerates ; the Cartel has to face both the run of many investors and a renewed inflexibility of the Bank of France. In 1926, Poincaré is appointed and the economy shifts once again to a more stable equilibrium. Why did those two shifts occured? What has changed between 1923 and 1924 ; and then between June 1926 and August 1926 ?

From 1919 to 1924 the "Bloc National" rules the State, the relationship between the State and the CB is characterized by what we called a solvability-active monetary policy. Indeed in 1921 the François-Marsal agreement between the Treasury and the Bank of France is signed and obliges the State to reimburse all the advances made from 1914. These advances represent about 30 billion of FF in 1920 when it was almost 0 in 1913. There was a consensus inherited from the previous century among the policy makers to run this deflationary policy (Blancheton, 2001). The solvability of the State was supposed to be guaranteed without any need for a fiscal adjustment due to the expectation that Germany will pay the full cost of the reparations. At that time the unanimous credo was "Germany will pay" ("L'allemagne paiera"). A solvent State, a solvability-active monetary policy : why should the investors fear? Indeed, investors didn't apparently fear too much. Theoretically an economy could always fall in a self-fulfilling crisis as shown in the first part. But from 1919 to 1924, there was no major financial stress. The interest rate on bond remained stable around 5.5 percent although sligty increased over time from 5% in 1918 to 6% in 1923. The advances of the CB to the State remained almost constant and equal to the upper limit fixed by law - but several times revised since 1914 - at 30 billions. The inflation rate fluctuates a lot which is more the consequence of the disequilibrium on the good markets due to the lack of supply at the end of the war and then in 1921 to the reconversion from a war to a peacetime economy than to the adjustment of the Balance Sheet of the CB (BSTPL) (Lutfalla and al. 1986 ; Blancheton, 2001). The French Franc sharply depreciated against the dollar in 1921 which is the direct consequence of the end of the multilateral agreements between the Allies. Then it slowly depreciated at a reasonable rate possibly reflecting the internal inflation.

[8]The figures are taken from Lutfalla, M. and J-P. Patat. (1986). *Histoire monétaire de la France au* XX^e *siècle*. Paris: Économica.

Figure 15: Financial Indicators between 1919 and 1926

	1919	1920	1921	1922	1923	1924	1925	1926	1927
FF/$	5.18	5.18	13.49	12.33	16.58	19.32	21.23	31.44	25.48
Surplus (mil.)	-26 688	-17 139	-9 275	-9 761	-11 866	-7 121	-1 507	+1 088	+ 217
M2	+ 12.4	+0.6	-0.7	+4.6	+8.7	+10.8	+25.4	+6.5	+ 6.4
Inflation	+22.6	+39.4	-13.2	-2.2	+8.9	+14.3	+7.1	+31.7	0.4
BDN (bil.)	30	•	•	60	•	•	50	•	•
Interest Rate	5.18	5.80	6.10	5.71	5.99	7.01	9.11	8.76	6.58
Advances BdF	+28.8*	+0.4	+0.9	-1.2	-	-1	+14.1	-0.9	-15.3
Debt/GDP		151	172	168	168	150	•	•	•

*Source : Patat. J-P and M. Lutfalla (1986) ; Blancheton B. (2001) ; * compared to 1913*

While the debt over GDP ratio continued to increase from 1919 to 1924 due to the large public deficit (-15 000 million on average), there was no major financial stress. In 1924, when the Cartel des Gauches took power, the ratio Debt over GDP decreased for the first time since the end of the war from 168% to 150%. The public deficit rapidly narrowed from -11 billion to -7 and then to -1.5 in 1925. Why then did the French economy shift from a financing-equilibrium with low interest to a - almost - non-financing equilibrium with high interest rate ? There are three main reasons. The first one is the taking up of power by the Cartel des Gauches itself. For reasons that are beyond the scope of this dissertation, the investors saw this election as a bad signal in itself, since it was associated with a possibly lower commitment to meet the financial obligations of the State. Another reason often mentioned is that the Cartel des Gauches wanted to create a new tax on capital which would have consequently lowered the net yield on French bonds. The second reason is also related to a solvability issue : at the same time, the illusion that Germany would pay dissipated. Signed in 1924, the Dawes Plan made clear the Germany would not pay the whole reparations agreed on in 1921 - 269 billion Marks. The final amount of the reparations was set at 132 billion which represents a division by two. This also contributed to reduce the signal about solvability. The third element is more subtle : the massagings of the account of the Bank of France made it possible for the monetary authority to threaten the Tresury to re-estabblish the liquidity-activity of the monetary policy. Since 1923, the account of the Bank of France had been falsified by the Treasury to hide the fact that the legal emission ceiling had been burst. In november 1924, when the Cartel started to face financing difficulty and before the Bank revealed the scandal, the money stock was 3% higher than the legal limit. This falsification gave the Bank of France the opportunity to resume the control over the creation of liquidity and to threaten the Treasury to shift from a rather liquidity-passive policy to a liquidity-active monetary policy.

Each of these three reasons may contribute to explain why despite an improving situation, a lot of investors decided not to roll-over their BDN - Bons de Défense Nationale - from the end of 1924 until the beginning of 1926 when Poincaré took power. Indeed, the interest rate on bonds increased from 6% in 1923 to 7% in 1924, and reached 9.1% in 1925 at the peak of the crisis ; and then started to decrease in 1926 at 8.7%. This is also confirmed by the fact that the issuing of new long-term debt and the conversion of BDN into bonds in 1924 by Clémentel was disappointing, despite its high return, 8% (Blancheton, 2001). After the scandal was reavealed, the issuing of new bonds became even more difficult. In the same way, in April 1925, 6.83 billion were due and not rolled-over by investors, while the new emission could only bring 5.16 billion : therefore in April, the government thus faced a deficit of 1.73 billion. The same happened in the following two months. The Bank of France made new advanced on June 25 1925. At the end of the year, a new long-term debt issuance led by Caillaux was also disappointing. That's why the advances to the Treasury and the money supply increased by 14% and 25% respectively in 1925. In parallel, the capital flight triggered a sharp exchange rate depreciation; on average, in depreciated by 30% from 1923 to 1924, and once again by 30% from 1925 to 1926.

The first two reasons may be understandable in the framework of our toy model as a decrease in the average signal about solvency, θ_1. The project of a capital tax also played in the same direction, by decreasing the promised net yield on public debt. For a constant monetary policy, these alone may have been sufficient to increase the proportion of runners δ - recall that the latter is decreasing in θ_1 and in r_S- and to push the French economy in the crisis zone and to a non-financing equilibrium. But we would like to insist on the third factor. From the beginning of the war in August 1914 to the scandal of the Bank account, the Bank of France was clearly liquidity-passive. Even if the board of the Bank always insisted on the fact that the deflation and the return to the 1913 parity was a necessity - which can be interpreted in the framework of our model as the willingness on the part of the Bank of France to guarantee the solvability-activity of the monetary policy - the Bank never refused any advances to the French Treasury. At that point, a legitimate question may be raised : did the Bank never refused because it was forced by the State and the Governor not to ? or is it because its objectives changed with the beginning of the war - a patriotic duty ? or did the Bank led an optimal liquidity-passive policy because it believed in the solvability-active policy and in the return to the secular parity (as suggested by proposition n°2) ?

Whatever the deep motivation, the Bank of France did follow a liquidity-passive monetary policy until 1924. But at that point, the Bank of France tried to shift to a more liquidity-active monetary policy. There are three main explanations of such a change. First of all, it could be that the Bank of France wanted to make the Cartel des Gauches fail for political reasons. Indeed there is evidence that de Wendel, one of the member of the board, did use all the means this position provided him with to fight its political opponents (Jeanneney, 1976). Secondly, it could simply be a optimal response to the belief that the Cartel des Gauches will probably abandon the deflation policy - partly because of the Dawes Plan - and therefore will put an end to the illusion of the solvability-activity of the monetary policy. Indeed we know from proposition n°2 that the optimal response of the CB to such a change is to increase the degree of liquidity-activity. The third explanation considers that the Bank of France was somehow forced to lend to the State from 1918 to 1924 ; and that the falsification of the accounts was the lever thanks to which it tried to change the balance of power and to release the creation of liquidity from the needs of the State. In the framework of our model, the increase in the degree of activeness implies that μ was no longer equal to 1 for any signal θ_1. This decrease in μ may have also contributed to the sharp increase in the number of runners in 1925. People ran also because they fear the Bank of France could stop supporting the Treasury up.

Finally, Poincaré took power in 1926, and the economy, almost magically, swithed to a financing equilibrium with low interest rate. Indeed the interest rate fell to 6.5% ; the number of runners sharply reduced ; this translated into a rapid reduction in the advances to the Treasury by -15 billion ; an appreciation of the French Franc to 25.5$ and a stabilization of prices. The taking up of power by Poincaré induced two main dynamics. First it may have improved the solvability signal θ_1 though a clear commitment to meet the obligation of the State. Secondly this better signal, associated with the higher political proximity to the board of the Bank of France, may have also contributed to decrease the degree of liquidity-activity of the Bank. These two elements may explain the sudden stop of the panic and of the roll-over crisis and the return to a financing equilibrium with low interest rate.

Notice however that another and maybe more elaborate and renewed reading of the stabilizing role of Poincaré can be made in the light of our model. In 1924, a solvent State meant a solvent State at the 1913 prices since the goal of the deflation was to come back to the secular parity. In 1926, Poincaré clearly put an end to this illusion and decided to set the new value of gold in terms of franc at a new parity - three fifth of its pre-war value. The truth is thus that Poincaré didn't come back to the secular parity and didn't give the Bank of France the solvability-activity it was claiming for. This can be seen as a simple debasement which considerably reduced the real value of the public debt and therefore improved the solvency of the State. According to this interpretation, the stabilizing role of Poincaré should not be attributed to a personal virtue of the man, or to its policy, but simply to the fact that it rapidly increased the solvency of the State by setting a new value of gold in franc. Actually he did not devalue the franc, he just set the new parity at the market parity of the time ; but if investors were expecting the franc to return to the old parity, its decision acted as a *de facto* devaluation of sovereign bonds.

4.4 Organization of the financial system

A developed financial market is often viewed as an efficient way to solve the credit constraint problem since the latter is the sign that the market to transfer wealth from tomorrow to today is missing. A part from the fact that the deep causes of financial market incompleteness - like for instance asymetries of information - may be hard to solve, our model suggests that developed financial markets does not prevent liquidity crisis from happening. The development of financial markets doesn't kill the strategic complementarities at the core of the run on liquidity that the model describes.

Our model allows us to go even further : the mobility of capital may not guarantee and might even go against the stability of the financing of the state. On the contrary, if for any reason, the investors are not totally free to liquidate their position, this would tend to kill the strategic behaviors at the core of runs since everybody knows that everybody knows - so on and so forth - that *we* are all "stuck" with *our* public bonds. Applying this prediction from the model to the second world war period and the after-war period, we would like to show that some institutional reforms made the likelihood of a liquidity crisis sharply decrease despite the high public debt and high public deficit.

At the end of the war, the french public deficit as a percentage of GDP amounted to 50% on average. This is the direct consequence of the war and the occupation costs imposed by the Nazis. This exploitation is a "triple suction pump" ("triple pompe aspirante", Bouthillier) : firstly the franc was overevalued compared to the Purchasing Power Parity by about 40% (Margairaz, 1991) ; secondly the French administration should pay to the occupying forces 400 million per day ("occupation fees") ; thirdly, according to the clearing aggreement, the Treasory had to bear the responsability of the functioning of the payment system, which meant that the Treasory had to supply the account of the occupier with sufficient amount of unit of account so that it can "pay" its imports - notice that as long as the account is not liquidate, which will never be the case, this is strictly equivalent to a forced loan.

With such a high public debt and deficit, liquidity crisis would have been highly probable. Still, the financing of the debt seems to have been successful and rather smooth. Moreover the interest rate at which the State could borrow have never been so low. Margairaz reports that the 3 months interest rate was about 1.5% during the war and the 1-year interest rate was 2.25% while they were respectively 4% and 5% during WWI. As reported by Lutfalla et al (1986), the long-term interest rate was stable round 3.5%. Of course, this seeming stability must not hide the rapid institutional reforms that have made it possible.

Figure 16: Financial Indicators between 1939 and 1944

	1939	1940	1941	1942	1943	1944
Deficit (in % GDP)	87 (21.1)	212 (47.5)	183 (44.2)	193 (43.6)	296 (56.7)	283 (52.5)
Financed through Long-Term debt	5		10	14	49	137
Monetary financing*	44.3	124.3	91.3	117.2	153.9	62.6
Treasury bills (CDC**)	37.7	77.7 (63)	81.7 (69.4)	61.8 (48)	93 (82)	83.4 (69)
M2 (growth)	255.4	370.6 (45.1)	447.4 (20.8)	589.3 (31.8)	741.6 (25.7)	847 (14.3)
Inflation		+17.3	+17.5	+20.3	+24.2	+76.9
Interest Rate	4.97	4.80	3.60	3.45	3.59	3.39

*Source : Patat. J-P and M. Lutfalla (1986) ; Blancheton B. (2001) ; * Advances of the Bank of France, Treasury bills bought by banks, and "comptes chèques postaux" ; ** : Bills bought by "la Caisse des Dépôts et des Consignations"*

We would like to argue that the absence of major liquidity crisis and the rather smooth financing conditions during and after the war in France are mainly due to the institutional characteristics of the time that had been inherited from past changes in the financial system and in the relation of the Central Bank with the State. The key dates for these turning points are 1914, 1936, 1940 and 1945. In 1914, the Bank of France massively intervened to support the State's finance during the war. In 1936, the status of the Bank of France were modified by the Front Populaire ; it reinforced the control of the government and of the public over the Bank. In 1940 and 1941, Bouthillier, the secretary of finance, set up the main devices that would constitute the financial policy of the Treasory until the end of the war and that would influence the after-war institutions (Margairaz, 1991). In order to avoid what they called at the time "a leak in the circuit", the Treasury implemented capital and exchange controls and a strict regulation of the Stock Exchanges ; it controled the private banks through the supervision of a new comity gathering important bankers - the Permanent Comity of Organization - and the Commission for the Control of Banks - directly run by the minister of finance, the director of the Bank of France and the president of the Comity of Organization - which urged them to buy Treasury bills on the money market ; it funded most of the deficit by short-term Tresury bills mainly bought by banks - 40% of the total deficit during the war is funded through bills, about one third is funded by advances from the Bank and the rest by taxes. Claims of public institutions amounted to 1/3 of the banks' portfolio in 1939, it rises to 4/5 in april 1942 until 9/10 at the end of 1943. The ceiling on deposits in savings bank are lifted at the beginning of the war and the interest rate on this product increased in order to give people the incentive to let their money on their account - recall that a large part of the deposits collected in savings bank were used to buy public debt. Although they may have also served other purposes, these institutional reforms of the financial system ensured that capital could not move easily and was guided towards - if not forced to buy - sovereign bonds.

Regarding the relationship between the Treasury and the Bank of France, the latter became, once again, the Bank of the State at war (Margairaz, 2002). The advances of the Bank to the State increased from less than 100 billion to 500 billion at the end of 1944 ; the stock of bank notes increased in parallel from less than 200 billion to more than 600 billion. The rise in the size of the balance sheet of the Bank is large from the beginning of the war in june 1940 to may 1941 - it has more than dubbled in a year - ; it went on rising but at a slower pace in the following year until mid-1942. From the beginning of the year 1942, it seems that, what they called, the "closing of the circuit" is made more and more difficult due to the return of Laval, the greater submission of the government of the Vichy France and the reinforcement of the exploitation of the occupied territories by the Nazis after the failure in the URSS and the starting of the war economy. The issuance of Treasury bills was not enough to finance the deficit and the Bank of France had to intervene more strongly to close the financing gap. Several times the Bank of France, in addition to its advances to the State, intervened on the open market in period of stress, in particular during the monetary tensions in september 1942 and in september 1943 after the bumbing of Nantes and the starting of a run on bank deposits. It is clear from this description that the monetary policy was liquidity-passive.

The greater control over capital movement reinforced by the liquidity-passive monetary policy served the same purpose : it ensured the closing of the circuit and the smooth financing of the public debt. Moreoer the "war economy" had not only a financial and monetary translation with the mobilization of all national ressources towards the war effort, but it had also shaped the institutions of the after-war period. The war economy had not entirely ended with the end of war. The French financial system has been permanently affected by the transformations that took place since the inter-war period. It is possible to classify the institutional features that characterized the after-war financial system - as far as the funding of public debt is concerned - into four categories :

- Relation with the Central Bank : since the shock in monetary policy in 1914, with the reforms of 1936 and the nationalization of 1945 the relationships between the State and the Bank of France have changed. Although the government respects, in practice and most of the time, the decisions made by the head of the Bank, the control of the government on the Bank is greater. The monetary policy is now a governmental task and the Bank is in charged of the implementation of the policy and not its definition.

- The Treasury becomes a banker of the economy through the ability it has to issue francs and the management of individuals accounts ("Compte chèques-postaux";"Comptables du Trésor"). As a deposit manager, the Treasury can use the liquid deposits to fund long-term debt, exactly like a bank.

- Some public, semi-public or even private entities, like the Savings Banks (Caisses d'Epargne), the Bank of Deposits (La Banque des Dépôts et des Consignations) or the SNCF, have to deposit their liquidity on their account in the book of the Treasury : those are the "correspondants au Trésor". They therefore provide quasi-free liquidity to the State because of their specific relationships with it. In the framework of our model, these investors could be seen as benevolent or at least myopic and non-strategic investors.

- Private banks has to hold compulsory public liquidity requirement either in the form of central bank liquidity or in the form of public debt.

- The interest rates are regulated.

In terms of our model, these four features that more or less directly inherit the characteristics of the Vichy France translates into four channels of action on liquidity availibity :

- The monetary policy is characterized by a lower degree of solvability-activity and liquidity-activity due to the new relations it has with the State and the submission to the Treasury. Since the monetary policy is designed by the Treasury, this means that the rate of preference that governs the optimal arbitrage is the one of the State, probably lower than the one of the CB before the war. Therefore, the new relations translate into a lower θ' and a higher availibility of CB liquidity to support the public debt market.

- The dependence on private external funds is reduced due to the decrease in the proportion of the debt that must be financed by private agents. Indeed, the State has now a direct access to liquidity since it has become a banker through the CCP or an indirect access to it through the "correspondants au Trésor".
- As explained above the new obligations that financial institutions have to hold public debt leads to a reduction in the mobility of capital and to a reduction of the vicious self-fulfilling dynamics.
- Since the interest rates are administratively regulated, they are by definition no longer determined by the market. This also may contribute to the stability of the financing.

These four channels are the properties of what one used to call the "Treasury Circuit" (le Circuit du Trésor). Consistently with our toy model, these four properties are also the ones that enabled the government to finance a very high public debt without experiencing any liquidity crisis and at a rather low interest rate - the real interest rate was even negative after the war for some periods. Notice also that the inflation rate was higher than in the previous and following decades which is also consistent with the new relationships between the CB and the State. As showed in Figure 18, the State never resorted to long-term debt issuance for more than 60% of the annual deficit. During some years - like 1955 or 1957 - the sum of the bank requirements, the liquidity brought by the "correspondants", the advances of the Bank of France and the CCP accounted for more than 80% of the financing of the deficit.

Figure 17: Financial Indicators between 1945 and 1960

	1945	1948	1951	1954	1957	1960
Deficit (bil.)	-311	-554	-399	-346.5	-655.1	-420
M2	1300	2191	3775	5465	7 535	105.8
Inflation	+48.5	+58.7	+16.2	+0.4	+ 3	+3.6
Advances of the BoF*	+ 445	+ 283	-57	+151	+ 233	+ 75
Interest Rate	4.3	5.86	7.02	6.63	7.1	5.5

*Source : Patat. J-P and M. Lutfalla (1986) ; * is the cumulative change over the last three years*

However it should be noticed that a rigorous assessment of the role of liquidity in the stability of financing of the public debt must not neglect the expectations about the sustainability of the public debt. Indeed in the framework of the model, a liquidity crisis hits when the signal about solvency is low enough. Therefore one could argue that the stability of financing could be simply due to optimistic expectations on the part of investors. To be clear, we don't think that the expectations especially during and just after the war were particularly good. *Ex post*, it seems obvious that France should be

Figure 18: "Le Circuit du Trésor"

	1952	1955	1956	1957	1958
Deficit (bil.)	706	626	938	1019	690
Long Term Bonds	215 (30.4)**	80 (12.8)	383 (40.8)	84 (8.2)	294 (42.6)
Priviledged Bonds*	71 (10)	149 (23.8)	74 (7.9)	110 (10.8)	114 (16.5)
Deposits of "correspondants"	204 (28.9)	391 (62.4)	295 (31.4)	226 (22.2)	185 (26.8)
Banks' purchases (legal requirement)	63 (8.9)	42 (6.7)	52 (5.5)	39 (3.8)	-5 (-0.7)
CCP, coins	40 (5.7)	94 (15)	125 (13.3)	78 (7.6)	107 (15.5)
Bank of France purchases or advances	113 (16)	-130 (-20.7)	9 (1)	482 (47.3)	-5 (-0.7)

*Source : Patat. J-P and M. Lutfalla (1986) ; * bonds with a specific fiscal regime ; ** as a percentage of the first line (deficit).*

liberated and even more obvious that the French economy should grow at 5% *per* year on average during the 30 years following the war. But at the time and *ex ante*, it was everything but obvious. Consequently we don't think that "good perspective" argument is sustainable.

For 40 years, reforms to liberalize financial markets have completely changed the structure of the financial system. Those reforms have restored the institutional conditions of possibility of liquidity crises. Three main processes have affected the features that characterized the "Treasury circuit" :

- Independence of the Central Bank : since 1993 the Bank of France is independent, since 1999 the European Central Bank has been created. This implies a higher degree of solvability-activity and the freedom, on the part of the Bank, to define itself its optimal degree of liquidity-activity - which obviously is higher than before the independence.
- Privatization : many of the "correspondants" have been privatized (La Banque des Dépôts et des Consignations, La Poste) so that they no longer provide the Treasury with free flows of liquidity. This modifies the second channel we had identified and increases the reliance on private funds.
- Deregulation : the interest rates are now equilibrium rates determined by the free forces of the markets. Deregulation also concerns the decrease in liquidity requirement either in public debt or in central bank liquidity. This modifies the third and fourth channels we had identified. It increases the discretion the financial institutions have to hold public debt ; and reopens the door to strategic and self-fulfilling behaviors.

The recent crisis and the increase in public debt have upset the previous consensus and according to Reinhardt (Reinhart, 2012), we may return to "financial repression" (Mac Kinnon, 1973). She identifies two main signs of a return to financial repression :

- Negative real interest rate and the massive purchases of public debts by Central Banks
- New macroprudential policies that require banks to hold a given proportion of their portfolio in public bonds.

The marketization and standardization of public debt instruments since the 1980s may help explain the higher number of self-fulfilling sovereign crisis. During the 1980s and 1990s the way the States in developing countries financed their debt have changed : in the 1960s and 1970s, the main private channel of financing was through loans made by syndicates of coordinated international banks ; in the 1990s and 2000s, public debt instruments have been standardized, issued on international markets and bought by a possibly very high number of anonymous private investors. This has clearly increased the liquidity of debt instruments for creditors but it may have also created the conditions of possibility of coordination failures and therefore self-fulfilling runs. In the 1960s, the international banks could not rapidly liquidate their position on the indebted States since these instruments were neither marketable nor standardized ; moreover the syndicate was designed in order to coordinate its members thus making a coordination failure unlikely to occur. On the contrary, the high mobility of capital for 30 years, has made the liquidation of position very easy ; but it has also made the coordination between market participants more difficult to sustain because of the higher number of participants and because of the higher liquidity of debt intruments. Once again, the prediction of the model seems to be verified : the increase in the number of self-fulfilling crisis for 25 years may be partly attributed to this marketization and democratization of international sovereign debt markets.

5 The Eurozone Crisis and the European Central Bank : an econometric study

The "crisis" of the European sovereign debts (2010-2012?) raises a sovereign rate puzzle. As we will show, and as it is well established by the recent literature, no reasonable econometric models of sovereign interest rates can account for the sharp increase in 2011 and 2012. In the same way, while the Anglo-Saxon countries are characterized by fundamentals similar or sometimes significantly worse than the countries of the south of the Eurozone, they pay the lowest interest rates in the world. How can we understand this situation? Different explanations have been proposed, although none of them rely on a rigorous model. De Grauwe *et al.* (2012) argue that the high interest rate are related to self-fulfilling dynamics. Di Cesare *et al.*(2012) argue that it is due to the expectation that the Eurozone could break up. Steinkamp *et al.* (2012) propose an alternative explanation based on the seniority of the Central Bank over private investors. We propose a very simple explanation: the Central Banks don't have the same degree of commitment to intervene on the market of the sovereign debts due to differences in the rates of preference for price stability, which gave rise in the Eurozone to a sharp increase in the sovereign yields of fragile countries to compensate investors for the risk of self-fulfilling run on the public debt as modeled in the first part.

In order to disentangle between different interpretations, we use an original identification strategy. We use three announcements of the ECB that have led to a greater commitment to act as a Lender of Last Resort (LLR) in the summer 2012 to identify the nature of the crisis. We first get from our toy model the vector of predicted theoretical changes of a set of key variables when the CB announces to commit to act as a LLR. We then compute the changes in the data by running a event-study analysis. The comparision between the different vector of theoretical changes and the vector implied by the event-study analysis allows to disentangle between different interpretations.

What is key is an event-study analysis is the window over which one compute the changes in the selected variables. The smaller the window the more convincing is the assumption that the policy announcement is the driving force of the changes in the variables. But a small window doesn't allow to capture cumulative effect, if for example the information is slowly processed and incorporated into prices. That's the reason why we first run a pure event-study analysis focusing on a very short window (1 to 4 days) and then turn to an event-study over the quarterly changes in the residuals of a regressions that aims at controlling for changes in fundamentals.

5.1 Different interpretations and associated predicted effects of the commitment of the CB to intervene

The theoretical model developed in the first part provides us with a rigorous modeling of different interpretations of the nature of the Eurozone crisis. These interpretations can be split into two broad categories: the first one can be labelled "fundamental" and the other "liquidity".

Within the category of the "fundamental" interpretations, we can distinguish :

1. The Fundamentalist explanation.

   The equilibrium is a financing equilibrium with low interest rate as defined in section 3.1.2. The interest rate only reflects the default risk - or more precisely the interest rate that makes compatible the risk premium and the credit risk itself implied by the interest rate. The increase in the spreads is only caused by the deterioration in fundamentals. As described in section 3., the intervention of the Central Bank does not change the equilibrium outcome in this situation.

2. The fundamental Multiple equilibria.

   The equilibrum is a financing equilibrium but with high interest rates while there exists another equilibrium with lower interest rates and lower risk. The risk premium - the spreads - reflects a vicious circle between the dynamic of public debt, the risk of default and the expectation of markets participants. If the markets participants expect the interest rate to be high, a vicious snow-ball effect is triggered which increases the default risk which justifies the high risk premium. The increase in the spreads is the result of a jump from a low interest / low risk equilibrium to a high interest / high risk equilibrium. As stressed in the first part of the dissertation, theoritically, this interpretation is problematic since the state should be able to select the "good" equilibrium" (Chamon, 2007). Notice also that although the intervention of the Central Bank as a Lender of Last Resort (LLR) would not change anything, a Lender of First Resort (Cohen and Portes, 2006) would help to select the "good" equilibrium.

   While the first category of interpretations are fondamental in the sense that liquidity does not matter - i.e. the flow of financing is not interrupted in equilibrium and the behaviors of investors are not affected by liquidity concerns and the commitment of the ECB to act as a Lender of *Last* Resort doesn't change the outcome of the game, although it can help to select a low equilibrium when acting as a Lender of *First* Resort -, the second category of interpretations is caracterized by a liquidity problem : in equilibrium there is not enough liquidity to refinance the debt or the interest rate also includes a liquidity premium. Within this category, one can distinguish two interpretations.

3. Banks hoarding liquidity

   The first one is the hoarding of liquidity by banks because of a fear of future liquidity difficulties. As explained in the theoretical part (3.1), the commitment on the part of the Central Bank to provide liquidity to the banks can remove this liquidity-hoarding behavior. Should this situation be the true one, the additional commitment to act as a Lender of Last Resort to the State should not change anything to the equilibrium yield.

4. Self-fulfilling run on public debt

   The second interpretation is a pure "run on the public debt" because of strategic complementarities - if everybody run, the State will not be able to rollover its debt and will cease its payments, which justifies the run. As described in the theoretical part, in such a situation, the premium not only reflects the solvability concerns but also the risk that the State could default because too many investors would have refused to roll-over the debt, themselves fearing that the State could fall short of liquidity, so on and so forth. The Central Bank can avoid this liquidity problem by commiting to intervene as a Lender of Last Resort of the State.

5. The exit of the euro interpretation / redenomination risk.

   According to this interpration of the situation which might be similar to the one President Mario Draghi had in mind in July, August and September 2012[9], a member of the Eurozone could exit and redominate its debt in a local currency. Following the standard Uncovered Interest Rate Parity, the interest rate should increase so as to compensate the investors for the expected depreciation of the currency[10]. Notice however that different versions of this view exist. We deal with this issue at the end of the dissertation.

Our identification strategy consists in comparing the change in variables over the period (day or quarter) during which the CB commits to act as a LLR as predicted by the theory to the actual changes. This is possible because the commitment of the Central Bank has different implications depending on the interpretation of the situation. The following covariance matrix summarizes the expected sign of the change in variables after the announcement of a commitment of the Central Bank to intervene as a LLR and LFR *of the State*[11]. The column "Expected Growth" allows to capture the effect of a default on the real economy and the loss in GDP as assumed previously. Recall that this assumption is consistent with the findings of Reinhardt and Rogoff (2010): a default is likely to imply a negative demand shock and a contraction of GDP. We denote the real interest rate by r, the nominal interest rate by i, the Risk of Default by RoD, the Nominal Exchange Rate by NER, the expected growth by g^e and the expected inflation by π^e.

[9]ECB Press Release, 2 August 2012 and also ECB Press Release, 6 September 2012
[10]We come back to this thorny issue in section 4.3.
[11]We stress to avoid the confusion with the commitment to act as a LLR of the banking system which doesn't have the same implications as stressed earlier.

	i	r	RoD	NER	g^e	π^e
Fondamentalist	(+)	.	.	(-)	.	(+)
fundamental Multiple Equilibria	-/(+)	-	-	(-)/+	+	(+)
Liquidity hoarding	.	.	.	.	.	.
Liquidity Crisis	-/(+)	-	-	(-)/+	+	+

Our toy model doesn't include the exchange rate so that one could argue that it is difficult to get clear predictions of the change in exchange rates. For the purpose of this section, we assume a consensual *ad hoc* determination of the exchange rate: $NER = f(\pi^e, g^e)$, where $f(.,.)$ is increasing in π^e and decreasing in g^e.

According to the Fundamentalist interpretation, the sovereign yields increased because investors required a higher premium for the risk of default implied by the degradation of the fundamentals - increase in fiscal deficit and in debt over GDP. There is no jump to a bad equilibrium and the economy stays on the "right" equilibrium. Our model predicts that the commitment to intervene and to buy bonds if necessary should not change anything to the situation. Nevertheless, because the model does not capture the complexity of reality, we leave open the possibility of an increase in expected inflation and consequently an increase in the nominal interest rate and a depreciation of the currency.

According to the fundamental Multiple Equilibria interpretation, the sovereign yields increased because the markets jumped from a low interest / low risk equilibrium to a high interest rate / high risk equilibrium. The commitment to intervene as a Lender of *First* Resort makes the economy jump from the "high interest rate - high risk" equilibrium to the "low equilibrium - low risk" equilibrium which makes the risk of default and the real interest rate decrease. The lowering of the risk of default makes the expected growth of GDP and the nominal exchange rate increase. Once again, even if our model doesn't predict any change in expected inflation, we let the possibility of an increase in expected inflation, a depreciation of the currency and an increase in the inflation premium.

According to the Liquidity Hoarding by Banks interpretation, the sovereign yields increased because banks hoarded liquidity to self-insur against a potential future liquidity shock. The commitment to intervene as a LLR of the State should not change anything since the Central Bank was, by assumption, already committed to act as a LLR of banks. The banks had already access to any amount of liquidity they needed in exchange for public bonds[12]. Therefore, the additional announcement should not change anything.

According to the Liquidity-Run on Public Debt" interpretation, the sovereign yields increased because investors feared that others investors would run which would make the State default, since it had no access to external liquidity. The commitment of the CB should change everything since it kills the strategic complementarities between private investors. Consequently we should see a decrease in the risk of default, a decrease in the real

[12]This supply of liquidity to banks was reinforced by the settling of the LTROs in the beginning of 2012.

interest rate, an increase in expected inflation - see the section on the inflation/stability trade-off for more details - and an increase in expected growth and nominal exchange rate.

The theoretical part suggests that there are more than one way to conceptualize a liquidity crisis depending on the information structure. Broadly speaking there are two main ways corresponding respectively to the setups (5) (and (1)) and (9). In the former the commitment of the Central Bank is a free lunch - the CB doesn't have to effectively buy bonds - and the risk of inflation is zero as long as its threshold of intervention is not lower than those on the markets. In the latter given the heterogeneous distribution of information, the CB always has to intervene - more or less - to buy part of the debt in order to clear the market, which could give rise to inflation if the bonds turn out to be worthless. The two main indicators that could enable us to distinguish between these two sub-interpretations of a liquidity-crisis are:

1. The share of the debt that the CB has to buy. If it is zero or one, then it supports the multiple equilibria - free lunch hypothesis. If it is a small positive number, it supports the second subinterpretation.
2. The change in the inflation expectations. This is a direct consequence of the first point: if the CB didn't have to intervene the expectations should not rise above the long term anchor - 2% in the Eurozone for example. If it has to intervene, like in the second interpretation, the inflation expectations should incorporate the positive probability of inflation above the long term anchor.

Two others explanations have been proposed. One is the seniority of the CB which implies that when the CB buys sovereign bonds it increased the riskiness of the bonds remaining in private hands which could have contributed to the increase in the sovereign yields (Steinkamp *et al.*, 2012). The other, much more difficult to deal with, is the euro breack-up hypothesis. We discuss the latter in a specific section.

Finally it is not possible to easily distinguish the "fundamental Multiple Equilibria" interpretation from the "Liquidity Crisis" interpretation since they have the same co-variance vector. There are two ways to deal with this ambiguity: either we exclude the fundamental Multiple Equilibria based on the theoritical argument according to which the State should be able to select the "good" equilibrium (Chamon, 2007) or we try to empirically distinguish between commitments to act as a LLR and commitments to act as a LFR since theoritically only the LFR can have an impact on the variables. This issue will come back later, but it will turn out that we will not be able to disentangle the two, unless we resort to the first and theoretical argument.

5.2 Announcements in the Summer 2012

The recent events in summer 2012 in the eurozone give us the opportunity to observe and study a situation arguably close to a quasi-natural experiment. It is obviously not a natural experiment strictly speaking since the market already incorporated the possibility that the ECB could react. However, at that time the policy of the ECB was anything but certain and perfectly forecastable and the announcements were welcome with surprise.

During the summer 2012 the ECB announced the OMT program and therefore increased its degree of commitment to act as a Lender of Last Resort to the State. In order to study the change in the variables the days of announcements by the ECB, we now turn to an event-study analysis. We consider the three more recent dates that has led to a stronger commitment to intervene on the secondary market of sovereign bonds (OMT). The content of the announcements are the following :

1. 26/07 : "The ECB is ready to do whatever it takes to preserve the euro."

   In a speech at the Global Investment Conference in London, President Mario Draghi said : "Within our mandate, the ECB is ready to do whatever it takes to preserve the euro." He added : "And believe me, it will be enough" and later "We think the euro is irreversible". While there was no concrete announcement, this announcement could be seen (and was perceived as such by financial markets) as a credible signal that the ECB was planning to restart the SMP. Indeed, it also said that "to the extent that the size of the sovereign premia hamper the functioning of the monetary policy transmission channels, they come within our mandate"[13].

2. 02/08 : Bond Buying could be restarted.

   President Mario Draghi announced that the ECB was working on plans to buy sovereign bonds. Consequently the sovereign bond-buying may be restarted. Indeed, on 9 May 2010, the EU finance ministers had agreed to create the EFSF and EFSM and the ECB to "conduct outright interventions in the euro area public and private debt securities markets". Officially the reasons of such intervention (SMP, Securities Markets Programme) was aimed at "adress[ing] the malfunctionning of securities markets and restor[ing] appropriate monetary policy transmission mechanism" in order to maintain price stability[14]. Purchases of bonds under the SMP were limited. SMP was reactivated in August 2011 when Italian and Spanish bonds yields increased to more than 6%. In August 2012, the total amount of bonds bought held by the ECB (and to be held until maturity) was €209.5bn [15]. On the 2 August 2012, President Mario Draghi said that the bond-buying plans would be "very different"[16] from the SMP : it would impose "conditionality" on the governments and the "concerns of private investors about seniority will be addressed"[17].

[13]Verbatim of the remarks made by Mario Draghi, Speech by Mario Draghi at the Global Investment Conference in London, 26 July 2012

[14]ECB Press Release, 10 May 2010

[15]ECB Press conference, 2 August 2012

[16]*Ibid*

[17]*Ibid.*

3. 06/09 : Outright Monetary Transactions

   Like the SMP, the official means and goals of such announcement are on the one hand the outright purchases of sovereign bonds on secondary markets and on the other the "safeguard[ing of] the monetary policy transmission mechanism". However the technical features make the OMT differ from its predecessor :

   (a) Conditionality.

   "A necessary condition for OMT is strict and effective conditionality attached to an appropriate European Fiancial Stability Facility/European Stability Mechanism (EFSF/ESM) programme"[18]. The necessity of a broader agreement with the ESM ensures a strict monitoring by the Commission and the IMF which, supposedly, may limit the risk of moral hazard and the degradation of the quality of the assets of the public institutions (ECB and ESM).

   (b) "No ex ante quantitative limits"[19].

   While the SMP was always meant to be limited and temporary, the OMT is a clear commitment to act as a Lender of Last Resort which by definition must be ready (and able) to provide any amount of liquidity.

   (c) Up to 3 years.

   The ECB will target bonds with maturity less than three years so that this is consistent with the "relevant horizon for monetary transmission"[20].

   (d) Sterilization.

   The purchases under the OMT programme will be sterilized in order to avoid an increase in base money and possibly in inflation or in asset prices. One way to do it for the ECB is to give the banks the incentives to shift their liquid holdings into fixed-term deposits. However, some commentators stressed that this is a purely technical measures only aimed at reassuring the German public opinion since it only change very liquid holdings into less liquid but still liquid and safe holdings.

   (e) Seniority.

   Following the announcement on 2 August 2012, the Central bank gives up its specific treatment : "it [the Eurosystem] accepts the same (pari passu) treatment as private or other creditors". This aims at reassuring private bond holders and at avoiding the perverse effect stressed by some commentators (Steinkamp and Westermann, 2012) of an increase in the quantity of risk remaining in bonds held by private investors. This might have contributed to

[18] *Ibid.*

[19] ECB Press Release, 6 September 2012

[20] *Ibid.*

the increase in the default risk premium required by market participants.

5.3 Event-study Analysis

5.3.1 Literature review

Our work is related to two main strands in the literature. The first one tries to econometrically predict a "fundamental" yield for sovereign bond and to quantify the degree of mispricing. These papers use mainly macroeconomic variables, at low frequency and panel regression techniques. Borgy *et al.* (2012) estimate a no-arbitrage term structure model and find that fiscal fundamentals up to mid-2011 can explain the divergence in the yields. Using panel regression with fixed effect on a larger sample of countries, Aizenman et al. (2011) do find evidence of mispricing for eurozone countries both before and after the crisis compared to their current "fiscal space". However they suggest that it could be due to the expectation of further deterioration in fiscal variables in the future that are not captured by actual data. Poghosyan (2012) uses cointegration techniques and finds in a similar way that fundamentals cannot explain the premia. Fratzscher and Beirne (2012) use interpolated quaterly data to predict "equilibrium" values of bond yields and CDS spreads based on a set of fundamentals, a spillover variable and allowing for changes in the parameters after 2008. They find that the main drivers of the increase in bond yields are a deterioration in fundamentals and an increase in the sensitivity of markets to fundamentals ("wake-up call"). They find no clear evidence of mispricing. Finally De Grauwe et al (2012) using panel regression techniques on a sample of developed countries and comparision with stand-alone countries find evidence of a mispricing of sovereign debt since 2010 compared to fundamentals. They also find evidence of a non-linear impact of public debt. More recently, Di Cesare *et al.* (2012) find clear evidence of deviations from fundamentals and argue the perceived risk of a Euro Area break-up can account for it.

The other stream uses event-study analysis and analyzes the effect of monetary policy announcements on high frequency financial data. Our own work is very much related to the one of Krishnamurthy A. and A. Vissing-Jorgensen (2011) on the Quantitative Easing announcements of the Fed. Swanson (2011) studies the Operation Twist implemented in 1961 and find a significant effect of announcements on Treasury bond yields. Dell'Erba (2012) shows that rating downgrades have large and persistent effects on public and private bonds yields. Kilponen et al. (2012) find a significant impact of monetary policies and European crisis resolution policies on sovereign bond yields and on financial market stress. Santoalha (2012) finds that sovereign yields fell and inflation expectation didn't rise following the SMP announcement while the announcements of the Fed or the Bank of England have almost no effect on sovereign yields. Our work is the first one to our knowledge to try to assess the channels of the recent ECB announcements that have led to the OMT.

5.3.2 Event-study Methodology

The objective is to capture and measure the reaction of a price to a policy announcement. Originally the event-study analysis comes from the corporate finance literature that studies the asset price effect of investment and earnings announcements of firms. The event-study analysis is theoretically based on the efficient market and rational expectation hypothesis and therefore relies on the statistical assumtion that asset prices should follow a random walk. The work of Fama et al. (1969) is an important contribution in this literature. According to this theoretical and statistical framework, asset prices follow a random walk and the change in asset prices from one period to the next is unforecastable and must be a white noise.

More precisely, let's denote $R_{i,t}$ the market yield of a sovereign bond i at time t, $NR_{i,t}$ the "normal" yield and $AR_{i,t}$ the "abnormal" yield. The "normal" return is the expected return, which is by assumption the best forecast, conditional on the available information : $NR_{i,t} = E[R_{i,t}|X_t]$ with X_t the available information. The "abnormal" return is defined as the difference between the actual return and the "normal" return :

$$AR_{i,t} = R_{i,t} - NR_{i,t}$$

Under the efficient market and rational expectation hypotheses, the "abnormal" yield should be therefore a white noise. The event-study analysis is meant to identify the effect of policy announcements on "abnormal" returns and to test whether the effect is significant or not.

If we knew and could estimate the true model of sovereign yield - the model that captures where the yield should be according to the available information and market rationality - we would simply collect all the available data to get the best forecast of the value of $R_{i,t}$: $NR_{i,t} = E[R_{i,t}|X_t]$. Consequently we would be able to compute an estimate of the abnormal return which could be attributed to the Central Bank announcement. But since we ignore this true model - if such a model exists - we must rely on assumptions.

The identifcation assumption is that the ECB announcement is the main shock within the window. The important point is to restrict our attention to a short enough window so as to be able to attribute the change in yield mainly to the announcement and not to another exogeneous shock. Consequently, the identification procedure is the following :

- Random Walk Hypothesis

  We follow the random walk hypothesis :

  $$E[R_{i,t}|X_{t-1}] = R_{i,t-1}$$

  so that

  $$AR_{i,t} = R_{i,t} - E[R_{i,t}|X_{t-1}] = R_{i,t} - R_{i,t-1} \sim i.i.d(0, \sigma)$$

.

- Set a window

  We choose a short enough window, η, so as for the following statement not to be too heroic : there is no other major shock within the window, η, apart from the policy announcement, that could also affect the yield. So that we could assume that the announcement is the dominant shock from t to $t+\eta$. Typically η is 2 days. But there is no rule. It mainly depends on the data availibility and on the model of the speed of information incorporation into asset price one has in mind. In Dell'Erba (2012), the window lasts 21 days. In Bernanke et al. (2004) it lasts one hour. We will do the analysis with four different windows from 1 to 4 days.

- Compute "abnormal" yields and run significance tests.

  We compute the change in yields over the window, $AR_{i,t,\eta}$, and, assuming $AR_{i,t,\eta} \sim \mathcal{N} i.i.d(0, \sigma_\eta)$, we test whether the change is significantly different from 0, positive or negative depending on the null hypothesis one wants to test.

5.3.3 Channels of transmission

We could have focused only on the set of financial variables that are necessary to the strict identification strategy. However we take the advantage of this occasion, to provide a broader analysis of the channels of the monetary policy announcements.

Following the recent literature, we distinguish different channels by which the ECB announcements can influence the markets of sovereign debt.

1. Signaling Effect or Expectational channel.

This channels predicts a decrease in bond yield via the anticipation of a decrease of future main refinancing rates. This channel has been stressed by Eggertson and Woodford (2003) who argued that unconventional policies can have an effect on long-term interest rates only to the extent that they act as credible commitment to lower future policy rates. Clouse et al. (2000) completed the argument by adding that a credible way to fulfill such a commitment is for the Central Bank to buy large amount of long term assets so that a future increase in policy rates would impose losses on the Bank balance sheet[21]. There is however no consensus on the last argument : there are many others ways the Central Bank can anchor expectations about future policy rates and moreover the ability of Central Banks to forecast future policy rates is almost null (Goodhart and

[21] Krishnamurthy and Vissing-Jorgensen, "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy", 2011

Wen Bin Lim, 2011 ; Goodhart and Rochet, 2011).

The identification strategy consists in studying the effect of the announcements on Forward Rate Agreement (FRA) at different maturities. These contracts are used to hedge future interest rate exposures and determine the rate to be paid on an obligation starting at a future date. The counterparties agree at date 0 on the interest rate that the buyer will have to pay to the seller on an obligation starting in j and maturing in j+t periods (the underlying asset has therefore a maturity of t units of time). According to the expectational view, the FRA should reflect the expected path of policy rate plus a term correcting for risk aversion. The change in FRA at different start dates and different maturities the day of ECB's announcements shoud therefore be attributed either to change in the risk aversion parameter or to the change in expectation about the future path of policy rate because the money market rate is mainly driven by the policy rate. The identification assumption is thus that the risk aversion term is not affected by the announcement so that the change the day of the announcement only reflects changes in expectations.

2. Portfolio Rebalancing Effect

This effect is based on a classical demand-supply model with imperfect sustituability between assets. The imperfect substuability argument may be based on a "preferred habitat demand" (Vayanos and Vila, 2009). According to this channel, the announcement of the ECB to buy bonds of maturity shorter than 3 years should increase the spread between the bond whose maturity is lower than 3 years compared to those whose maturity is longer than 3 years.

We will therefore study the change in the spread between bonds of maturities longer than 3 years (4 years, 5 years) and bonds with shorter maturities (2 years and 3 years). The key assumption here is that all others factors affecting the slope of the yield curve between 3 and 4 years remain constant. In particular, according to the expectational view, the expectation that the policy rate could remain low longer than expected (especially beyond the 3-years horizon) should tend to make the yield curve flatter ; therefore reducing the spread between 3 years and 4 years bond yields. Consequently, one should pay attention to the interaction between these two channels.

3. Inflation Effect and internal value of money

According to this channel, an increase in the commitment to intervene increases the likelihood of inflation and decreases the likelihood of deflation. As we have already presented above, a stronger commitment means a decrease in the risk of deflation and recession. Moreover this increase in the probability that the Centrak Bank has to intervene makes the inflation rate likely to increase according to either the quantitative theory of the price level or the fiscal theory of the price level - either the pure fiscal theory or the generalized fiscal theory of the price level (Leeper , 1991 ; Sims, 1994 ;

Woordford, 1994, 1995, 2001). Consequently the channel predicts an increase in expected inflation and therefore an increase in the nominal yield - for a given real rate.

To study the effect on inflation expectations, we will analyse the change in Inflation Indexed Swap. In a inflation swap agreement, the "inflation payer", that bears the risk, pays the realised inflation over the period of the swap and receives the inflation swap rate from the "inflation receiver". This is considered as a unbiased measure of inflation expectation over the medium run. Because of data availibity, we could not analyse the effect on inflation-indexed bond yield which is the second market-based measure of inflation expectation. Because of their low frequency, we can't apply the event-study analysis to the inflation expectations as measured by the Consensus survey. The advantages of inflation swap rates is that they are available at high frequency for a wide range of tenors (up to 30 years). They are not subject to liquidity premia but can incorporate counterparties risk and time-varying inflation risk premia if, for instance, volatility in realised inflation is expected to increase (Devlin et al., 2012). Consequently our identification assumptions are that counterparty and volatility in inflation risk are kept constant.

3. Devaluation Effect and External value of money

The CB announcement affects the exchange rate through different channels. According to the PPP, the external value of money is exactly the mirror of the internal value of money ; therefore, an increase in inflation expectations should directly translate into a depreciation of the currency. Moreover according to the UIP, the exchange rate should depreciate if one expect the policy rates to decrease in the future. Finally, the perspective of a default and a recession makes the exchange rate depreciate ; on the contrary when the risk of a default is removed this should makes the exchange rate appreciate. The final effect depends on the combination of these three channels.

4. Default Risk

The stronger commitment to act as a Lender of Last Resort may imply an improvement in the risk of default. As underlined previously, there are two main theoretical reasons - related to the liquidity and the solvency dimensions of the crisis - to which this can effectively be attributed :

- Either the stronger commitment to intervene makes the economy jump from a high-risk high-interest rate equilibrium to a low-risk low-interest rate equilibrium.
- Or the stronger commitment kills the run on the public debt and the self-fulfilling crisis, which makes the risk of default decrease.

We measure the risk of default with the Credit Default Swap (CDS) spreads. A CDS is a swap in which the protection buyer pays on a regular basis a premium, called "spread", to the protection seller. In the event of default, the latter must give to the former usually the face value of the underlying bond. In theory, CDS spread and bond yield spread should be equal. Consider an investor that has a long position in a bond with yield i_t. If it buys a protection against the risk of default, it will pay a premium of cds_t. Therefore the return of the portfolio will be certain and equal to $i_t - cds_t$. By arbitrage, this must be equal to the risk-free rate : $i_t - cds_t = i_{t,rf}$ which is equivalent to $i_t - i_{t,rf} = cds_t$.

However, there is limit to arbitrage. The higher liquidity of the CDS market is often mentionned to explain this lack of arbitrage. Some papers try to analyse the links between the two markets and to determine which is the leader in the price discovery process. Coudert and Gex (2012) showed that the CDS market leads both for corporate and sovereign bonds. It therefore seems that the information is more rapidly incorporated in the CDS spread than in the bond spread. Apparently, we should focus on the CDS spread in order to study the default risk channel.

Nevertheless our own results for sovereign bonds spread seems to contradict their conclusions. Indeed, as shown in appendix, we have run a Vector Error Correction Model (VECM), as standard in this literature (Coudert and Gex, 2012 ; ECB 2004), with CDS and bond spreads for each country of the eurozone and we have found that the bond spread unambiguously leads. This shows, at least, that it is not so clear that the CDS market leads as it is often believed. In spite of the absence of a consensus on the best way to measure the risk of default, we will use 10-years and 5-years CDS spreads, since those are the most traded maturities, relative to a benchmark (German or US sovereign CDS) in order to control for global risk aversion changes.

Notice finally that some default risk are not priced in the CDS. The Greek "default" revealed that a State could force investors to bear losses on their bond holdings without triggering a credit-event and to activate the insurance provided by the CDS contracts. We provide evidence of such hidden credit risk in appendix 4.

Finally in order to study the effect on the "real" economy, we will also study the impact of the announcements on the average 5-years CDS spreads relative to a benchmark (US sovereign CDS) of large private companies in each country. This will allows to capture the change in perceived default risk of private companies and indirectly the expected growth of the economy. Because of data availibility we restrict our attention to Germany, France, Spain and Italy.

6. Price of Risk

In most models, the risk aversion is an exogeneous parameter. In the model sketched in the first part, for instance, we assumed that investors were risk neutral. By contrast, some recent models endogenize it. The intervention of the Central Bank could have an positive impact on risk-taking behavior by decreasing the risk aversion and the price of risk (Dubecq et al., 2009). This should decrease the risk premium asked by financial markets and decrease the sovereign bond yields. There is still no consensus - and even no clear theory of the mechanism - on this channel but the empirical evidence seems to be numerous (Bekaert et al., 2011 for the VIX ; Alturbas et al, 2010 ; Jimenez et al., 2009 ; Maddaloni et al., 2010 for the behavior of banks). Bekaert et al (2011) shows that a lax monetary policy predicts a decrease in risk aversion and in the price of risk after about five months.

The main issue at this stage is the absence of consensus on how to capture risk aversion (see Coudert and Gex (2008) for a survey of the measurement). In absence of a consensus, we focus on a popular measure of the implied volatility of S&P 500 index options: the Chicago Board Options Exhcnage Market Volatility Index alias the VIX. We could also have studied the credit spread, the yield curve, the equity risk premium or an composite indices of different variables. As it has been stressed (Bekaert et al. 2011), the VIX is both a measure of risk aversion and of uncertainty so that we don't really know if the change in the VIX is due to the change in the first factor or in the second. The attempt to decompose the index into those two components however suggests that the two are very much correlated.

Remarks on the new channels highlighted by the model.

- Information revealing effect

  According to this channel and as shown in section 2.3, the announcement to intervene reveals Central Bank private information to the private investors, which is supposedly good - otherwise the Central Bank would not commit. Since the investors are bayesian, they will update their beliefs and this should be reflected in a improvement in expectations about determinants of debt sustainability (growth of debt, deficit, growth of GDP). However, it is hard, if not impossible, to identify this channel and distinguish it from the others - default risk for example. This effect predicts an improvement and reduction in uncertainty of the expectations of future economic conditions.

- Self-fulfilling run on public debt and/or jump to a low-risk/low-interest rate equilibrium.

  These channels have been already presented in the "default risk" section but we would like to stress the precise mechanism by which the CB announcement cna affect the risk of default according to our model.

  As we have shown in our toy model, by commiting to act as a LLR the Central Bank can avoid a self-fulfilling liquidity crisis to occur. In such a situation, the announcement of the Central Bank works through the expectation of market participants : since everybody knows that the Central Bank would intervene if needed, the self-fulfilling dimension (the fear that the State could encounter a lack of liquidity because the other participants would not have roll over their claim) is eliminated. The interest rate only reflects fundamentals variables and no longer fear of a default caused by a lack of liquidity. Therefore this channels predicts a decrease in the probability of default. However, as stressed earlier, a second interpretation, in terms of multiple fundamental equilibria, is also possible. In this interpretation, only a commitment to act a Lender of First Resort can make the economy jump to a low-risk/low interest rate equilibrium.

5.3.4 Results

The event-study analysis leads us to compute the change in the forward rates of different maturities and reference dates, in the difference between the 4-months and 3-months sovereign bond yields, in the inflation swaps from 1 to 30 years, in the exchange rate of the euro against nine currencies of developed countries [22], in the bond yields of different maturities, in the difference between the sovereign CDS and the US sovereign CDS at 5-years and 10-years maturity, and in the average of CDS of private and national corporations based on available information.

The countries in the sample are Austria, Belgium, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal and Spain. When information is available, we also study Malta, Cyprus, Slovekia and Slovenia. We run the analysis with four different windows from 1 day to 4 days and with two different starting date: firstly the day of announcement and secondly the day before. All the series come from Datastream. The VIX is taken from the website of the Chicago Board Options Exchange [23]. For space reasons, we don't report all the results. For illustrative purposes, we report the table for the 5-years sovereign CDS in appendix 3.

- Signaling Effect or Expectational channel

  There is clear evidence that the first announcement on July 26 made the forward rates decrease. However, although significant at the 10% level, it remains small. It affected all forward rates agreement whose maturity is less than 15 months. We observe no change in the contracts maturing more than 15 months after the announcement, suggesting that the investors updated - downward - their expectation about the path of future policy interest rates but not after a horizon of 15 months. The other announcements had no significant impact of forward rates. We thus conclude that only the first announcement had a significant effect and that it decreased the expected future policy rates at least up at a 15 months horizon.

[22]Dollar, Canadian Dollar, Pound Sterling, HK Dollar, Dennish Krona, Swedish Krona, Swiss Franc, Australian Dollar, Japanese Yen

[23]www.cboe.com

- Portfolio Rebalancing Effect

  Focusing only on September 6, which is the day of announcement of the OMT program, most countries remained unaffected, except Ireland which experienced a large positive change in the spread, as predicted, and Spain which experienced a large decrease in the spread - which is more surprising. As stressed earlier this could be due to the interference with others determinants of the yield curve, but we don't have convincing and rigorous explanation of this surprising result. This suggests that, overall and on average, there is no clear evidence of a rebalancing effect.

- Inflation Effect

  The announcement on July 26 had a significant positive impact on inflation expectations in the long-run - at a horizon longer than 6 years - but not in the short-run. On the contrary, the announcement of August 2 seems to have had a significant positive impact on short term expected inflation, but not in the long-run. The announcement on September 6 had a significant and positive impact at all maturities.

  This difference between the first two dates may be attributed to the nature of the announcement. While the first one was clearly related to the Eurozone problem the second one was partly related to the conventional monetary policy since investors learnt that the policy interest rate would remain unchanged at 0.75%. Probably the short horizon of the conventional monetary policy, compared to the commitment to stabilize the euro which spreads on a longer time horizon can account for this difference.

- Exchange Rate

  Due to the interaction of different effects, the bilateral exchange rates didn't exhibit extreme movement in one direction or in another. We find a significant appreciation at the 10% level, sometimes at the 5% level, only for the bilateral exchange rates against one fourth of the sample (Hong-Kong and the US dollars) on July 26 and against three fourth of the sample on September 6 (HK dollar, UK pound, US dollar, Swiss franc, Yen). We don't observe any significant depreciation of the euro after the announcement except for the euro-swedish krona rate on September 26.

  As already mentioned, several effects may interact and offset each other. In particular the decrease in expected policy rates and the increase in expected inflation should make the exchange rate decrease, while the decrease in CDS should make it increase. Overall, we don't find any depreciation of the euro, on the contrary we observe a small appreciation, in particular on September 6 which tends to suggest that the decrease in the probability of default had more than offset the increase in inflation expectations and the decrease in expected policy rates.

- Default Risk.

  Sovereign CDS spreads relative to US decreased significantly and substancially for Ireland, Spain, Cyprus, Italy and Portugal around the three days of announcement. For Germany, France, the Netherlands and Belgium we observe a significant but smaller decrease around the first and second announcement, but not around September 6 [24]. This tends to show that investors perceived a large decrease in the default risk of the States of the South of the Eurozone, but also in the "core countries" - which should probably be attributed to the spill-overs that a default would have had on these countries.

  As for the economy as a whole, we also find a very large and significant decline in average CDS of large companies in Spain, Italy and France for all announcements[25]. For Germany, the effect is smaller and significant only for the first announcement on September 26. This suggests that the investors' expectations on the future economic conditions of those countries significantly and largly improved, which is closely related to the fact that a sovereign default would have had a large negative impact on the national economy.

- Price of Risk

  There is no clear evidence that the price of risk as captured by the VIX decreased significantly at the dates of announcements. The analysis suggests that no significant change occured on July 26. On August 1 and September 6, only the two-days change is significantly negative at the 10% level. Overall, there is no evidence that the price of risk / risk aversion responded to the announcements.

5.3.5 Interpretation and partial conclusion

Overall, the ECB announcements had a significant and large impact, in the Southern countries, on the default risk of sovereign as well as on the default risk of a panel of large national corporations ; it also increases the expected inflation, which we interpret as a decrease in the risk of deflation ; it leads to a slight appreciation of the euro ;

[24]For Malta and Greece, we don't find any significant change around any of the three dates. This might be due to either measurement errors or the functioning of trading platforms for these countries - with possibly low liquidity and few transactions - or to the high standard deviation of the series.

[25]Portugal, Greece and Ireland among others are absent, not because they did not observe a decline in CDS spreads, but because we didn't have enough data to construct a meaningful aggregate measure of default risk.

and it decreases the forward rate which tends to suggest that investors revised their expectations about the future path of policy rates.

The data allow to assess the empirical validity of each of the several interpretations of the financial tensions that occurred in 2012. The data strongly reject the Fundamentalist as well as the liquidity hoarding interpretation. On the contrary it supports, without being able to distinguish them, both the multiple fundamental equilibria and the liquidity crisis interpretations.

	i	r	RoD	NER	g^e	π^e
Fondamentalist	(+)	.	.	(-)	.	(+)
fundamental Multiple Equilibria	-/(+)	-	-	(-)/+	+	(+)
Liquidity hoarding	.	.	.	.	.	.
Liquidity Crisis	-/(+)	-	-	(-)/+	+	+
Data	**-**	**-**	**-**	(+)	+	+

5.4 Panel Data Estimation of Equilibrium Bond Spreads, Inflation Expectations and Exchange Rates

5.4.1 Limits of the event-study analysis

The event-study analysis run relies on the crucial assumptions that asset prices follow a random walk. In particular, the new information released by the policy announcement should be incorporated in the price within the window. If this is not the case, the new information is progressively and slowly incorporated into the price. First this implies that the efficient market (EMH) and rational expectation hypothesis (REH) no longer hold since the future path of prices would be predictable and that there would exist huge gains from arbitrage ; second, this would mean that the 2-days window is a conservative assessment of the effect of the policy announcement.

In the data, we find very strong evidence that the random walk hypothesis doesn't hold, at least for the year 2012[26]. We find that it was possible to beat the market on average during the year 2012 by simply following the prediction of a AR(1) process for the first difference of bond yield. Indeed the MSE of out-of-sample of an AR(1) is on average 80% smaller than the one of a random walk (RM). This allows us to reject the RM hypothesis and consequently the theoretical hypothesis on which it was based (EMH and REH). Our evidence implies that there is momentum and persistence in change in bond price. A window of 2 days is therefore a conservative way of identifying the effect of policy announcement since the new information is progressively incorporated into prices.

Another limit of the event-analysis is that it only allows economists to capture the unexpected part of policy announcement and the surprise of financial markets. If

[26] See appendix 2.

financial markets perfectly anticipated the announcements, then nothing, significantly different from a noise, should happen the day of announcement. That's could also be an explanation why yields usually decrease the days before Board meetings. When this is true, the event-analysis doesn't provide us with a pure measure of the reaction of investors to a change in the degree of liquidity-passivity as we had implicitely assumed until now. The event-analysis gives a measure of the reaction to the unexpected part of the announcement. For this second reason, we could also consider a broader window that would start before the announcement (Dell'Erba, 2012).

Finally, the widenning of the window is also justified by the very fact that the three announcements we are studying are actually, viewed from *ex post*, part of the same policy process that has led to a greater commitment to intervene as a LLR for the State. Since these three announcements occurred at the beginning and at the end of the summer, it makes sense to consider the "summer 2012" as if it was a single period.

Nevertheless considering a wider window, either by starting before the announcement or by going further after the announcement, in order to capture the total cumulative effect has a cost : the key identification assumption that the policy announcement was the major shock within the window becomes more and more heroic as the length of the window expands. Therefore when incorporation of information takes time, there is a trade-off between capturing the total cumulative effect and identifying a pure announcement effect.

5.4.2 Identification assumptions and panel regression

We need a new identification assumption if we want to enlarge the window. The assumption that no other news during a quarter arise is no longer tenable. The world is now changing within the window. We need a new assumption that allows us to say that "all other things -a part from the monetary policy- are held constant". To build such a stationary world, we will assume that we control, in a econometric meaning, for the changes in the fundamental determinants of yields, inflation expectations and exchange rates. The methodology consists in estimating equilibrium values of the three variables by regressing - over a period prior to the crisis - the actual values of the variables on a set of fundamentals.

We now turn to a panel estimation of equilibrium variables from which we will be able to get an estimation of the quaterly deviations of actual spreads from equilibrium. The methodology is very simple and standard in the literature: we use a standard panel model with country fixed effects (see Fratzscher and Beirne, 2012 ; Hauner *et al.*, 2010 ; Aizenman *et al.*, 2011 for equilibrium bond spreads).

$$Y_{i,t} = \alpha + \beta X_{i,t} + u_i + \epsilon_{i,t}$$

where $Y_{i,t}$ represents the bond spread relative to Germany, the inflation expectations as measured by the difference between nominal and inflation indexed bonds, and the nominal effective exchange rate, $X_{i,t}$ a set of economic fundamentals, potentially different for the three dependent variables, u_i a country fixed effects.

In order to state that the fitted values of the linear panel regressions are a relatively good approximation - i.e. an unbiased estimator - of the "fundamental" or "equilibrium" values, four assumptions need to hold. The first three are the traditional identification assumptions according to which $\hat{\beta}$ is unbiased. We add the fourth assumption called the exhaustivity variables assumption which is specific to the estimations of equilibrium variables. In the following discussion of these assumptions, we argue that although the assumptions are debatable there are good reasons to think that they are reasonable.

1. The linearity assumption

   One could argue that both our toy model and the theoretical and empirical literature point out the possible non-linearities of bond yield dynamics as the fundamentals deteriorate[27].

   More precisely, consider the following non-linear specification :

   $$\epsilon_{i,t} = \gamma 1_{X_{i,t} \in \mathbb{C} \cap noLLR} + \eta_{i,t}$$

   $$Spread_{i,t} = \alpha + \beta X_{i,t} + u_i + \epsilon_{i,t}$$

   with $\eta_{i,t} \sim i.i.d(0, \sigma)$ and $\mathbb{C}$ the crisis set. This model is clearly non-linear due to the term $1_{X_{i,t} \in \mathbb{C} \cap noLLR}$. But conditional on having a LLR, the true model is also linear, therefore the econometric model is well identified from the data and without any bias :

   $$E(Spread_{i,t}/LLR) = \alpha + \beta E(X_{i,t}/LLR) + u_i + \gamma \underbrace{E(1_{X_{i,t} \in \mathbb{C} \cap noLLR}/LLR)}_{=0} = \alpha + \beta E(X_{i,t}) + u_i$$

   Nevertheless the linear model is biased in the unconditional case. By omitting the indicator function, we would wrongly attribute to β what should be attributed to γ. For example, if the public surplus is negatively related to bond spreads as it is expected to be, not including the indicator function would tend to increase in absolute value the marginal effect of public surplus on bond spreads.

   How to deal with it ? We could create this indicator function, but this would first of all require to specify *a priori* not only the countries and periods of time with a LLR but also the periods of liquidity crisis which would contradict our inductive approach. Secondly the exact form of non-linearities is not known, why should it be of the form $\gamma 1_{X_{i,t} \in \mathbb{C} \cap noLLR}$?. Finally we want to be able to caracterize and study the specificities of liquidity crises and of policy announcement country by

[27] De Grauwe et al. (2012) estimate a similar model on quaterly data with a sample made of developed countries and including the square of the gross debt over GDP and find evidence of a non-linear effect of debt.

country instead of estimating an average marginal effect.

Moreover one could question the assumption that the non-linearities only come from the absence of a LLR. For example, Bi (2011) has a DSGE model where the risk premium is non-linear in the fundamentals.

Consequently, in order to deal with these thorny issues we do the following two statements : the bias, if any, should play *against us* since it tends to attribute too much of the increase in bond spreads to change in fundamental. If the bias plays against us, it implies that we might miss positive deviations from fondamental spreads but if we detect a substancial positive deviation from fundamentals, this will be for sure a crisis. We may miss some periods of financial tensions but it is unlikely that we mistakenly mark periods of financial troubles as liquidity crisis. Moreover we exclude the year 2010-2011-2012 from the sample period and get static fitted values for the crisis period which avoid biasing the regression coefficients.

2. The independence assumption

   One could argue that the indicator function $1_{X_{i,t} \in \mathbb{C} \cap noLLR}$ is an ommited variable correlated with the explanatory variables. It is undoubtedly correlated with the latter, but it does not raise an endogeneity issue because the causality goes from the explanatory variables to the indicator function, and not the other way round. Therefore, we are back to the previous discussion : if we omit the indicator function, we don't create an endogeneity problem, but we wrongly attribute to β what should be attributed to γ. This may artificially increase the marginal effect of the fundamentals. But it is not an endogeneity issue.

3. Homoskedasticity and no serial correlation of errors

   These two assumptions don't bias the estimator if violated. But it can make the variance of the estimator no longer minimum. We deal with homoskedasticity by using robust standard errors.

4. The exhaustivity of fundamentals assumption

   This assumption is related to the precise purpose of this estimation. It says that all relevant fundamentals must be included ; otherwise what we would label "deviations from equilibrium" could reflect a missing fundamental variable. If one wants to get an estimation of the deviations from equilibrium, one needs to make sure that the deviations are not simply the consequence of the absence of a key variable in the regression. In order to deal with this issue, we will run all possible models that take into account all possible combinations of seven explanatory variables standard in the literature[28] Obviously we can't make sure that no variable is missing. In

[28] We report the charts of the analysis in the appendix 5.

particular, the policy decisions are to a certain extent absent from the model. However for the periods and the country we want to study - 2010-2011-2012 in the Euro area -, the assumption that no important variable that could drive the residuals is missing is reasonable and we will argue that no other policy decisions, except the ECB announcements, can arguably account for the change in the summer 2012. [29]

The sample covers 51 developed and developing countries from the first quarter of 1980 to the first quarter 2010. The annual data are from the World Economic Outlook (IMF). All data are taken from Datastream, except the VIX. We interpolate them to get quaterly data using standard cubic interpolation methods.

We could have chosen CDS spread as a measure of default risk instead of sovereign yields spreads relative to Germany. However, CDS markets have developed very recently and the data are not available before 2000. Moreover, as shown previously, there are absolutely no evidence that the CDS spread is more informative than the bond yield spread.

Finally the choice of a good indicator of inflation expectations for the members of the Eurozone is a thorny issue. The Consensus Forecasts provide interesting data, unfortunately they are plagued with the usual problem of being survey data. Moreover we did not access to such data. The others surveys or market data do not consider members of the Eurozone individually but only the aggregate area. The only remaining indicator is the difference between the nominal and the inflation-linked bond yield. Unfortunately the countries for which such a variable exists are very few: for the Euro Area, Bloomberg provides only data for France, Germany and Italy.

5.4.3 Results for Sovereign Yield Spreads

The estimation results are almost in line with the *a priori* predictions: all variables have the predicted sign and almost every of them are significant (we drop the current account

[29] In particular, on could argue that we don't take into account "spillovers". We deliberately don't include spillovers effects nor we allow for a shift in parameters during the crisis contrary to Fratzscher and Beirne (2012) for two reasons. First, imagin that Spain and Portugal are affected by the same shocks and are both in a situation of "run on public debt", then including a spillover variable would artificially increase the "equilibrium" bond spread simply because the neighbour is also in the same situation. Second allowing for a change in the parameters before and during the crisis is also debatable : why should the pricing of risk change before and after the crisis once one controls for the increase in risk aversion ? Fratzscher and Beirne (2012) allow for this shift because they want to show that there has been a "wake-up call" of financial markets in 2009 : financial markets used to underprice sovereign risk in southern european countries before the crisis, and they suddenly woke up at the beginning of the crisis. We argue that if the sample (of country and the time length) is large enough, it should not be necessary to include such a variable to see the underpricing in the euro area between 1999 and 2008 and then the wake-up call. These criticisms are related to those made by De Grauwe et al (2012) : including too much variables, like spillovers, on the right hand side on the equation artificially improves the fit of the regression and consequently attributes to "fundamental" too much of the variations in the spreads.

which is not significant at all). The first column reports the estimated coefficients over the sample period 1980Q1-2013-Q2. We then do some robustness checks: the second column reports the results estimated over the period 1980Q1-2010Q1 (pre-crisis period), the third over the period 2010Q1-2013Q2 (post-crisis period), the fourth restricts the sample to the developed countries, the sixth to the emerging countries and the seventh to the members of the Euro Area. The results are very robust. Notice one interesting feature: debt over GDP is no longer significant when we restrict the sample to the pre-crisis period, suggesting that financial markets did not price debt before the sovereign debt crisis. This confirms the traditional result in the literature (Fratzscher and Beirne, 2012).

Figure 19: Panel Estimation of Equilibrium Bond Spreads

Yield Spread	(1)	(2)	(3)	(4)	(5)	(6)
Constant	-0.844801	0.653291	-4.756394	-1.16744	-2.189682	-2.636053
	(0.000)	(0.0032)	(0.0388)	(0.000)	(0.000)	(0.000)
Debt	0.007721	-0.001153	0.062472	0.007268	0.029040	0.017213
	(0.000)	(0.5494)	(0.000)	(0.000)	(0.000)	(0.000)
Surplus	-0.033632	-0.046742	-0.015249	-0.044571	-0.060477	-0.053757
	(0.000)	(0.000)	(0.6696)	(0.000)	(0.000)	(0.000)
Inflation	0.44004	0.440148	0.172954	0.442916	0.426136	0.525686
	(0.000)	(0.000)	(0.0085)	(0.000)	(0.000)	(0.000)
Growth	-0.118557	-0.078806	-0.063150	-0.067694	-0.152971	-0.179631
	(0.000)	(0.000)	(0.0108)	(0.000)	(0.000)	(0.000)
GDP/Capita	-2.46E-05	-4.91E-05	-3.34E-05	-2.21E-05	5.67E-05	-4.27E-06
	(0.000)	(0.000)	(0.6316)	(0.000)	(0.000)	(0.620)
Unemploy	0.169784	0.076231	0.358315	0.164740	0.148196	0.180954
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R^2	72%	77%	83%	68%	79%	66%
Observations	3095	2644	487	2544	682	1314

Notes: p-values are reported in brackets.

In addition to the traditional panel regression analysis, we estimate all models that take into account all possible combination in a set of eight variables (debt, deficit, GDP growth, GDP per capita, unemployment, VIX, current account, inflation). This allows us to get the minimum gap to equilibrium at each point in time which gives us a lower bound for the gap between actual and equilibrium spreads and therefore an idea of the uncertainty surrounding the model on which we focus - reported in column 2 - which is estimated over the pre-crisis period. The charts of the gap from equilibrium are reported in appendix 4. The lower straight line gives us the lower bound of the possible gaps.

The first important result is that none of the 150 econometric models are able to account for the inverted U-shaped behavior of sovereign yields of countries under stress in 2012. The analysis allows to distinguish three groups of countries. The first one is made of States whose - predicted - interest rate deviated for more than 7 percentage points from equilibrium value (5 percentage points for the lower bound)[30]. Those are almost excluded from markets and bailed out by public institutions (Greece, Ireland, Portugal). The second group is made of countries that were at a critical point in the summer 2012, for which the interest rate deviated by 2 percentage points (1 percentage point) from equilibrium, for which the expectations were not well-anchored and that could have fallen into a bad equilibrium, had the CB not intervened (Slovenia, Spain and Italy). Finally the others (Autria, France, Germany, the Netherlands, Belgium) is made of countries that did not experience any significant deviations from equilibrium.

The second important result is that the yields were converging back to their equilibrium values starting in the summer 2012 - when the CB made its commitment - until the very end of 2012. The very strong tendency for the gap to close from the summer 2012 to the last observation is confirmed by figure 20 in which we report the sign as of 2012Q2 and the change in the gap between actual and equilibrium spreads between 2012Q2 and 2012Q3, 2012Q2 and 2012Q4 and 2012Q2 and 2012Q4. We argue that this allows us to capture the effect of the policy announcement that took place in 2012Q3 since the moves of the spreads caused by changes in the major determinants have been controlled for. This analysis allows us to better capture the cumulative effect of the announcement.

Figure 20: Sign and Change in Gap from Equilibrium Spreads

Country	Q2	Q3-Q2	Q4-Q3	Q4-Q2
Austria	+	-0.34	-0.05	-0.39
Belgium	+	-0.60*	-0.26*	-0.87*
Malta	+	-0.20	-0.16	-0.37
Finland	-	-0.22	0.05	-0.17
France	+	-0.53*	-0.13	-0.66*
Germany		0	0	0
Greece	+	-1.53***	-7.40***	-8.92***
Ireland	+	-1.21**	-1.08**	-2.30**
Italy	+	-0.16	-0.97**	-1.13**
Netherlands	+	-0.27	-0.16	-0.44
Slovakia	+	-0.36*	-0.04	-0.40
Slovenia	+	1.01	-1.10***	-0.09
Portugal	+	-1.63***	-1.68***	-3.30***
Spain	+	0.28	-0.88**	-0.60

[30] We report in brackets the deviations estimated from the most "optimistic" model - the one for which the predicted deviation is the lowest - ; this gives an idea of a lower bound for the deviations.

Because of serial correlation, the normal distribution of errors is not a tenable assumption. Therefore usual tests of significance can't be applied. We thus do our own non-parametric "test" which consists in a cross-sectional comparision to control for common factors affecting all countries at the same time and that could have driven down the bond spreads for all countries. One star means that the decrease in abnormal return is larger in absolute value than two third of the sample, two stars means that it is larger than the first fifth and three stars larger than the first tenth.

The table suggest that there have been a closing in the deviations from equilibrum yield around the summer 2012. A part from Finland, all countries were underpriced by financial markets in 2012Q2 with an interest rate higher than equilibrium. The values in the last column exhibit a negative sign for all countries: between spring 2012 and fall 2012, deviations from equilibrium spreads relative to Germany have declined in every country in the eurozone. Moreover this decline is for three fourth of the euro-sample superior in absolute value to the median decline. According to the assumptions that the main determinants of bond spreads are captured by the estimated linear model and that the announcements of the ECB were the major policy shocks affecting abnormal returns during this third quarter [31], we interpret these results as a clear sign that the commitment policy - the increase in the the degree of liquidity-passivity - made the yield decrease and return to "fundamental" values while they previously incorporate a high premium for self-fulfilling liquidity crisis risk - as modeled in proposition n°4.

Moreover and as previously mentioned, we can distinguish different types of countries : the Netherlands, France, Belgium and Austria on the one hand which experienced small decline and mostly between the second and the third quarters. On the other hand, Italy, Portugal, Greece, Spain, Ireland and Slovakia which experienced a sharp decline in spreads not only between the second quarter and the third but also between the third and the last quarter of the year. This also goes towards the direction of the effectiveness of the policy : the countries that were in the "crisis zone" should be very much affected by the commitment of the Central Bank to intervene because it makes this country go from a run on liquidity equilibrium to a "fundamental" equilibrium - at the end of Q4 the estimated deviations were back to zero for all countries.

[31] We studied the timeline of events during the summer 2012 and found no policy announcement that could have led to a significant decrease in yields. The major other events are the announcement by the European Council that it was working on a plan for the ECB to become the regulator of banks and on a plan to build a deposit insurance program at the European level on June 16. On June 9, European leaders agreed to inject €100 billions in Spanish banks in needed. On June 29, the ESM was allowed to recapitalize the banks. On July 27, 18 billions have been negociated to support four important Greek banks. On September 12, the Commission proposed a banking union. On October 19, European leaders agreed on a single banking supervision running by early 2013. On December 13, they reached an agreement to form a banking union.

5.4.4 Results for Inflation Expectations and Nominal Effective Exchange Rates

We estimate, in the same way as for the default risk, an "equilibrium" value for inflation Expectations and Nominal Effective Exchange Rate based on a panel regression over the period prior to the crisis. As already mentionned, it is hard to get inflation expectations data on individual countries of the Eurozone. We didn't have access to the Consensus Forecast database and the spread between nominal and inflation-linked bond yield are available for very few countries. That's the reason why the number of observations shrink enormously for the inflation expectations regression. The results are reported in appendix 7. In the following figures we show the sign of the gap as of 2012Q2 and the following quaterly changes.

Regarding the Inflation Expectations, the actual expectations were clearly below their equilibrium value in 2012Q2, suggesting that the markets put a great weight on the probability of deflation in Italy but also in France and in Germany. The ECB announcements made the expectation rise in the third quarter, but then decrease in the fourth quarter with an overall positive effect from 2012Q2 to 2012Q4. This increase is nevertheless not higher than 2 standard deviations except for Italy, suggesting that the announcement did not cause a huge shift in expectations but rather a decrease in the probability of deflation.

Figure 21: Sign and Change in Gap from Inflation Expectations Equilibrium

Country	Q2	Q3-Q2	Q4-Q3	Q4-Q2
France (5 years)	-	0.7282	-0.1585	-0.5696
Germany (5 years)	-	0.69417	-0.21955	0.47461
France (10 years)	-	0.4608	-0.1459	0.3148
Germany (10 years)	-	0.3405	-0.1557	0.1848
Italy (10 years)	-	0.7313*	0.2579	0.9893*

Regarding the change in the log-difference of the nominal effective exchange rate, the announcements made the euro, starting from a negative gap position in 2012Q2, appreciate from 2012Q2 to 2012Q4. Notice that the differences in the evolution of the NEER between the eurozone countries result only from the heterogeneity in weights - stemming from the differences in the structures of trading partners - in the computation of the effective rate.

Figure 22: Sign and Change in Gap from NEER Equilibrium

Country	Q2	Q3-Q2	Q4-Q3	Q4-Q2
Austria	-	-0.00413	0.01818	0.01405
Belgium	-	-0.00443	0.02303	0.01860
Malta	-	-0.00505	0.02924	0.02419
Finland	-	-0.00802	0.02993	0.02191
France	-	-0.00264	0.02326	0.02062
Germany	-	-0.00567	0.02920	0.02352
Greece	-	-0.00628	0.016693	0.01040
Ireland	-	-0.00587	0.03512	0.02925
Italy	-	-0.00375	0.02255	0.0188
Netherlands	-	-0.00287	0.02398	0.02110
Slovakia	-	-0.00604	0.01665	0.01061
Slovenia				
Portugal	-	-0.00294	0.01351	0.01056
Spain	-	-0.00604	0.01961	0.01694

5.4.5 Second partial conclusion

The panel analysis clearly confirms the results we get in the event-study analysis. The ECB announcements had a significant and large impact, in the Southern countries, on the default risk of sovereigns ; it also increased the expected inflation, which must be understood as a decrease in the risk of deflation and it led to a slight appreciation of the euro.

Once again, the data strongly reject the Fundamentalist as well as the liquidity hoarding interpretation. On the contrary it supports, without being able to distinguish them, both the multiple fundamental equilibria and the liquidity crisis interpretations.

Regarding the two sub-interpretations of a liquidity-crisis, the evidence supports the multiple equilibria - free lunch hypothesis. Indeed, the CB didn't have to buy any sovereign bond to clear the markets and the inflation expectations have not risen above the long term anchor.

	i	r	RoD	NER	g^e	π^e
Fondamentalist	(+)	.	.	(-)	.	(+)
fundamental Multiple Equilibria	-/(+)	-	-	(-)/+	+	(+)
Liquidity hoarding	.	.	.	.	.	.
Liquidity Crisis	-/(+)	-	-	(-)/+	+	+
Data	**-**	**-**	**-**	(+)	.	+

5.5 Limits to the Self-Fulfilling Roll-over Crisis Interpretation and Redenomination Risk

The first limit is related to the identification assumption we made in the econometric part. We implicitly assume that the announcement did not change the perception of the fundamentals - what we label the information revealing effect. Indeed according to our toy model, the CB announcement affects the markets participants through two channels. The first one is the usual LLR channel. And the second one is the informational channel by which the CB reveals its own information to the market participants and can therefore influence the way investors see the fundamentals. The effect we measured in the event-analysis was a mixture between a pure LLR commitment and an informational signal about the perceived fundamentals of the States by the Central Bank. The question is how to disentangle the two channels. This is a point that would be interesting, although difficult, to try to deal with.

The second limit is related to the so-called redomination risk. The official justification of the greater commitment on the part of the ECB was the ill-founded beliefs that the euro might be reversible. The OMT is thus justified by the fact that the interest rates incorporated a currency risk premium coming from the positive probability - in the mind of investors - that some countries could exit the euro and redominate their debt in a local currency. The renomination risk should be understood as a combination of a devaluation risk and of a default risk.

In a first approximation, one could think that the two are equivalent and that a redenomination is a way to default. However, the devaluation doesn't have the same reputational impact for the State ; moreover the devaluation concerns the whole economy and not only the State. For these reasons, it would be interesting to distinguish between the devaluation and the default risk. Because of lack of data and the complexity of the law[32], we could not find out a convincing identification strategy to isolate the redenomination risk. And the indicators introduced by policy-makers are not more convincing[33] .

[32]Indeed, the redenomination process is not regulated by law. For example, if Italy exited the euro, it could redenominate only the bonds in euros that have been issued under its jurisdiction. It could not redenominate the bonds that have been issued under the German or the English jurisdiction. Notice also, that if Italy redenominated its bonds in a local currency, it would not trigger an credit event because Italy belongs to the G7 ; but if Spain or Greece did it, it would because they don't belong to the G7. It implies that investors cannot use CDS of Italian bonds to hedge against the risk of Italian redenomination, bu they can for Spanish and Greek bonds.

[33]The measurement - and the detection - of the redenomination risk is a thorny issue. According to Patrick Honohan, the Governor of the Central Bank of Ireland, at the BIS Conference "Sovereign risk - a world without risk-free assets", there are three ways to measure this risk: the first one is to use econometric estimates of the cross-sectional determinants of sovereign spreads for foreign currency-denominated borrowing to predict current spreads in stressed euro area countries: a positive residual might suggest a redenomination risk premium. The second way is to compare the current spreads of euro area sovereigns in euro and in foreign currency-denominated borrowings provides for an alternative approach. The third one is to look at the co-movement in the time series of euro area country spreads. Some of this co-movement can be attributed to fluctuations in market risk-appetite; the remainder could be interpreted as a system-wide redenomination premium.

6 Conclusion

The starting point of this dissertation is the intuition that the "crisis" of the European sovereign debts (2010-2012?) was not entirely driven by fundamentals. This intuition has been confirmed by the "Lower Bound" estimation : no reasonable traditional econometric models of sovereign interest rates can explain the sharp increase in sovereign spreads in 2011 and 2012. Our simple explanation to this "sovereign rate puzzle" comes from the comparision between the Euro Area and the Anglo-saxon countries : due to differences in the preference for inflation, Central Banks don't show the same degree of commitment to intervene on the sovereign bond market - what we label the degree of liquidity-passivity - and this in turn implies different probability of crisis and inflation.

While the literature until now have concentrated mostly on the relations between the State and the Central Bank from the angle of solvency, we wanted in this dissertation to underline the idea that the liquidity dimension of the relation is as important. We provide a clear framework to conceptualize the two dimensions of the relationship between the Central Bank and the State (liquidity and solvency) and that aims at understanding how they interact. In particular we clearly model a trade-off between price stability and financial stability.

Our theoretical contribution is to show how the monetary policy influences the determination of the equilibrium of a simple roll-over game by acting as a lender of last resort ; we show that the CB faces a trade-off between stability of the financing of the State and inflation when the information is not complete, that the solution to this trade-off - the degree of liquidity-passivity - is decreasing with the preference for inflation and decreasing with the degree of solvability-activity. When the interest rate is allowed to be endogeneously determined, we show that it decreases with the degree of liquidity-passivity and increases with the preference for price stability of the CB. We also show why the ECB policies before the summer 2012 failed : the diagnosis was wrong, the liquidity hoarding behavior of Banks was not the main sources of the problem ; therefore trying to influence the sovereign debt market through the refinancing of Banks was not a sufficient solution.

We then propose a historical perspective to test our model and to introduce renewed - but tentative - interpretations of some key financial events, in the light of our model. The history of Central Banks and State and of their relationship clearly reveals firstly that the Central Bank has been the lender of last resort of the State before being the one of the banking system and secondly that the nature of their relationship and the context - for example, the solvability of the State, the mobility of capital - has had an significant impact on the probability of sovereign crisis and inflation.

In order to distinguish between the different interpretations of the Eurozone crisis, we focus on the ECB announcements of the summer 2012 with the implicit idea that the reaction of markets to these events should give us information about the nature of the crisis. The econometric section is made of two parts that are both event studies but over two different horizons - or windows - and that don't rely on the same set of assumptions. In the pure event-study analysis, the window is between one and four days around the event. We show that the announcements made the sovereign yields decrease mainly through the decrease in default risk, and not trough a rebalancing effect nor through the expected decrease in future policy rates - although we find evidence of a small but significant decline in forwards rates on July 26. The expected inflation slightly but significantly increased in the three announcements. The second part is motivated by the need to enlarge the window in order to consider the cumulative effect of the announcement over a quarter and consists in computing quarterly changes in residuals of a panel regression that aims at controling for changes in the fundamentals. The two parts leads exactly to the same conclusion: the fundamentalist and the liquidity hoarding interpretation of the crisis are rejected by the data contrary to the liquidity-crisis interpretation. Finally regarding the two sub-interpretations of a liquidity-crisis, the evidence supports the multiple equilibria - free lunch hypothesis. Indeed, the CB didn't have to buy any sovereign bond to clear the markets and the inflation expectations have not risen above their long term anchor - 2%.

7 Appendix

7.1 Demonstrations of Propositions n°1, n°2 and n°4

Proposition n°1

If $P(Crisis;\theta'_{opt})$ and $E(\pi;\theta'_{opt})$ are twice differentiable in θ', then θ'_{opt} is increasing in a.

Proof

Assuming the densities are differentiable, the First Order Condition to the minimization program writes

$$a\frac{\partial E(\pi;\theta'_{opt})}{\partial\theta'}E(\pi;\theta') + \Delta y\frac{\partial P(Crisis;\theta'_{opt})}{\partial\theta'}P(Crisis;\theta') = G(a;\theta'_{opt}) = 0$$

According to the implicit function theorem - assuming the densities are twice differentiable -,

$$\frac{\partial\theta'_{opt}}{\partial a} = -\frac{\frac{\partial G}{\partial a}}{\frac{\partial G}{\partial\theta'_{opt}}} = -\frac{\frac{\partial E(\pi;\theta'_{opt})}{\partial\theta'}E(\pi;\theta')}{\frac{\partial G}{\partial\theta'_{opt}}}$$

We want to show that $\frac{\partial\theta'_{opt}}{\partial a} > 0$. Since a necessary second order condition for θ'_{opt} to be a minimum is exactly that the second derivatives be strictly positive, we therefore have

$$\frac{\partial G}{\partial\theta'_{opt}} > 0$$

Consequently the sign of $\frac{\partial\theta'_{opt}}{\partial a}$ is the same as $-\frac{\partial E(\pi;\theta'_{opt})}{\partial\theta'}$. It thus remains to show that the latter is indeed positive.

$$E(\Pi,\theta') = \int_{\theta'}^{+\infty}\int_{\theta_1}\int_{-\infty}^{\delta(\theta_{BC},\theta_1)}\log(\delta(\theta_{BC},\theta_1)) - \log(\theta_2)f(\theta_2,\theta_1,\theta_{CB})d\theta_2 d\theta_1 d\theta_{CB}$$

$$\frac{\partial E(\pi;\theta'_{opt})}{\partial\theta'} = -\int_{\theta_1}\int_{-\infty}^{\delta(\theta',\theta_1)}\log(\delta(\theta',\theta_1)) - \log(\theta_2)f(\theta_2,\theta_1,\theta')d\theta_2 d\theta_1 < 0$$

.

Since $E(\Pi,\theta')$ is decreasing in θ', we have $\frac{\partial\theta'_{opt}}{\partial a} > 0$.
Q.E.D

Proposition n°2

θ'_{opt} is a decreasing function of z. In words, the optimal degree of liquidity-passivity of the CB is decreasing with the degree of solvability-activity.

A shift from a solvability-passive monetary policy to a solvability-active monetary policy decreases the optimal threshold of intervention by the CB. A solvability-active monetary policy implies that a liquidity-passive monetary policy is optimal. Conversely, a solvability-passive monetary policy implies that a higher degree of liquidity-activity is optimal.

Proof

According to the implicit function theorem - assuming the densities are twice differentiable -,

$$\frac{\partial \theta'_{opt}}{\partial z} = -\frac{\frac{\partial G}{\partial z}}{\frac{\partial G}{\partial \theta'_{opt}}} = -\frac{\frac{\partial^2 E(\pi;\theta'_{opt})}{\partial \theta' \partial z} E(\pi;\theta'_{opt}) + \frac{\partial E(\pi;\theta'_{opt})}{\partial \theta'}\frac{\partial E(\pi;\theta'_{opt})}{\partial z}}{\frac{\partial G}{\partial \theta'_{opt}}}$$

We want to show that $\frac{\partial \theta'_{opt}}{\partial z} < 0$. Since a necessary second order condition for θ'_{opt} to be a minimum is exactly that the second derivatives be strictly positive, we therefore have

$$\frac{\partial G}{\partial \theta'_{opt}} > 0$$

Consequently the sign of $\frac{\partial \theta'_{opt}}{\partial z}$ is the same as the sign of $-\left[\frac{\partial^2 E(\pi;\theta'_{opt})}{\partial \theta' \partial z} E(\pi;\theta'_{opt}) + \frac{\partial E(\pi;\theta'_{opt})}{\partial \theta'}\frac{\partial E(\pi;\theta'_{opt})}{\partial z}\right]$. It thus remains to show that the latter is indeed negative.

We know from the demonstration of proposition n°1 that

$$\frac{\partial E(\pi;\theta'_{opt})}{\partial \theta'} = -\int_{\theta_1}\int_{-\infty}^{\delta(\theta',\theta_1)-z} \log(\delta(\theta',\theta_1)) - \log(\theta_2+z) f(\theta_2,\theta_1,\theta') d\theta_2 d\theta_1 < 0$$

We also have that

$$\frac{\partial}{\partial z}\int_{\theta_1}\int_{-\infty}^{\delta(\theta',\theta_1)-z} \log(\delta(\theta',\theta_1)) - \log(\theta_2+z) f(\theta_2,\theta_1,\theta') d\theta_2 d\theta_1 < 0$$

hence

$$\frac{\partial^2 E(\pi;\theta'_{opt})}{\partial \theta' \partial z} = \frac{\partial}{\partial z} - \int_{\theta_1}\int_{-\infty}^{\delta(\theta',\theta_1)-z} \log(\delta(\theta',\theta_1)) - \log(\theta_2+z) f(\theta_2,\theta_1,\theta') d\theta_2 d\theta_1 > 0$$

Moreover

$$\frac{\partial E(\pi;\theta'_{opt})}{\partial z} = \frac{\partial}{\partial z}\int_{\theta'}^{+\infty}\int_{\theta_1}\int_{-\infty}^{\delta(\theta',\theta_1)-z} \log(\delta(\theta',\theta_1)) - \log(\theta_2+z) f(\theta_2,\theta_1,\theta') d\theta_2 d\theta_1 d\theta_{CB} < 0$$

Therefore

$$\frac{\partial E(\pi;\theta'_{opt})}{\partial \theta'}\frac{\partial E(\pi;\theta'_{opt})}{\partial z} > 0$$

and finally

$$\frac{\partial^2 E(\pi;\theta'_{opt})}{\partial\theta'\partial z}E(\pi;\theta'_{opt})+\frac{\partial E(\pi;\theta'_{opt})}{\partial\theta'}\frac{\partial E(\pi;\theta'_{opt})}{\partial z}>0$$

which shows that $\frac{\partial\theta'_{opt}}{\partial z}<0$.

Proposition n°3
r_{eq} is a non-increasing function of μ.

Proof
The State solves the following program:

$$\min_r \frac{1}{2}\Big[r+bE(\delta(r,\mu))\Big]$$

s.t. $E(\delta(r,\mu))-\mu\leq 0$.

If the set of solutions to the constraint is empty, as usual, we set $r_{eq}=+\infty$. In this case, an increase in μ cannot make r_{eq} increase.

If the set of solutions is not empty, then we look for a maximum over this set. Denoting λ the lagrange multiplier associated with the constraint, the F.O.C writes:

$$r+(b-\lambda)\frac{\partial E(\delta(r,\mu))}{\partial r}E(\delta(r,\mu))=0$$

Assuming $E(\delta(r,\mu))$ is twice differentiable, we have:

$$\frac{\partial r_{eq}}{\partial\mu}=-\frac{(b-\lambda)\Big[\frac{\partial^2E(\delta(r,\mu))}{\partial r\partial\mu}E(\delta(r,\mu))+\frac{\partial E(\delta(r,\mu))}{\partial r}\frac{\partial E(\delta(r,\mu))}{\partial\mu}\Big]}{\frac{\delta^2 L}{\partial r}}$$

The denominator is positive due to the second order condition for a minimum. Therefore we focus on the sign on the numerator. We want to show that $\frac{\partial r_{eq}}{\partial\mu}<0$, therefore that $(b-\lambda)\Big[\frac{\partial^2E(\delta(r,\mu))}{\partial r\partial\mu}E(\delta(r,\mu))+\frac{\partial E(\delta(r,\mu))}{\partial r}\frac{\partial E(\delta(r,\mu))}{\partial\mu}\Big]>0$.

If the constraint doesn't bind, $\lambda=0$ and we have that the numerator is equal to

$$b\Big[\underbrace{\frac{\partial^2E(\delta(r,\mu))}{\partial r\partial\mu}}_{\geq 0}\underbrace{E(\delta(r,\mu))}_{\geq 0}+\underbrace{\frac{\partial E(\delta(r,\mu))}{\partial r}}_{<0}\underbrace{\frac{\partial E(\delta(r,\mu))}{\partial\mu}}_{<0}\Big]$$

therefore $\frac{\partial r_{eq}}{\partial\mu}<0$.

If the constraint binds, $\lambda>0$ and we have that the numerator is equal to

$$(b-\lambda)\Big[\underbrace{\frac{\partial^2E(\delta(r,\mu))}{\partial r\partial\mu}}_{\geq 0}\mu\Big]$$

which is positive if and only if $b - \lambda > 0$. We show that this is indeed the case. From the F.O.C, we have

$$r + (b - \lambda)\frac{\partial E(\delta(r,\mu))}{\partial r}E(\delta(r,\mu)) = 0 \iff \lambda = \underbrace{\frac{r}{\frac{\partial E(\delta(r,\mu))}{\partial r}E(\delta(r,\mu))}}_{<0} + b < b$$

Therefore $\frac{\partial r_{eq}}{\partial \mu} < 0$.

We have shown that $\frac{\partial r_{eq}}{\partial \mu} < 0$ in all cases when the set of solutions of the constraint is not empty, which ends the proof.

7.2 Bond vs CDS spreads : which one leads the other in the price discovery process?

The methodology is standard in the literature.In order to capture the long-term relationship and the short-term dynamics between the two variables, we run a vector error correction model (VECMs) using Johansen's procedure to test for the existence of cointegrating relationship for each country. The estimated model is as follows :

$$\Delta CDS_{i,t} = \lambda_1(CDS_{i,t-1} - \alpha - \beta i_{i,t-1}) + \sum_{j=1}^{q} \gamma_{1,j} \Delta CDS_{i,t-1} + \sum_{j=1}^{q} \delta_{1,j} \Delta i_{i,t-1} + \epsilon_{1,t}$$

$$\Delta i_{i,t} = \lambda_2(CDS_{i,t-1} - \alpha - \beta i_{i,t-1}) + \sum_{j=1}^{q} \gamma_{2,j} \Delta CDS_{i,t-1} + \sum_{j=1}^{q} \delta_{2,j} \Delta i_{i,t-1} + \epsilon_{2,t}$$

The long term relationship is captured by the first term $CDS_{i,t-1} - \alpha - \beta i_{i,t-1}$ with λ_h the speed at which the spread reacts and adjusts to the gap to the long term relationship. We also add a number of lags (determined on the basis on the Akaike information criterion) of the dependent variables. A significantly negative and high -in absolute value- λ_1 would mean a relatively high speed of correction of the CDS spread to the long term relationship. This is interpreted in the literature as the sign that the interest rate is leading and the CDS following. If λ_2 is significantly positive and high the reverse is true : the interest rate adjusts rapidly to a gap to the long term relationship which means that the CDS spread is leading.

Most of the time λ_1 and λ_2 are significantly, respectively, negative and positive, which means that both the bond spread and the CDS spread react and adjust to the gap. Gonzalo and Granger have proposed a measure of contribution to price discovery : $GG = \frac{\lambda_2}{\lambda_2 - \lambda_1}$. If $GG < 0.5$, the contribution of the bond spread to the adjustement to the long term relationship is low, this is the sign that the bond markets leads.

Here are the results. We see clearly that the GG statistics is always very low which allows us to conclude that the bond market is likely to lead in the price discovery process.

	Correlation	α	β	λ_1	λ_2	GG
France	0.8787	24	-31***	-0.0276964***	0.00035**	0.01247932
Greece	0,7902	12606	-8452***	-0,0026511**	0,000000889**	0,00033522
Ireland	0,4201	2258	-1675***	0,0009604	0,00000684***	-0,00717312
Italy	0,9561	17	-75***	-0,1980124	0,004226	0,020896131
Netherlands	0,7595	-4	-2,5***	-0,0129663***	0,0012744	0,089489983
Portugal	0,7879	127	-101***	-0,0093464**	-0,0000209	-0,002241167
Spain	0,9287	12	-56***	-0,017866**	0,0001165	0,006478521

7.3 Can we beat the Random Walk on sovereign bond markets?

Consider the simplest model one can imagin, an AR(1) :

$$\Delta i_{i,t} = \rho \Delta i_{i,t-1} + \epsilon_t$$

For every i and t this gives us a $\rho(i,t)$ which is then used to make out-of-sample forecast (OSF) :

$$OSF_{i,t}(t+1) = \rho(i,t)\Delta i_{i,t}$$

. The next step is to compute the forecast errors (FE)

$$FE_{i,t}(t+1) = \Delta i_{i,t+1} - OSF_{i,t}(t+1) = \Delta i_{i,t+1} - \rho(i,t)\Delta i_{i,t}$$

To compare the prediction power of this simple model to those of the random walk as implied by the EMH and the REH, the Mean Squared Error (MSE) is computed over the year 2012. Here are the results, the MSE of the Random Walk is systematically much larger -on average 80% larger- than the MSE of the AR(1) :

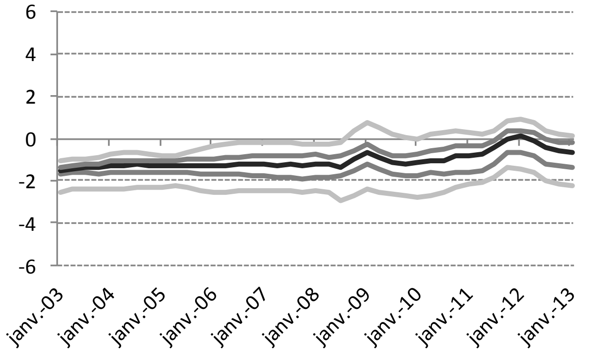
Country	Random Walk	AR(1)
Austria	2.93	1.88
Belgium	3.61	2.20
Malta	12.49	5.65
France	3.28	2.00
Germany	3.74	1.89
Greece	2927.66	1677.35
Ireland	15.63	7.33
Italy	18.18	10.42
Netherlands	3.88	2.07
Portugal	80.49	47.58
Spain	24.49	14.95

7.4 Event-study Analysis: 5-years Sovereign CDS Spreads

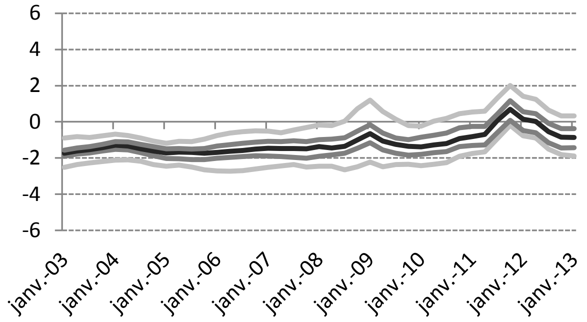
For every country, every window, and every date we compute the change in 5-years Sovereign CDS Spreads (relative to US sovereign CDS). We also compute the standard deviation of the associated series. We finally compute the Student ratio $\frac{\Delta CDS}{\sigma_{CDS}}$ and run one-sided tests for $\Delta CDS > 0$ and $\Delta CDS < 0$. The following table summarizes all the results. "-1" (resp. -2 ; -3) means that the CDS decreased by a amount statistically significant at the 10% (resp. 5% ; 1%) threshold.

Country		25-07	01-08	05-09	26-07	02-08	06-09
Austria	1-day						
	2-days						
	3-days						
	4-days						
Belgium	1-day		-3	+1			
	2-days	-3				-1	
	3-days	-3				-2	
	4-days	-2				-2	
Cyprus	1-day						
	2-days			-3			
	3-days			-3			-3
	4-days			-3			-3
France	1-day		-2		-1		
	2-days	-1			-2	-1	-1
	3-days		-1		-2		
	4-days					-1	
Germany	1-day	-1					
	2-days						
	3-days	-2					
	4-days	-2			-1		
Greece	1-day						
	2-days						
	3-days						
	4-days						
Ireland	1-day	-1	-3				-1
	2-days	-1	-2	-1	-2		
	3-days		-1	-2	-1		-2
	4-days			-2	-1		-2
Italy	1-day	-2	-3		-3	+3	-3
	2-days	-3	-3		-3		-3
	3-days	-2	-3		-3	-1	-3
	4-days	-1	-3	-2	-3	-1	-3
Malta	1-day						
	2-days						
	3-days						
	4-days						
The Netherlands	1-day	-1			-1		-2
	2-days	-1			-2		
	3-days	-1	-1		-2		-2
	4-days	-2	-1		-2		-1
Portugal	1-day	-2				+2	
	2-days	-1	-2		-1		-1
	3-days		-2		-1		
	4-days		-2				
Spain	1-day	-3	-3		-3	+3	-3
	2-days	-3	-3	-1	-3		-3
	3-days	-2	-3		-3		-3
	4-days	-2	-3	-2	-3		-3
Slovakia	1-day						
	2-days		-1				
	3-days		-3				
	4-days		-3			-3	
Slovenia	1-day		+3	-2			-2
	2-days		+3	-3		+2	-3
	3-days		+2	-2		+2	-3
	4-days		+1	-2		+1	-3

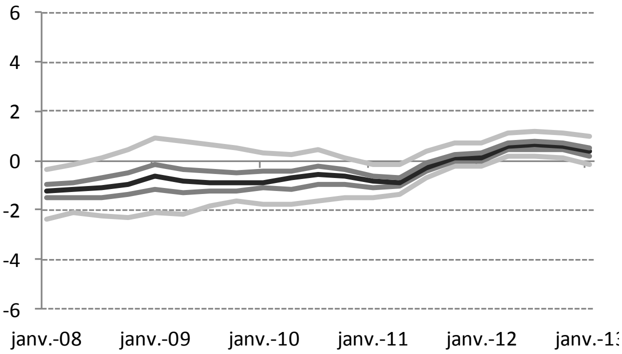
7.5 Lower Bound Estimation of Gap from Equilibrium



Austria



Belgium



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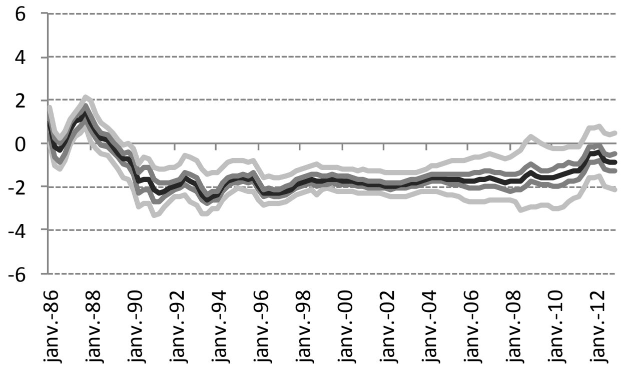
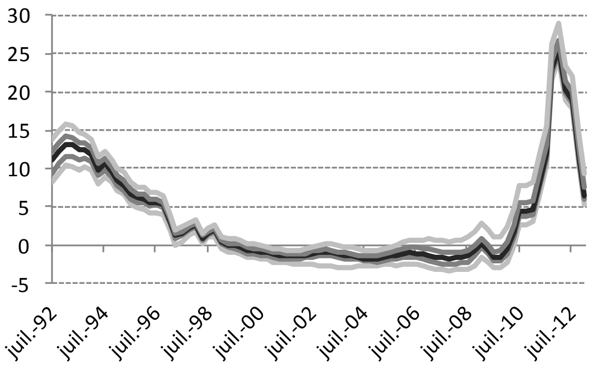
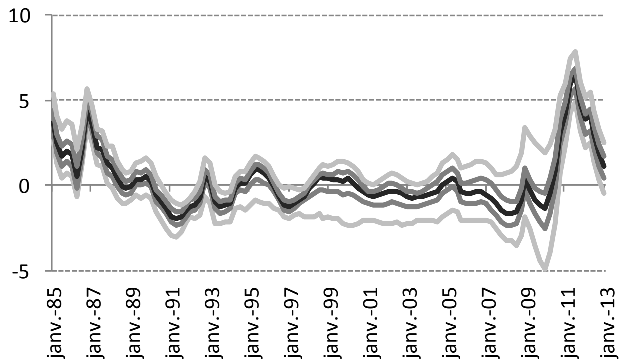


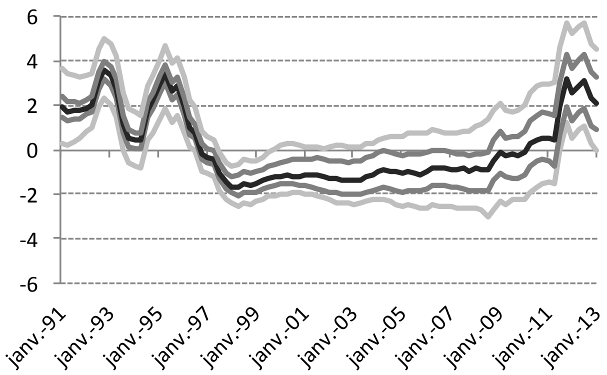
France



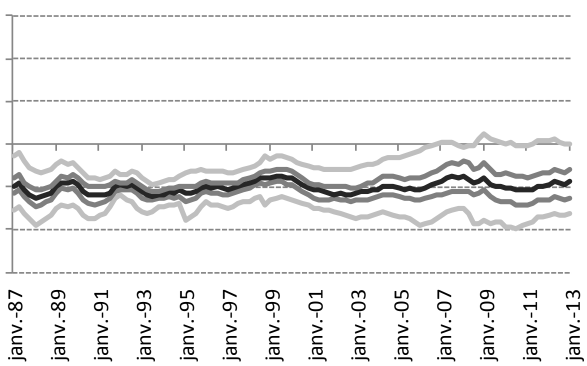
Greece



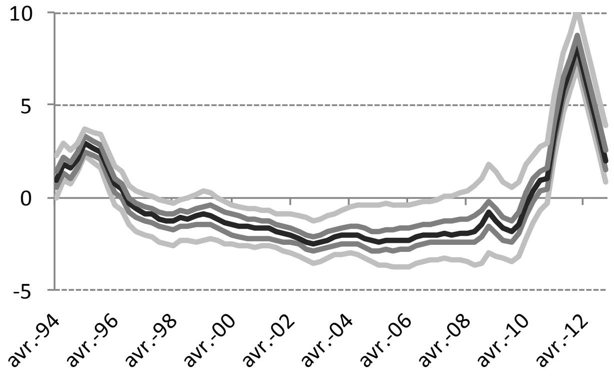
Ireland



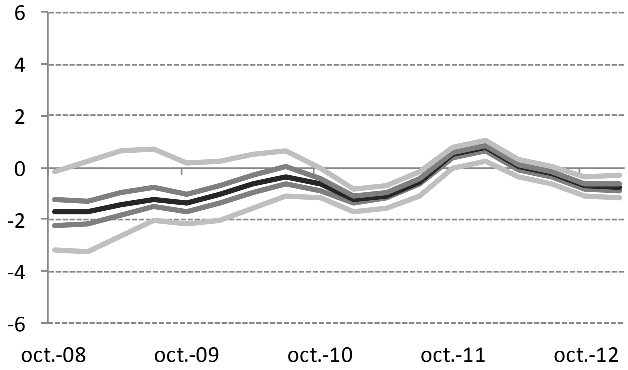
Italy



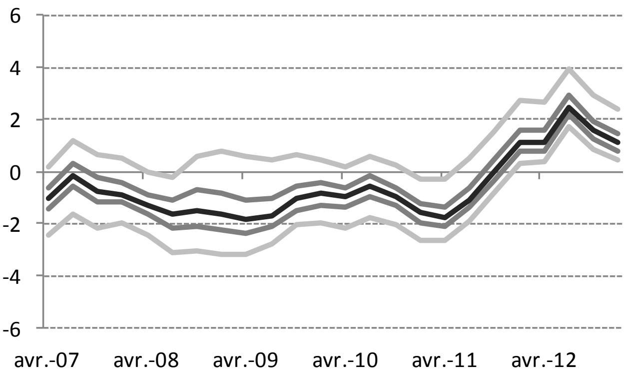
Netherlands



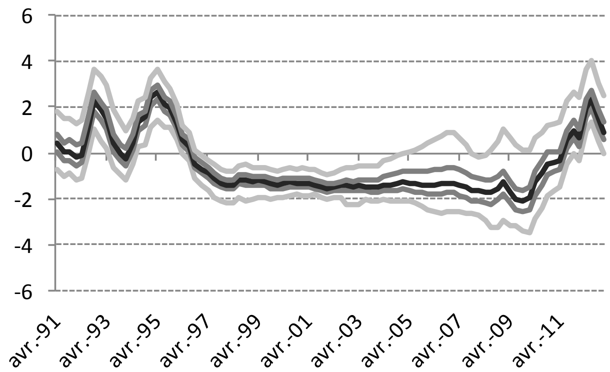
Portugal



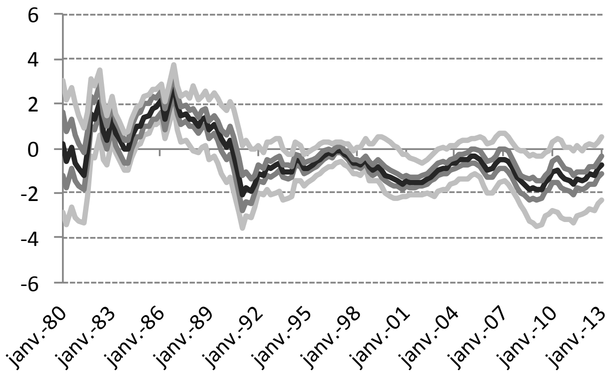
Slovakia



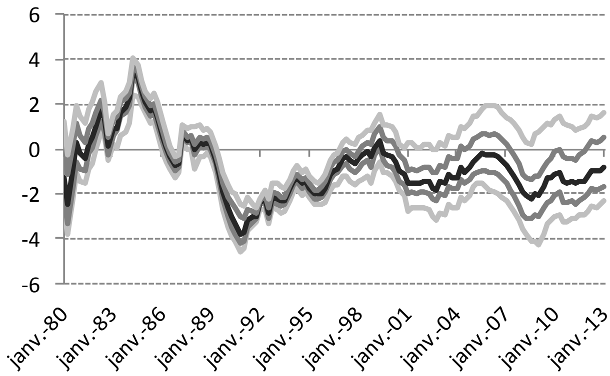
Slovenia



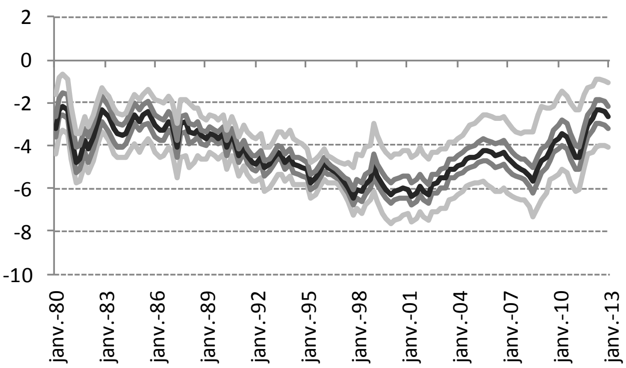
Spain



Great Britain



US



Japan

7.6 Panel Estimation of Inflation Expectations and Nominal Effective Exchange Rate

In figure 23, the first column shows the results of the regression of the inflation expectations at a 5-years horizon on a set of variables ; the second restricts the sample to the pre-crisis period - i.e. until 2010Q1 and the third to the crisis period. In the last three columns, the dependent variable becomes the inflation expectations at a 10-years horizon.

Figure 23: Panel Estimation of Inflation Expectations

Inflation Expectations	(1)	(2)	(3)	(4)	(5)	(6)
Constant	1.770395	2.613291	-5.820407	3.163122	2.361470	1.328956
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.011)
Debt	0.001408	-0.01177	0.058260	-0.020119	-0.010377	0.001306
	(0.688)	(0.2010)	(0.000)	(0.1816)	(0.000)	(0.7676)
Surplus	0.057501	0.000915	-0.168720	-0.065203	-0.090678	-0.027214
	(0.002)	(0.974)	(0.000)	(0.000)	(0.000)	(0.194)
Inflation	0.041135	0.023223	0.054416	0.067131	0.031450	-0.090277
	(0.270)	(0.616)	(0.305)	(0.057)	(0.454)	(0.006)
Growth	0.063602	0.096058	0.051203	0.036128	0.111831	0.055321
	(0.000)	(0.000)	(0.012)	(0.018)	(0.000)	(0.000)
R^2	70%	68%	89%	82%	79%	97%
Observations	213	129	90	299	187	120

Notes: p-values are reported in brackets.

In figure 24, we report in the first column the results when all variables are included and in the second column the model where all variables are significant. The third one restricts the sample to the pre-crisis period and the fourth to the crisis period.

Figure 24: Panel Estimation of the log-difference of Nominal Effective Exchange Rates

dln(NEER)	(1)	(2)	(3)	(4)
Constant	0.001426	-0.009550	-0.010705	-1.16744
	(0.686)	(0.0032)	(0.0388)	(0.000)
Debt	-6.67E-05			
	(0.065)			
Surplus	-0.000209			
	(0.288)			
Inflation	-0.001025	-0.009550	-0.000280	0.442916
	(0.000)	(0.000)	(0.000)	(0.000)
Growth	0.000871	0.001404	0.001484	-9.15E-05
	(0.000)	(0.000)	(0.000)	(0.851)
GDP/Capita	4.53E-08			
	(0.6158)			
Unemploy	0.000206			
	(0.500)			
CA	0.000656	0.000828	0.000841	0.001643
	(0.000)	(0.000)	(0.000)	(0.0151)
R^2	7%	41%	42%	9%
Observations	3518	4846	4366	520

Notes: p-values are reported in brackets.

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