Optimal Interest Rate Tightening with Financial Fragility*

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Abstract

Recent events have reignited concerns about the financial stability implications of monetary policy. We show empirically that monetary tightening exacerbates financial stress after supply shocks, through declines in asset prices, bank equity and increased run risks. We then develop a tractable model in which intermediaries face occasionally binding leverage constraints and endogenous risks of runs, while producers face price adjustment frictions. Interest rate tightening, by lowering asset prices, exacerbates both financial distortions when intermediaries' equity is sufficiently low. We use the model to characterize the constrained efficient use of interest rate policy, credit policy, equity injection, macroprudential policy and deposit insurance during periods of supply-driven inflation and fragility. When other tools are costly, optimal monetary policy tightening should be less aggressive in the presence of financial fragility. If other tools were not costly, the right combination of tools could perfectly separate financial stability objectives.

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1 Introduction

Recent events of financial market turbulence, including the collapse of Silicon Valley Bank and other regional banks in the US and the pension funds and liability-driven investment (LDI) crisis in the UK, have renewed concerns that sharp monetary policy tightening to curb rising inflationary pressures could exacerbate financial instability. These events have raised important questions for policymakers. First, under what conditions can monetary policy tightening trigger or amplify financial instability? Second, how should monetary policy respond to inflation when there are tensions with financial stability goals? Third, how should other policy tools be used to better separate financial stability concerns?

Our paper makes three contributions that shed light on these questions. First, we document that monetary policy tightening exacerbate financial stress after supply shocks, through lowering asset prices and bank equity, and increasing the risk of depositor runs. Second, we develop a New Keynesian model with a financial sector subject to a leverage constraint and a risk of depositors run to rationalize how monetary policy tightening exacerbates financial instability during periods of supply-driven inflation. Third, we characterize the constrained efficient combination of interest rate tightening and other tools, including credit policy, equity injections, deposit insurance and macroprudential policy. The tractability of our model allows us to derive simple intuitive formula for optimal policies, clarifying the determinants of the optimal policy mix.

The model focuses on the essential ingredients needed to analyze the trade-offs between price and financial instability in times of rising inflation and interest rate tightening. First, firms and workers face adjustment costs to price changes. This implies that prices are sticky in the short run, giving rise to an *aggregate demand externality*. Sticky prices generate a meaningful role for monetary policy in stabilizing prices. Second financial intermediaries face an occasionally binding leverage constraint due to moral hazard in the spirit of Gertler and Kiyotaki (2010) or due to macroprudential regulation. A binding constraint limits the financial sector's ability to arbitrage and opens up *a wedge between the deposit and the lending rate* (the return on real assets). Third, intermediaries face a *risk of bank runs* due to coordination failures among depositors, which we microfound using a global games approach (Morris and Shin, 2003; Goldstein and Pauzner, 2005). Coordination failures arise from the inefficient drop in asset prices when intermediaries are forced to quickly liquidate their assets in a situation of depositors' run.

We show that interest rate tightening in times of rising inflation can exacerbate financial instability by dampening asset prices and returns.[1] Lower asset returns and prices deteriorate intermediaries' net worth. The decline in net worth tightens intermediaries' leverage constraints which increases the spread in returns and raises the probability of a run by increasing the likelihood that banks become insolvent in case of a widespread panic. Policymakers, including central banks, should strive to close these financial wedges—the spread between the deposit and the lending rate, and the risk of bank run—and address financial distortions.

We then characterize the constrained efficient combination of interest rate tightening and other tools in times of rising inflation and financial fragility. Our baseline approach derives the welfare-maximizing combination of tools, but we also consider the case of a central bank with a strict inflation targeting mandate. Importantly, we allow other tools to be costly as argued in the recent literature: the use of equity injections, credit policy and deposit insurance have fiscal costs, encourage excessive risk-taking, and impair market development. The paper then analyzes the implications of the costs of other tools for the optimal combination of policies.

Crucially, optimal monetary policy should respond less aggressively to inflation, when other tools are costly. This is to account for the impact of interest rate tightening on the intermediaries' net worth though asset prices and returns. We derive a formula clarifying that in the case where the leverage constraint binds, the optimal policy rate is decreasing in the levels of spread and in the sensitivity of spreads to changes in the interest rate. Similarly in the case where there is a risk of a run, the interest rate is decreasing in the likelihood of a run and in the sensitivity of the probability to changes in the interest rate. Importantly, the qualitative findings also hold for a central bank with a strict inflation targeting mandate.

Other policy tools can help alleviate these trade-offs and can allow policymakers to separate financial stability objectives. We derive a complete separation result under the extreme assumption that other tools have no costs. In this case, financial fragility should be addressed with other tools and interest rate policy can ignore its impact on financial variables and focus on inflation stabilization and output maximization. In general, the optimal mix of policy tools depends on several factors. The more fragile the financial system and the higher the marginal costs of other tools, the more interest

[1] Loan delinquencies is another important channel in practice. This is not modeled explicitly, but it is encapsulated by a drop in asset returns.

rate policy should internalize its effect on financial instability and accept to deviate from its conventional stance. Taken together, these findings suggest that Tinbergen Rule (1952) – that achieving every policy target requires an equal number of tools – is a necessary but not sufficient condition for mitigating the trade-offs between price stability and financial stability.

Related literature. This paper contributes to a long-standing literature analyzing macroeconomic fluctuations in an environment with financial frictions and their implications for the optimal design of policies (Bernanke and Gertler, 1989; Bernanke et al., 1999; Kiyotaki and Moore, 1997; Gertler and Kiyotaki, 2010; Cúrdia and Woodford, 2011; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Boissay et al., 2016; Cúrdia and Woodford, 2016; Collard et al., 2017; Drechsler et al., 2018) and more recently Adrian and Duarte (2020) Di Tella and Kurlat (2021),Akinci et al. (2021) as well as Boissay et al. (2022). Like Curdia and Woodford (2010) we find that interest rate policy should take into account credit spreads when the balance sheet constraint of intermediaries bind. Like Gertler and Karadi (2011) and Karadi and Nakov (2021) the use of additional tools such as credit policy can improve welfare, even outside of the ZLB. The structure of the model most closely relates to Gertler and Kiyotaki (2015) and Gertler et al. (2020) who examine the implications of bank runs in a model with balance sheet constraints and sticky prices.[2]

Our overall contribution to this rich literature is threefold. First we provide a global games microfoundation to the probability of a run which we incorporate into a more conventional New Keynesian model with a balance sheet constraint. Second we analytically characterize the constrained efficient combination of interest rate policy and other financial policies in this environment. For example, Adrian and Duarte (2020) look at the implications of financial instability only for the design of interest rate policy and in an environment without runs. Third, we allow for occasionally binding balance sheet constraints. This is similar to Akinci et al. (2021) and also to Boissay et al. (2022) which has occasional endogenous credit market freezes. But instead of comparing Taylor rules, we characterize the non-linear constrained efficient use of interest rate and other tools.

Relative to the lean-against-the-wind literature, we highlight new trade-offs

[2]We also build on the literature looking at the interest rate exposure of banks and the transmission of monetary policy. This literature emphasizes the role of the cash-flow exposure of banks to interest rate risk (Gomez et al., 2021), of uninsured deposits (Drechsler et al., 2023), of profit margins and net worth (Abadi et al., 2023) and balance sheet and interest rate risk management (Di Tella and Kurlat, 2021).

between price and financial stability faced by central banks. This literature was concerned by the build-up of financial imbalances in times of low inflation and low interest rate and has focused on the risk-taking channel of monetary policy, the development of bubbles and the search for yield (Svensson, 2014; Gerdrup et al., 2017; Ajello et al., 2019; Bauer and Granziera, 2017; Abbate and Thaler, 2023). We are instead motivated by financial instability triggered or exacerbated by interest rate tightening in times of rising inflation. In this situation, the trade-offs for monetary policy are different: asset price drops, leverage constraints bind and run risks rise. In the debate whether monetary policy should take into consideration financial stability concerns, we highlight that it crucially depends on the costs of non-interest rate tools. When these other tools are costly, the constrained efficient solution is to implement less aggressive rate hikes. When they are not costly, full separation of objectives is implementable and optimal.[3]

Moreover, our paper adds to recent empirical evidence that monetary tightening can exacerbate financial stability (Demirguc-Kunt and Detragiache, 1998; Schularick et al., 2021; Jiménez et al., 2022; Boissay et al., 2023).[4] We build on the findings of Schularick et al. (2021) and show that rate hikes have potent implications primarily following supply shocks. Our finding that rate hikes entail greater financial stress during supply-driven shocks is consistent with Boissay et al. (2023), who uses a high-frequency identification approach on a recent sample period. Additionally, we document that rate hikes are associated with lower asset prices and bank equity as well as heightened risk of depositor runs. These findings indicate the importance of these channels in explaining how monetary policy tightening affects financial stability.

We contribute to the literature on the optimal use of policies to address financial fragility stemming from coordination failures. The literature has looked at the optimal use of macroprudential tools and bank regulation, public liquidity provision, deposit insurance, and bank resolution in models with bank runs (Vives, 2014; Phelan, 2016;

[3]The papers considering the coordination between monetary policy and other tools usually abstract from their costs (Paoli and Paustian, 2017; Martinez-Miera and Repullo, 2019; Carrillo et al., 2021; Van der Ghote, 2021). In addition, they focus on macroprudential tools while we consider a broader set of other tools, such as credit policy, equity injection and deposit insurance, and analyze the implications of the risk of run, which is empirically important.

[4]The literature primarily highlights that prolonged periods of low interest rates increase financial instability due to risk taking and reach-for-yield by financial institutions (Jiménez et al., 2009; Ioannidou et al., 2009; Maddaloni and Peydro, 2011; Altunbas et al., 2010; Dell'Ariccia et al., 2017; Paligorova and Santos, 2017; Grimm et al., 2023).

Tella, 2019; Dávila and Goldstein, 2023; Schiling, 2023; Ikeda, 2024; Kashyap et al., 2024; Porcellacchia and Sheedy, 2024).[5] We draw on this literature and consider a large set of financial policies. The novelty of our approach is to consider the optimal combination of these tools with monetary policy in times of rising inflation. We do so by embedding a two-period run model in a new keynesian framework with nominal frictions.

Methodologically we relate to Ajello et al. (2019) and even more closely to Basu et al. (2023) who also study optimal policy in a 2-period model with nominal frictions. Like the former, which looks at the joint optimal interest rate policy, foreign exchange interventions and capital flow management, we look at the constrained efficient combination of interest rate, credit policy, equity injection, deposit insurance and macroprudential tools in a fully non-linear setting.

The rest of the paper is organized as follows. Section 2 documents the empirical relationship between monetary policy and financial stability. Section 3 describes the model. Section 4 characterizes the equilibrium path of the economy and show how interest rate tightening gives rise to price and financial trade-offs. Section 5 examines the optimal policy mix of conventional interest rate policy and additional tools. Section 6 concludes.

2 Empirical relationship between monetary policy and financial instability

This section presents empirical evidence of the link between monetary policy and financial instability. In particular, we show that monetary policy tightening can exacerbate financial stability risks, particularly when fluctuations are driven supply-driven shocks. We also find that rate hikes increase the likelihood of both equity crashes and banking panics, indicating that both intermediary constraints and bank runs are important sources of financial instability. Moreover, rate hikes predict lower real stock prices, house prices, and bank lending, suggesting that these are important channels through which monetary policy impact financial stability risk.

Data. We draw on historical data on financial crises and monetary policy from the

[5]These papers and ours build on a long-standing literature analyzing bank runs with a global game approach (Diamond and Dybvig, 1983; Carlsson and van Damme, 1993; Morris and Shin, 2003; Rochet and Vives, 2004; Goldstein and Pauzner, 2005; Bebchuk and Goldstein, 2011).

Jorda-Schularick-Taylor (JST) Macrohistory Database (Jordà et al., 2017) and Baron et al. (2021). The merged dataset spans 18 advanced economies over the period 1870–2016.[6]

We proxy financial stress using three complementary measures. The first is the financial crisis indicator from Schularick and Taylor (2012), which identifies episodes in which "a country's banking sector experiences bank runs, sharp increases in default rates accompanied by large losses of capital that result in public intervention, bankruptcy, or forced merger of financial institutions." The other two measures are from Baron et al. (2021). One measure is the occurrence of bank equity crashes, defined as annual declines in bank equity prices exceeding 30 percent. The other measure captures banking panics, drawing on narrative sources to identify episodes of "severe and sudden withdrawal of funding by bank creditors from a significant part of the banking system," including both solvent and insolvent banks (Baron et al., 2021).

To differentiate between high-inflation episodes driven by supply versus demand shocks, we follow the approach in Jump and Kohler (2022). Shocks are identified based on the signs of residuals in reduced-form regressions of real GDP growth and inflation. Further details on this methodology are provided in Appendix E.

2.1 Impact of monetary policy on financial instability

Our baseline specification for evaluating the systematic impact of monetary policy on financial stability is given by:

$$C_{i,t+h} = \alpha_{i,h} + \beta_h^S SS_{i,t-1} \times \Delta r_{i,t} + \beta_h^D (1 - SS_{i,t-1}) \times \Delta r_{i,t} + \sum_{l=0}^{L} \mathbf{\Gamma}_{h,l} \mathbf{X}_{i,t-l} + \epsilon_{i,t+h} \quad (1)$$

where $C_{i,t+h}$ is indicates whether country i experienced a financial crisis in year t or in any of the following two years, $\alpha_{i,h}$ are country fixed effects, $\Delta r_{i,t}$ is the change in nominal short-term interest rates, which measure yields on three-month government securities and money market rates, and $SS_{i,t-1}$ is an indicator for whether economic fluctuations in country i in year $t-1$ are dominated by supply shocks.

The set of control variables $\mathbf{X}_{i,t}$ include four lags of the following variables:

[6]The countries included are Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

per capita real GDP growth, per capita real consumption growth, per capital real investment growth, CPI inflation, world GDP growth, changes in short-term and long-term interest rates, growth in real stock prices, real house prices, and real bank loans, the current account-to-GDP ratio, as well as the crisis dummy. The set of controls include contemporaneous values of these variables, except for the crisis dummy, and changes in short and long-term interest rates. We also interact all controls with $SS_{i,t-1}$ to allow for state-dependent effects.

The rich set of controls aims to hold fix other channels that explain the correlation between short-term rates and incidence of financial crisis. For example, the growth rate in real stock prices, real house prices, and real bank loans, control for the role of risky credit build-ups, which could explain the tightening of short-term rates and subsequent rise in financial crisis risk. Similarly, real per capita GDP growth and real per capita consumption growth hold fix differences in real economic activity that could explain both changes in the policy rate and the probability of a financial crisis.

Another source of endogeneity concern is the possibility that central banks internalize financial stability risk when setting the policy, which would lead to a downward bias on the impact of rate hikes on financial stress. To this end, we instrument changes in nominal rates with the Trilemma instrument from Jordà et al. (2017). The instrument is based on the economic intuition that, under perfect capital mobility, maintaining an exchange rate peg requires a country to adjust their domestic interest rates to match those of the base country's. Changes in the country's interest rates would thus be plausibly exogenous to local economic and financial conditions.

The trilemma IV is given by

$$z_{i,t} \equiv \left(\Delta r_{b(i,t),i,t} - \Delta \hat{r}_{b(i,t),i,t} \right) \times PEG_{i,t} \times PEG_{i,t-1} \times KOPEN_{i,t}$$

where $r_{b(i,t),i,t}$ is short-term nominal rate of the base country for country i in period t, $PEG_{i,t}$ is an indicator for whether the country's currency is fixed with respect to base b, and $KOPEN_{i,t}$ is an index of financial openness. The effect of monetary policy tightening on financial crisis is estimated using the LP-IV approach. This involves estimating specification (1), but instrumenting $\Delta r_{i,t}$ with $z_{i,t}$.

Results. Figure 1 shows the impact of an interest rate hike on the probability of a financial crisis. The left panel is consistent with the results from Schularick et al. (2021) and shows that a one pp increase in short-term nominal rates increase the probability of a crisis by 1.9 percent on impact. The risk of a financial crisis peaks

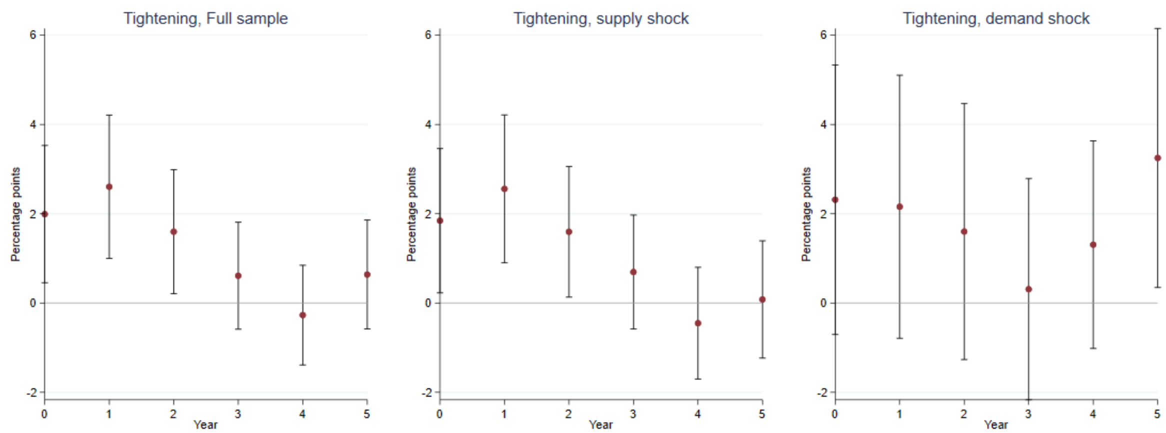


Figure 1: Annual probability of financial crisis following a one percentage point increase in short-term nominal rates. Solid bars denote 90 percent confidence intervals. The full sample depicts the unconditional effect. The other two panels show the effect of a tightening in period t conditional on a supply or demand shock in period $t-1$.

at 2.6 percent one year after the initial rate hike, and remains elevated up to two years after the crisis. Compared to an unconditional annual probability of a crisis of around 3 percent, the magnitudes of the changes in crisis risk are large.

The second and third panels show the impact of rate hikes in periods following supply shocks versus demand shocks. We see that the effect of rate hikes are primarily observed in the periods following supply shocks. The magnitude of the impact is similar to the unconditional sample, with financial crisis risk peaking at 2.5 percent one year after the rate hike. On the other hand, in the periods following demand shocks, financial crisis risk rises but is largely insignificant.[7]

2.2 Channels of transmission

We now investigate the potential channels through which monetary policy affects financial stability. First, we show that monetary policy tightening leads to both heightened risk of bank equity crashes and banking panics. The significance of these two channels suggest important roles of intermediary capacity constraints and

[7]This finding complements those from Boissay et al. (2023), who document that rate hikes exacerbate financial stress in the presence of supply-driven inflation, whereas it dampens financial stress following demand-driven inflation. While we do not find that rate hikes lead to lower financial crisis risk following demand-driven shocks, our findings indicate that monetary policy has significant financial stability implications following supply-driven shocks.

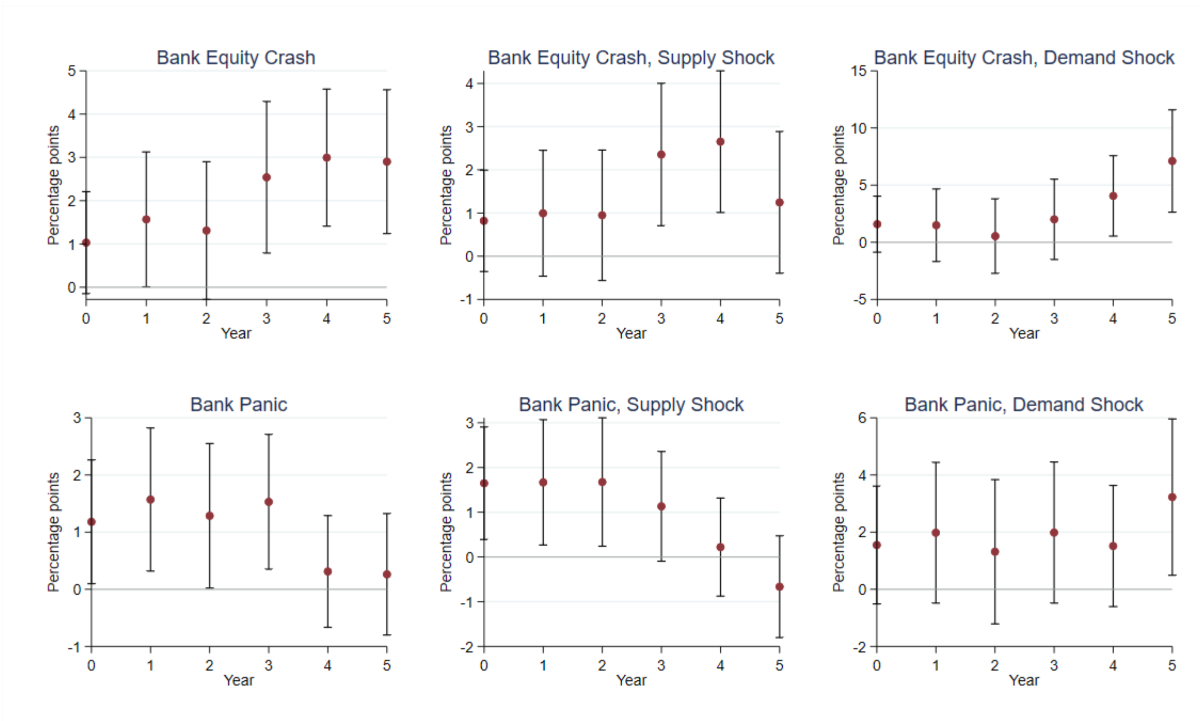


Figure 2: Annual probability of bank equity crashes (top panel) and banking panics (bottom panel) following a one percentage point increase in short-term nominal rates. Solid bars denote 90 percent confidence intervals. The second and third columns show the effect of a tightening and loosening in period t conditional on a supply or demand shock in period $t-1$.

the risk of depositor runs as sources of financial instability. Second, we show that tightening leads to large declines in real stock prices, house prices, and the bank credit. The impacts of rate hikes on house prices and bank credit are not reversed even five years after the initial hike. These findings highlight that monetary policy can have negative implications for asset prices and bank profitability, which in turn matter for financial stability.

Equity crashes and banking panics – Figure 2 shows the impact of a one percent increase in short-term nominal rates on the probability of bank equity crashes and banking panics.[8] The empirical specification is similar to (1), except with lagged controls for the crisis substituted with lagged controls for both indicators of financial stress.

The figure shows that rate hikes lead to a significant increases in the probability of both bank equity crashes and banking panics. Interestingly, the timing for which both sources of financial stress become elevated differs. The likelihood of banking panics rises by 1 percent point in the same year of the rate increase. On the other hand, the likelihood of bank equity crashes become significant three years after the initial rate hike. Given an average unconditional annual probability of a bank equity crash of 3.5 percent and 3.4 percent respectively, these represent sizable increases in financial crisis risk.

The second and third panels show that the effects of rate hikes on these two measures of financial stress is more immediate in the periods following supply shocks. In the years following demand shocks, the probability of bank equity crashes and bank panics is elevated four and five years, respectively, after the initial rate hike. By contrast, supply shocks are associated with increases in bank equity crash risk three years after the rate hike, and with bank panics in the same year of the rate hikes.

Bank credit and asset prices – Next, we examine the impact of rate hikes on bank credit and asset prices. The empirical specification is similar to (1), but with the left-hand variables replaced with cumulative log changes of these variables. The specification excludes contemporaneous values of the dependent variables as controls, but includes for contemporaneous value of the financial crisis indicator to isolate the direct impact of short-term rate hikes on these variables.

Figure 3 shows that a short-term rate hike leads to significant declines in bank

[8] The correlation between these two indicators of financial stress is 0.34.

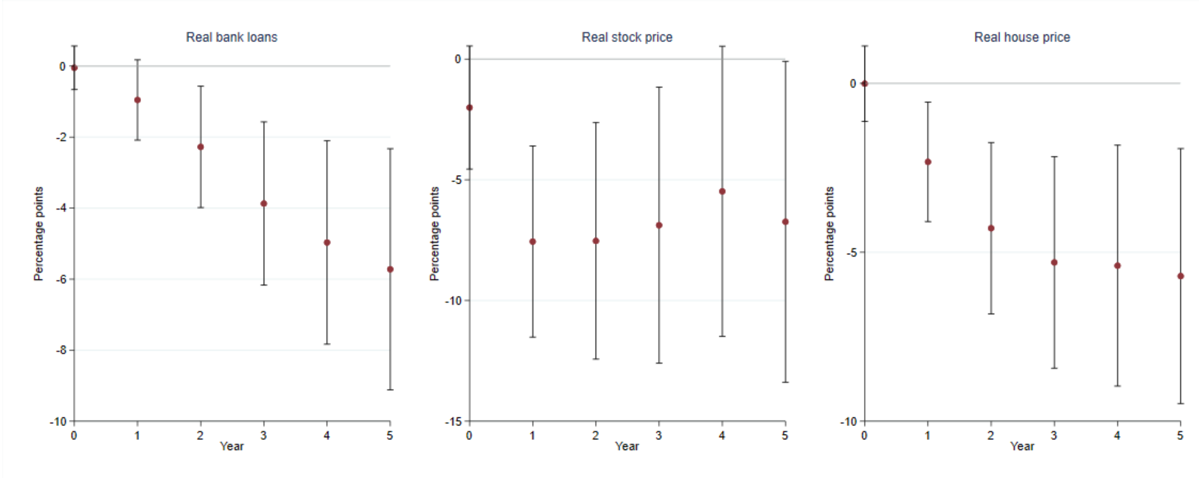


Figure 3: Cumulative log percentage change in real bank loans, real stock price, and real house price following a one percentage point increase in short-term nominal rates. Solid bars denote 90 percent confidence intervals. The second and third columns show the effect of a tightening and loosening in period t conditional on a supply or demand shock in period $t-1$.

credit, stock prices, and house prices. The effect of rate hikes on stock prices tends to be largest one year after the initial rate hike, and dissipates in the subsequent years. On the other hand, short-term rate hikes continue to have an impact bank credit and housing prices several years after the initial rate hike, with a sizable cumulative impact five years after the initial rate hike. The model described in the subsequent section shows that declines in asset prices and lending capacity are key channels through which rate hikes exacerbate financial instability.

3 A Model with Sticky Prices, Constraints to Intermediation and Panics

In this section, we introduce a two-period model of an economy with firms subject to nominal frictions and financial intermediaries subject to financial frictions, including constraints to intermediation capacity and panic-driven runs. The model is closely related to the setting in Gertler and Kiyotaki (2015) and Gertler et al. (2020). The goal is to analyze the optimal conduct of monetary policy after a shock that increases inflation when there are financial fragilities.

3.1 Environment and Sequence of Events

There are 2 periods indexed by $t = 1, 2$ and the economy is populated by five agents: households, financial intermediaries, final good firms, intermediate good producers and the government which includes a central bank.

In the model, there are three frictions. First firms face adjustments cost to changing their goods' prices and nominal wages are fixed in the first period. In the second, all prices, including the real wages, can freely adjust. This captures the notion that prices are sticky in the short run, giving a role to monetary policy in stabilizing prices, but flexible in the medium run. In addition, there are two financial frictions. A leverage constraint, which could arise from an incentive compatibility constraint a la Bernanke and Gertler (1989) or regulatory capital requirements, generates the traditional financial accelerator. A coordination problem among depositors can lead to panic-led runs.

There are three financial instruments: equities of firms, long-term government bonds, and banks short-term deposits. Equities are claims on the residual income of firms after they have paid workers. We abstract from capital accumulation and assume that there is an exogenous mass of capital K which are used for production both in period 1 and 2. In the remainder of the paper, we call this asset "capital". Capital turns into final goods and is consumed by their owners at the end of period 2. The supply of long-term government bonds B is exogenous and controlled by the government. Deposits are endogenous to the financial system's and households' decisions.

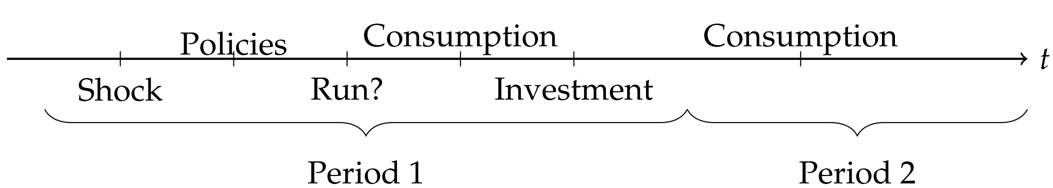


Figure 4: Timeline

As described by Figure 4, at the beginning of the first period, shocks are realized, then the government announces its policies. Knowing the shock and the policies, depositors decide to withdraw their deposits or not. After the outcome of the run game is realized, there is no more uncertainty. At this point, consumers decide how much to consume and work, and firms choose their level of production and prices.

Subsequently households and banks decide their investment and portfolio allocations. In the second period, firms produce, households supply labor and consume their earnings.

3.2 Households: Consumption and Investment Decisions

There is a continuum of mass one of households indexed by $j \in [0,1]$. Lower case letters denote individual variables, while upper case letters denote aggregate variables. We omit the j subscript when no confusion results. Households have the following preferences over the stream of consumption and labor supply:

$$\max_{c_1, c_2, \ell_2} \left\{ \log(c_1) - v_1(\ell_1) + \beta \left(\log c_2 - v_2(\ell_2) \right) \right\}. \tag{2}$$

with $v_t(.)$ is an increasing and concave function for $t = 1, 2$.

Households enter period 1 with a portfolio of investments in long-term bonds b_{h1}, capital k_{H1} and deposits d_1. From these investments they collect returns $\frac{Q_{L1}+r_B}{Q_{L0}}$, $\frac{Q_{K1}+r_{K1}}{Q_{K0}}$ and R_1 respectively, where r_B, Q_{L1} are the exogenous interest rate and the endogenous price of long-term bonds, and r_{k1}, Q_{K1} are the endogenous dividends per unit capital and the price of capital in period 1. Returns on assets depend on the entire equilibrium, including on the outcome of the run.

At the beginning of the period, the run game occurs. We define the households strategies and the equilibrium of the game later in section 3.5. The outcome of the run determines the return on deposits R_1. If no run occurs or doesn't lead to the intermediaries' liquidation, returns on deposits are simply the promised rate at the end of the previous period, $\bar{R}_1$. If it is successful and banks have to be liquidated, the return on deposits R_1 is denoted R_1^*. From the perspective of households, this is a random variable, whose realized value depends on how many depositors withdraw their deposits and on the asset value of the bank relative to its liabilities in the run equilibrium. We define R_1^* formally in the above-mentioned section 3.5. The returns on other assets also depend on the entire equilibrium path. Finally, households collect income from their supply of labor, $W_1 \ell_1$, and receive net-of-tax lump-sum transfers from the government, T_1.

After the outcome of the run is realized and income is collected, households choose how much deposits d_2 to hold at banks, how much long-term bonds to invest in b_{h2}, how much capital to hold k_{H2} and how much to consume c_1. Accordingly, their budget constraint in period 1 is given by

$$P_1 c_1 + d_2 + Q_{L1} b_{h2} + Q_{K1} k_{H2} = R_1 d_1 + (Q_{L1} + r_B) b_{h1} + (Q_{K1} + r_{k1}) k_{H1} + T_1 + W_1 \ell_1 \tag{3}$$

In period 2, households collect income from their portfolio of investments in long-term bonds b_{h2}, capital k_{H2} and deposits d_2, as well as labor earnings $W_2 \ell_2$, and transfers T_2. Importantly, we follow GKP in assuming that households are less efficient at holding capital and bonds than intermediaries.[9] More formally, we assume that the returns on their direct holdings are decreasing with the amount they hold in period 2.[10] For tractability we assume these costs are quadratic and given by $\left(\frac{\beta_K}{2} \frac{P_2 k_{H2}^2}{K}\right)$ for capital holdings and $\left(\frac{\beta_B}{2} \frac{P_2 b_{h2}^2}{B}\right)$ for bonds holdings. In period 2, the budget constraint is thus given by

$$P_2 c_2 = R_2 d_2 + \left(1 + r_B - \frac{\beta_B}{2} \frac{P_2 b_{h2}}{B}\right) b_{h2} + \left(1 + r_{k2} - \frac{\beta_K}{2} \frac{P_2 k_{H2}}{K}\right) k_{H2} + T_2 + W_2 \ell_2 \tag{4}$$

Consistent with the assumption that wages are fixed in nominal terms and prices are sticky in the first period, households supply labor perfectly elastically to firms. In the second period, we assume that $v(\ell_2) = 0$ for $\ell_2 < \bar{\ell}$ and $v(\bar{\ell}) = +\infty$, which implies that they supply inelastically $\bar{\ell}_2$. Households are price-takers in all markets: they take the path of wages W_1, W_2, final goods prices P_1, P_2, asset prices $Q_{L1}, Q_{L2}, Q_{K1}, Q_{K2}$, dividends per unit capital r_{k1}, r_{k2} and of the interest rate on deposits R_1, R_2 as given.

Optimality conditions. Taking the first-order conditions for c_1, c_2, d_2, b_{h2} and k_{H2}, the household' optimality conditions are given by

$$1 = \beta \frac{R_2}{1 + \pi_2} \frac{c_1}{c_2}, \qquad R_2 = \frac{1 + r_B - \beta_B \frac{P_2 b_{h2}}{B}}{Q_{L1}} \quad \text{and} \quad R_2 = \frac{1 + r_{K2} - \beta_K \frac{P_2 k_{H2}}{K}}{Q_{K1}} \tag{5}$$

where $\pi_2 \equiv P_2/P_1 - 1$. The first condition is the traditional Euler equation

[9] This cost also rationalizes why intermediaries exist.

[10] Given that portfolio holdings from period 0 to period 1 are exogenous, we abstract from these costs in the first period.

governing the allocation of consumption between period 1 and 2, the second and third are the no-arbitrage conditions for the long-term bonds and capital respectively. Note that there is no intratemporal optimal condition for period 1, which is consistent with our assumption of fixed wages and elastic labor supply. In period 2, labor is simply $\ell_2 = \bar{\ell}$.

3.3 Final Good Firms

Final good firms buy intermediate goods to produce final goods which they sell to households. They are competitive and take the price of the final good P_1, P_2 and of intermediates $\{P_{1i}, P_{2i}\}_i$ as given. The technology to produce the final good has constant elasticity of substitution, ϵ. They seek to maximize profits subject to the technological constraint. Their problem is given by

$$\max_{\{Y_{ti}\}_i} P_t Y_t - \sum_i P_{ti} Y_{ti} \quad \text{subject to} \quad Y_t = \left(\int_i Y_{ti}^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Optimality conditions. The solution to their problem is given by

$$Y_{ti} = \left(\frac{P_{ti}}{P_t} \right)^{-\epsilon} Y_t. \tag{6}$$

In equilibrium, free entry ensures that final goods producers earn zero profits. Final output Y_t is determined by the goods market clearing condition.

3.4 Intermediate Good Firms

Intermediate goods are differentiated and produced by firms which are in monopolistic competition. These firms combine labor and capital and sell their variety to final goods firms. Following Rotemberg (1984), they face quadratic adjustment costs when choosing their price in the first period $\theta_1 > 0$, but not in the second $\theta_2 = 0$. To simplify the analysis, physical capital K cannot be accumulated and is firm-specific, so that it cannot be moved across firms. Taking wages in both periods (W_1, W_2) as given, their problem is given by:

$$\max_{Y_{ti},P_{ti},\ell_{ti}} P_{ti}Y_{ti} - W_t\ell_{ti} - \frac{\theta_t}{2}\left(\frac{P_{ti}}{P_{t-1i}} - 1\right)^2 P_tY_t \tag{7}$$

$$\text{subject to} \quad Y_{ti} = \ell_{ti}^{\alpha}K_{ti}^{1-\alpha} \quad \text{and} \quad Y_{ti} = \left(\frac{P_{ti}}{P_t}\right)^{-\epsilon} Y_t \tag{8}$$

Optimal pricing decisions. Taking the first-order conditions, their optimal pricing decisions are given by

$$(\epsilon_1 - 1)\left(\frac{\epsilon_1}{\epsilon_1 - 1}\frac{MC_{1i}}{P_{1i}} - 1\right) = \theta_1\pi_1(\pi_1 + 1) \quad \text{and} \quad P_{2i} = \frac{\epsilon_2}{\epsilon_2 - 1}MC_{2i} \tag{9}$$

$$\text{with} \quad MC_{ti} = \frac{W_t}{\alpha}\left(\frac{Y_{ti}}{K_{ti}}\right)^{\frac{1-\alpha}{\alpha}} \tag{10}$$

Labor earnings and capital returns. We assume that two types of lump-sum transfers are enforced by the government: (i) the cost of price changes $\frac{\theta_t}{2}\left(\frac{P_{ti}}{P_{t-1i}} - 1\right)^2 P_tY_t$ are transferred to the government; (ii) each worker receive $\frac{P_{ti}-MC_{ti}}{MC_{ti}}W_t$ per unit of labor from their employer, which means that their post-transfer labor earnings are given by

$$W_t\ell_t = \alpha P_{ti}Y_{ti} \tag{11}$$

These transfers ensure that the returns on capital are a constant fraction of aggregate output. As a result, each period firms distribute to their shareholders—a mix of intermediaries and households—the dividends which are equal to $(1-\alpha)P_{ti}Y_{ti}$. The dividends per unit equity r_{kt} and the total ex post return R_{kt} are given by

$$r_{kt} = (1-\alpha)\frac{P_{ti}Y_{ti}}{K_{ti}} \quad \text{and} \quad R_{kt} = \frac{(1-\alpha)\frac{P_{ti}Y_{ti}}{K_{ti}} + Q_{Kt+1}}{Q_{Kt}} \tag{12}$$

3.5 Financial Intermediaries and the Risk of Run

Financial intermediaries enter period 1 with a portfolio of investments in long-term bonds B_{F1} and capital K_{F1}, and they owe deposits D_1 to households. After the shocks are realized and policies are announced, each household enters the period with a portfolio of assets, (d_{1j}, k_{1j}, b_{1j}), and decides whether to withdraw or to keep their deposits. This section briefly explains the run game and its global game microfoundation. We refer the reader to Appendix B for a more detailed exposition.

Information structure and posterior beliefs. Depositors know their own individual holdings of deposits, d_{1j}, and the overall size of the banks balance sheet, but they don't perfectly observe the composition of their liability, between deposits D_1 and equity N_0. They have two pieces of information to form beliefs about N_0. They know that it is drawn from a log-normal distribution around the end-of-period 0 net worth $\bar{N}_0$ with dispersion σ_N:$\log N_0 \sim \mathcal{N}(\log \bar{N}_0, \sigma_N)$. They also receive a private signal η_j, centered around N_0 with dispersion σ_η: $\log \eta \sim \mathcal{N}\left(\log N_0, \sigma_\eta\right)$. For future reference, we denote $F(\eta|N_0)$ the CDF of this distribution. The posterior belief of household j about N_0 is thus also log-normally distributed:

$$\log N_0 \sim_j \mathcal{N}\left(\mu_{N_0}(\eta_j, \bar{N}_0), \sigma^2_{NP}\right) \tag{13}$$

where μ_{N_0} is an average of η_j and $\bar{N}_0$ weighted by the signal's precision $\sigma_\eta^{-1}, \sigma_N^{-1}$. We denote the density of this posterior distribution $p(n|\eta_j, \bar{N}_0)$.

Condition for a successful run. If a sufficiently large fraction of depositors decide to withdraw, banks aren't able to repay them all and have to be liquidated. Denoting δ the share of depositors who run in equilibrium, a run is successful if:

$$R^*_{k1} Q_{K0}(K - K_{H1}) + R^*_{L1} Q_{L0}(B - B_{H1}) < \bar{R}_1 D_1 \delta \tag{14}$$

where the asterisk * denotes variables in the "run" equilibrium—the equilibrium in which banks have to be liquidated. In such an equilibrium, all assets have to be held by households or the government, i.e. $K^*_{F2} = B^*_{F2} = 0$, which leads to a drop in their prices (which justifies the run ex post).[11] Asset prices and returns on assets are

[11] In GKP banks' equity recovers slowly as new banks enter, we make the simple assumption that no bank enters in period 2.

given by

$$R_{k1}^* = \frac{r_{K1}^* + Q_{K1}^*}{Q_{K0}} \quad \text{and} \quad R_{L1}^* = \frac{r_B + Q_{L1}^*}{Q_{L0}} \tag{15}$$

$$Q_{K1}^* = \frac{1 + r_{K2} - \beta_K \frac{P_2(K-K_G)}{K}}{R_2} \quad \text{and} \quad Q_{L1}^* = \frac{1 + r_B - \beta_B}{R_2} \tag{16}$$

where we used the households optimal conditions (5) to pin down asset prices. Using the previous equation (14), a run is successful if and only if the share of depositors running exceeds a threshold: $\delta > \bar{\delta}(N_0)$ with $\bar{\delta}(N_0) = \frac{R_{k1}^* Q_{K0}(K-K_{H1}) + R_{L1}^* Q_{L0}(B-B_{H1})}{\bar{R}_1[Q_{K0}(K-K_{H1}) + Q_{L0}(B-B_{H1}) - N_0]}$.

Payoffs and equilibrium. If a depositor runs and the run is successful, it gets a share of the bank liquidation value proportional to its deposits d_{j1}. If they don't run, they lose all their deposits.[12] A depositor who runs always incurs a small exogenous utility cost of running ζ. Without this utility cost, running would always be a dominant strategy. Denoting the after-run indirect utility of a depositor with initial deposits d_1, $U(R_1 d_1)$ (and omitting the dependence on k_{H1j}, b_{H1j} for simplicity)[13], Table 1 summarizes the payoffs for each action and in each equilibrium outcome.

	In equilibrium the run is ...	
Individual action	Successful	Unsuccessful
Run	$U\left(\frac{R_{k1}^* Q_{K0}(K-K_{H1}) + R_{L1}^* Q_{L0}(B-B_{H1})}{\bar{R}_1[Q_{K0}(K-K_{H1}) + Q_{L0}(B-B_{H1}) - N_0]\delta} d_1\right) - \zeta$	$U(\bar{R}_1 d_1) - \zeta$
Don't run	$U(0)$	$U(\bar{R}_1 d_1)$

Table 1: Payoffs in Four Cases

In equilibrium, depositors adopt a trigger strategy: they run if their private signal is below a threshold $\bar{\eta}$. The equilibrium share of depositors running is the share of those receiving a signal below this threshold, $\delta^*(N_0) = F(\bar{\eta}|N_0)$. Importantly, the threshold $\bar{\eta}$ is endogenous and should be such that a depositor with signal $\bar{\eta}$ is indifferent between running and not running:

[12] The setup assumes that deposits are uninsured. When considering optimal policy, we allow for the possibility that governments provide deposit insurance to prevent runs.

[13] R_2 enters into depositors' payoffs only through R_{K1}^* and R_{L1}^*. Note that, if the run is unsuccessful, the return on deposits is $\bar{R}1$ because these are deposits held by banks between period 0 and 1.

$$\int_0^{\max(\bar{N},0)} \left[U \left(\frac{R^*_{k1} Q_{K0}(K - K_{H1}) + R^*_{L1} Q_{L0}(B - B_{H1})}{\bar{R}_1 \left[Q_{K0}(K - K_{H1}) + Q_{L0}(B - B_{H1}) - n \right] F(\bar{\eta}|n)} D_1 \right) - U(0) \right] p(n|\bar{\eta}) dn = \zeta \tag{17}$$

where $\bar{N}$ is the level of net worth such that even if all depositors run, the run is unsuccessful $\bar{N} = \frac{(\bar{R}_1 - R^*_{k1}) Q_{K0}(K - K_{H1}) + (\bar{R}_1 - R^*_{L1}) Q_{L0}(B - B_{H1})}{\bar{R}_1}$.

3.6 Financial Intermediaries if No Run Happened

If a run doesn't happen or if the run isn't successful, financial intermediaries continue to operate from period 1 to period 2. Their period-1 equity depends on the returns on assets and payments on liabilities $R_1 = \bar{R}_1$ made at time 1:

$$N_1 = \bar{R}_1 N_0 + (R_{k1} - \bar{R}_1) Q_{K0} K_{F1} + (R_{L1} - \bar{R}_1) Q_{L0} B_{F1} + N_G \tag{18}$$

where $\bar{R}_1, B_{F1}, K_{F1}, Q_{K0}$ are all exogenous in period 1 and N_G denotes equity injection by the government. Returns on both types of assets R_{k1} and R_{L1} are endogenous.

Unconstrained portfolio allocation and no-arbitrage. At the end of period 1, financial intermediaries collect households deposits, D_2, and invest in capital, K_{F2} and in long-term bonds B_{F2}. Taking all asset prices and returns as given, they seek to maximize the end of period-2 N_2. When intermediaries can freely allocate their portfolio they would arbitrage away any differences in returns:

$$R_2 = R_{K2} = R_{L2} \tag{19}$$

Incentives-compatible balance sheet constraint. Following the financial accelerator literature, we assume that due to an agency problem the intermediaries' overall investments cannot exceed a multiplier of their equity. Denoting the maximum leverage ϕ^P, the constraint is given by

$$\phi^P N_1 \geq Q_{L1} B_{F2} + Q_{K1} K_{F2} \tag{20}$$

We assume that $\phi^P > 0$ is an exogenous parameter, like for example in Di Tella and Kurlat (2021). One could endogenize ϕ^P, in which case the maximum leverage ϕ^P

would increase with the spread R_{k2}/R_2 which would partly mitigate the amplification stemming from the constraint.[14] Given that this second round mechanism doesn't qualitatively affect our results, but would substantially complicate the analysis, we keep ϕ^P exogenous in the rest of the paper.

When this balance sheet constraint binds, returns on bonds and capital are determined by the households conditions (5) and rise above the interest rate on deposits, $R_{K2} = R_{B2} > R_2$. This is because households are the marginal buyer and they require a compensation for holdings these assets.

Macroprudential policy Consistent with the development of macroprudential tools since the Great Financial Crisis, the government in the model can implement an equity-based balance sheet constraint, ϕ^G which takes the exact same form as the incentive-based constraint (20). We can write both the incentive-based and the macroprudential-based constraints together by replacing ϕ^P in the inequality (20) with $\phi = \min(\phi^P, \phi^G)$.

3.7 Government, Central Bank and the Costs of Tools

The central bank controls the rate on deposits R_2. The government can issue short-term deposits D_{G2}, purchase equities K_G, inject equity into the banking system N_G, and sets transfers T_1, T_2. In addition, the government has to pay interest on long-term bonds B in period 1, and repay the principal in period 2. These choices have to be consistent with the following budget constraints:

$$D_{G2} = T_1 + \frac{\theta_t}{2}\pi_1^2 P_t Y_t + r_B B + Q_{K1} K_G + N_G \tag{21}$$

$$\left(1 + r_{k2} - \frac{\beta_G}{2}\frac{P_2 K_G}{K}\right) K_G - \frac{\beta_N P_2}{2} N_G^2 = T_2 + (1 + r_B)L + (1 + R_2) D_{G2} \tag{22}$$

where T_1, T_2 may be negative or positive and where we have assumed that all firms were symmetric.

Importantly, we assume that the use of tools such as equity injections, credit policy, and deposit insurance, are costly. We model these costs in a reduced-form way

[14] Following Gertler and Karadi (2011), and consistent with our assumption that bankers go back to being households in period 2, ϕ^P would be given by: $\phi^P = \frac{\eta}{\lambda - \nu}$ with $\eta = 1$ and $\nu = \frac{R_{k2}}{R_2} - 1$ where λ is the fraction of funds the banker could divert (see equations 11 and 13 in their paper).

as quadratic pecuniary losses in the government's budget constraint. These costs capture several implementation challenges and negative implications associated with the use of these tools. First, these tools can have fiscal costs. For example, purchase of risky assets exposes the central bank to financial losses. Deposit insurance by governments may trigger a large unexpected spending in case of a run (Allen et al., 2011). These fiscal costs entail real costs when taxation is distortionary. Second, generous central bank intervention, public equity injection and deposit insurance can introduce moral hazard and incentivize risk-taking (Cooper and Ross, 2002). Additional costs have been highlighted in the literature. For example, large scale credit policy could lead to mispricing of risk premia and can become "addictive" (Steeley, 2015; Karadi and Nakov, 2021).

We define the joint objective of the government and the central bank in the section 5 on optimal policies.

3.8 Market Clearing

The labor and goods market clear. Given that price adjustments consume real resources in the first period, the market clearing for final goods in the first period is given by

$$Y_1 \left(1 - \frac{\theta}{2}\pi_1^2\right) = C_1 \tag{23}$$

where $C_1 = \int c_{1j} dj$ where each individual household is indexed by j. Given that real wages are flexible in the second period, the level of the price level is indeterminate. We thus normalize $P_2 = P_1$ and let W_2 adjust so that the real wage clears the labor market. This assumption allows us to abstract from inflation in goods prices from period 1 to period 2 and focus on inflation from period 0 to period 1 only.

$$\pi_2 = 0 \tag{24}$$

In addition, the capital, long-term bond and short-term deposits markets also clear:

$$K = K_{H2} + K_{F2} + K_G, \quad B = B_{H2} + B_{F2} \quad \text{and} \quad D_2 = D_{F2} + D_{G2} \tag{25}$$

where $D_2 = \int d_{2j} dj$, $B_{H2} = \int l_{H2j} dj$ and $K_{H2} = \int k_{H2j} dj$ and j indexes an individual household.

4 Price, Output and Financial Stability Trade-offs

We now characterize the equilibrium path of the economy with a small number of equations: the Phillips Curve ($\mathcal{PC}$), the Euler Equation ($\mathcal{EE}$), and in the parts of the state space where banks' equity is low, the Balance Sheet Constraint ($\mathcal{BSC}$) and a Run Equation ($\mathcal{RE}$). We highlight the wedges capturing the distortions implied by the two financial frictions and analyze how interest rate tightening, by increasing the risk of run and exacerbating the balance sheet constraint, gives rise to two price-financial stability trade-offs, adding to the well-known output-inflation trade-off.

4.1 The Trade-off Between Output and Inflation

In period 1, the set of equations pinning down the equilibrium can be split into two subsets. First, the equations determining consumption, output and prices. Second, the no-arbitrage conditions across assets, the run condition and the balance sheet constraint, which together determine the equilibrium holdings of assets by financial intermediaries and households. We start with the former to highlight the trade-off between output and inflation.

We derive the Phillips Curve from combining the intermediary firm's optimal pricing condition (9), the optimality condition of final goods firms (6), the symmetry of intermediary firms and the production function of final goods firms $Y_{i1} = Y_1$, the definition of marginal cost (10) and the assumption that wages are fixed in period 1:

$$(\epsilon_1 - 1)\left(\frac{\epsilon_1}{\epsilon_1 - 1}\frac{\underline{W}}{(1+\pi_1)P_0\alpha}\left(\frac{Y_1}{K}\right)^{\frac{1-\alpha}{\alpha}} - 1\right) = \theta_1\pi_1(\pi_1 + 1) \quad (\mathcal{PC}) \tag{26}$$

The Phillips Curve relates the level of inflation π_1 to the level of output Y_1 in period 1. The second important equation is the Euler equation, the optimality condition governing the intertemporal allocation of consumption of households which is given by (5):

$$Y_1 \left(1 - \frac{\theta}{2}\pi_1^2\right) = \frac{C_2}{\beta R_2}. \quad (\mathcal{E}\mathcal{E}) \tag{27}$$

where we used the goods market clearing condition in period 1 (23).

The Phillips Curve and the Euler Equation determine inflation π_1 and output Y_1 in period 1 as a function of C_2 and the policy rate R_2. The other variables related to production in period 1 are directly implied by the equilibrium level of Y_1 and π_1: consumption C_1 is closely related to output Y_1 through the market clearing condition, and labor supply adjusts to accommodate the needs of firms: $\ell_1 = \left(\frac{Y_1}{K^{1-\alpha}}\right)^{\frac{1}{\alpha}}$.

Trade-offs between price and output stabilization. The Phillips Curve and the Euler Equation are sufficient to illustrate the well-known static trade-off between output and inflation central banks face when setting the interest rate R_2 following a markup shock (an increase in $\frac{\epsilon_1}{\epsilon_1 - 1}$). From the Phillips curve $(\mathcal{PC})$, we see that when a markup up shock hits the economy, inflation π_1 increases for a given level of output Y_1. If the central bank responds by increasing R_2, it is clear from the Euler Equation $(\mathcal{E}\mathcal{E})$ that consumption C_1, hence output Y_1 should decrease. We will illustrate this point in the calibrated model in the next subsection.

4.2 The Trade-off Between Inflation and Intermediation Capacity

We now analyze how policy rate increases in response to a rise in inflation can negatively affect the intermediation capacity of banks. We show that this trade-off between inflation and intermediation capacity has both an extensive and an intensive margin. High interest rates can cause the balance sheet constraint to bind and a wedge to open up between the deposit rate and returns on assets. Once the economy is in this "constrained" zone additional rate hikes further decrease intermediation capacity and widen the wedge. In this zone, the model is simply described by a Phillips Curve, an Euler Equation and a Balance Sheet Constraint.

Costs of Limited Intermediation and Returns Spread. The costs of limited intermediation appears in the resource constraint in the second period:

$$C_2 = \bar{\ell}^{\alpha} K^{1-\alpha} + K - \left(\frac{\beta_B}{2}\frac{B_{H2}}{B}\right) B_{H2} - \left(\frac{\beta_K}{2}\frac{K_{H2}}{K}\right) K_{H2}$$

where we used the labor supply ℓ and production technology $Y_2 = \bar{\ell}^{\alpha} K^{1-\alpha}$ and assumed no government interventions.[15] These costs are strictly increasing in the two state variables inherited from period 1: K_{H2}, B_{H2}. These costs, which lower welfare, arise when the balance sheet constraint of intermediaries bind in the first period and households hold part of the capital stock and long-term bonds.

When the balance sheet constraint of intermediaries doesn't bind, intermediaries hold the entire stock of assets and they arbitrage away any spread between the policy rate and the rate of returns on assets $R_{K2} = R_2$. In that case financial variables are irrelevant to the real allocation and welfare. When the balance sheet constraint binds, households hold part of the capital stock and a wedge opens up between the deposit rate and the returns on asset, $R_{k2}(1-\sigma) \equiv R_2$. Using the first order condition of households (5), the wedge σ is strictly increasing in K_{H2} and equal to 0 when $K_{H2} = 0$:

$$\sigma \equiv 1 - \frac{R_2}{R_{k2}} = \beta_K \frac{P_2 K_{H2}}{K(1+r_{K2})}.$$

This returns spread is the wedge capturing the distortions implied by the balance sheet constraint which policymakers would like to close.

Trade-off (extensive margin). A first illustration of the trade-off faced by monetary policy when hiking the interest rate in response to a rise in inflation is that the balance sheet constraint is more likely to bind. This is because the drop in the asset value depletes their net worth. To see this, recall that a necessary and sufficient condition for the constraint to bind is that intermediaries are able to hold all assets in the

[15] For future reference, we can also determine W_2 from the assumption of flexible prices and wages in the second period and the normalization $P_1 = P_2$, $W_2 = \left(\frac{\epsilon_2}{\epsilon_2 - 1}\frac{1}{\alpha P_1}\left(\frac{\bar{\ell}^{\alpha} K^{1-\alpha}}{K}\right)^{\frac{1-\alpha}{\alpha}}\right)^{-1}$

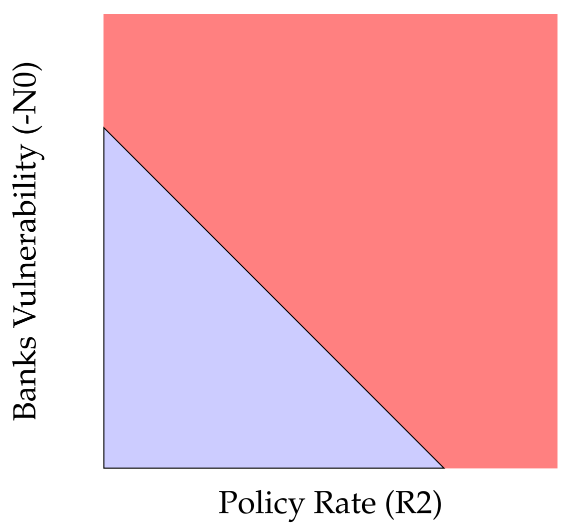
economy, namely

$$\bar{R}_1 N_0 + (R_{k1} - \bar{R}_1) Q_{K0}(K - K_{H1}) + (R_{L1} - \bar{R}_1) Q_{L0}(B - B_{H1}) + N_G > \frac{Q_{L1}B + Q_{K1}K}{\phi} \tag{28}$$

with $Q_{K1} = \frac{1 + r_{K2}}{R_2}$, $Q_{L1} = \frac{1 + r_B}{R_2}$, $R_{k1} = \frac{r_{K1} + Q_{K1}}{Q_{K0}}$, and $R_{L1} = \frac{r_B + Q_{L1}}{Q_{L0}}$.

From this equation, we see that the drop in asset prices caused by rate hikes leads the right-hand side to decrease faster than the left-hand side up to the point where the inequality holds. The following lemma formalizes the idea that the constraint is more likely to bind whenever, for a given level of N_0, the policy rate R_2 is high enough. Figure 4.2 illustrates the split of the state space $(-N_0, R_2)$ between the "constrained" zone in red and the "financial stability" zone in blue.

Lemma 1. *Under regularity conditions, there exists a strictly increasing and continuous function $\bar{R}_2(N_0)$ for $N_0 \geq 0$ such that (28) holds if and only if $R_2 < \bar{R}_2(N_0)$.*



The regularity conditions are as follows: households' holdings are not too large relative to the leverage ratio $\frac{\phi-1}{\phi} > \max\left(\frac{K_{H1}}{K}, \frac{B_{H1}}{B}\right)$, households hold positive deposits $D_1 > 0$ and the Phillips curve is upward sloping (inflation increases with output at period 1) and not too steep. The assumptions of the lemma are in general true. For example, if households don't hold any asset from time 0 to time 1, $K_{H1} = B_{H1} = 0$ it simply says that banks are allowed to have a positive leverage,

$\phi > 1$. The second assumption simply says that banks enter period 1 with some positive leverage.

Trade-off (intensive margin). Besides this extensive margin of the trade-off implied by high policy rate, additional increases in the interest rate further lowers intermediation capacity. To illustrate this intensive margin of the trade-off between preserving intermediation capacity and inflation faced by interest rate policy R_2, we derive the Balance Sheet Constraint, which together with the Euler Equation and the Phillips Curve, characterize the equilibrium path of the economy in the constrained zone. Using the optimality portfolio conditions of households (5) and the definition of returns in period 1 (12) to substitute for the equilibrium asset prices and returns in the balance sheet constraint (20), we obtain:

$$\begin{aligned}
&\bar{R}_1 N_0 - \bar{R}_1 \left(Q_{K0}(K - K_{H1}) + Q_{L0}(B - B_{H1})\right) + N_G + r_{K1}(\pi_1, C_1)(K - K_{H1}) \\
&+ r_B(B - B_{H1}) + \frac{1 + r_B - \beta_B \frac{P_2 B_{H2}}{B}}{R_2} \left(B - B_{H1} - \frac{B - B_{H2}}{\phi}\right) \\
&+ \frac{1 + r_{K2} - \beta_K \frac{P_2 K_{H2}}{K}}{R_2} \left(K - K_{H1} - \frac{K - K_{H2} - K_G}{\phi}\right) = 0 \quad (\mathcal{BSC})
\end{aligned}$$

This $\mathcal{BSC}$ equation pins down the equilibrium portfolio holdings of households K_{H2} and B_{H2} as a function of Y_1, π_1, and policy interventions, including the policy rate R_2. For future reference, we thus denote this function $\mathcal{BSC}(K_{H2}, K_G, R_2, N_0, N_G)$. The split of portfolios between K_{H2} and B_{H2} is in turn given by combining the no-arbitrage condition of banks: $\frac{1+r_{K2}}{Q_{K1}} = \frac{1+r_B}{Q_{L1}}$ with the no-arbitrage conditions of households (5), which gives

$$\frac{\beta_B B_{H2} K}{B \beta_K K_{H2}} = \frac{1 + r_B}{1 + r_{K2}} \tag{29}$$

Figure 5 illustrates the equilibrium outcomes as a function of the interest rate policy R_2.[16] When interest rates are sufficiently low, the leverage constraint BSC is slack. In this region, raising interest rates dampens inflation π_1 but it also leads to lower output $Y1$. Higher interest rates also lead to lower price of capital Q_K and long-term government bonds Q_B. While this reduces the net worth of intermediaries, the leverage constraint remains slack and intermediaries remain the marginal buyer

[16] Appendix D discusses the calibration of the model.

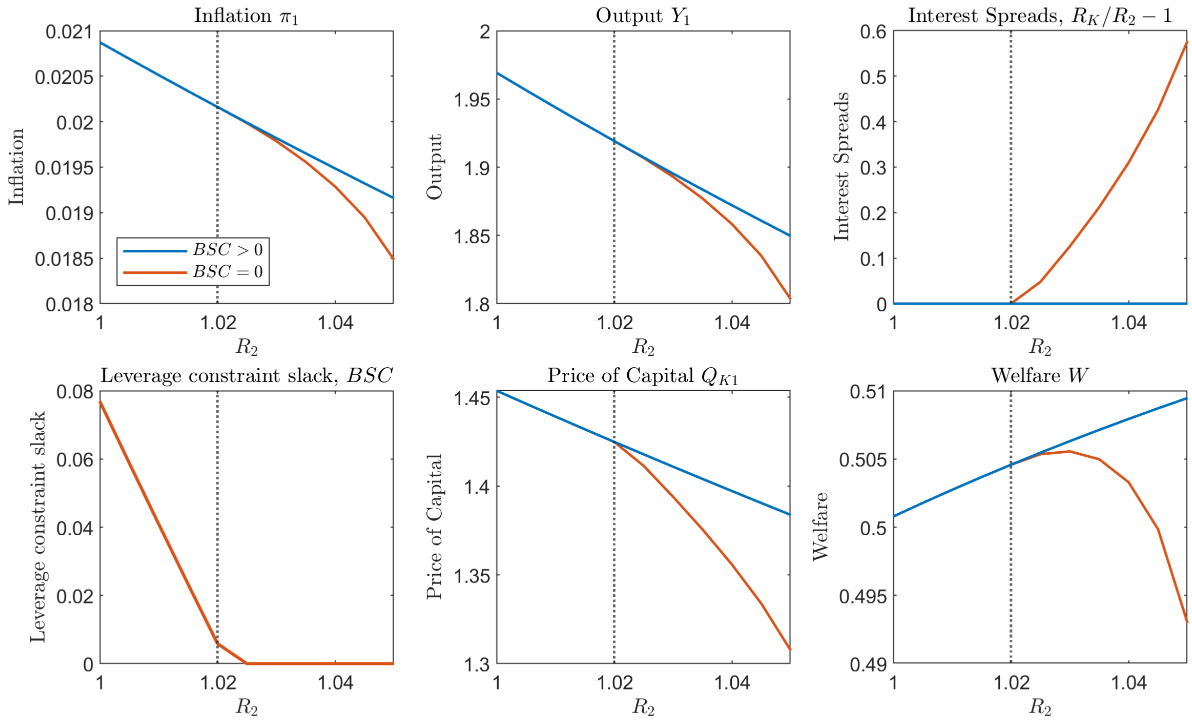


Figure 5: Equilibrium as a function of R_2. Blue line shows the equilibrium where the balance sheet constraint does not bind $BSC \equiv (\phi N_1 - Q_{L1}B_{F2} - Q_{K1}K_{F2}) > 0$ (or never binds counterfactually). The red line shows the equilibrium where the constraint binds $BSC = 0$. The vertical dashed line shows the interest rate above which the constraint binds.

of capital. This keeps interest rate spreads at zero.

When interest rates are sufficiently high, the balance sheet constraint binds. In this situation, intermediaries are unable to fully intermediate all assets in the economy and households become marginal buyers. Since households need to pay an efficiency cost for holding assets, there is a positive spread between between the policy rate and the return on capital $R_K - R_2$ and long-term bonds $R_B - R_2$ in equilibrium. The positive spreads further depresses asset prices Q_{K1} and exacerbates the balance sheet constraint. Relative to a counterfactual situations where the balance sheet constraint does not bind, both inflation and output in period 1 are lower. Notably, this happens through an intertemporal effect as households save more today in anticipation of the efficiency losses associated with households' holding of capital in period 2.

4.3 The Trade-off Between Inflation and the Risk of a Run

We now analyze how higher interest rates increases the risk of a run. Like for intermediation capacity, we show that the trade-off has two related manifestations. On the extensive margin, high enough interest rates make the probability of a run—the wedge capturing the coordination failures—positive. In this "run zone", further rate hikes exacerbate the coordinate failures and increase the risk of a run. In this part of the state space, the model boils down to a Phillips Curve, an Euler Equation and a Run Equation.

The likelihood of a run. The likelihood of a successful run ξ is the second wedge related to financial frictions that policymakers would like to close. It is the probability that the share of depositors who run δ is above some threshold:

$$\xi = P\left(\delta \bar{R}_1 D_1 > R^*_{k1} Q_{K0}(K - K_{H1}) + R^*_{L1} Q_{L0}(B - B_{H1}) | \bar{N}_0\right) \tag{30}$$

where $D_1 = Q_{K0}(K - K_{H1}) + Q_{L0}(B - B_{H1}) - N_0$ is unknown since policymakers, like households, don't perfectly observe N_0. As a result, the probability is conditional on the information set policymakers have at the beginning of time 1, which is only that the true value is drawn from a log-normal distribution with mean $\bar{N}_0$.

The equilibrium share of depositors who run is equal to those who receive a signal lower than a threshold $\bar{\eta}$, namely $\delta = F(\bar{\eta}|N_0)$ (see section 3.5). This threshold is endogenous to the values of asset returns in the run equilibrium and to the interest rate, as shown in equation (17). The higher the interest rate the stronger the incentives to run, the lower the threshold, the higher δ, everything else equal.

The probability ξ, together with the equation determining $\bar{\eta}$ (17), is the third equation of the model when a run happens with positive probability. We call this (pair of) equation the Run Equation ($\mathcal{RE}$). It is easy to see that it is a function of the period-0 equity of banks N_0 and government policies, including the policy rate R_2. We denote this function $\xi(R_2, N_0, K_G, N_G)$. It is differentiable and increasing in R_2 and decreasing in N_0.

Trade-off (extensive margin). For any given level of banks equity, a run is more likely to occur with positive probability when the policy rate is high enough. This is the first illustration of the trade-offs interest rate policy faces when tightening. To formalize this idea, recall from section 3.5 that if N_0 is above $\bar{N} = (1 - R^*_{k1}/\bar{R}_1)Q_{K0}(K - K_{H1}) + (1 - R^*_{L1}/\bar{R}_1)Q_{L0}(B - B_{H1})$, the run is unsuccessful even if all depositors run. An immediate corollary is that runs can occur with positive probability whenever intermediaries are less well capitalized or when the policy rate R_2 is high enough.

Lemma 2. *Under regularity conditions, there exists a strictly increasing and continuous function $\tilde{R}_2(N_0)$ for $N_0 \geq 0$ such that $\xi = 0$ if $R_2 < \tilde{R}_2(N_0)$.*

The intuition is as follows: when interest rates increase, the asset value drops, which decreases banks equity relative to deposits. This makes it more likely that banks won't be able to repay all their depositors if it were to be liquidated. The split of the state space between "run" zone in red and "financial stability" zone in blue would be qualitatively similar to Figure 4.2. However, the location where the split occurs depends on model parametrization. Intuitively, the leverage constraint is likely to bind at lower levels of interest rates and bank vulnerability compared to when the risk of run becomes positive. However, there could be situations where run risk turns positive even when the leverage constraint is slack. For example, Silicon Valley Bank consistently reported capital ratios above its minimum regulatory requirements before its failure in March 2023 (FRB, 2023).

The regularity conditions for the lemma to hold are that households hold positive deposits $D_1 > 0$ and the Phillips curve is upward sloping (inflation increases with output in period 1) and not too steep. The proof of this result is simple. Under the regularity conditions R^*_{k1} and R^*_{L1} are continuous and strictly decreasing in R_2. $\bar{N}$ is strictly decreasing and differentiable in R^*_{k1} and R^*_{L1}. We can thus define a strictly increasing, differentiable function $\bar{N}(R_2)$. Given it is strictly increasing, we can invert it, and define $\tilde{R}_2(N_0)$.

Trade-off (intensive margin). Once the economy lies inside the "run" zone, further increases in the policy rate R_2 increases the probability of a run ξ. We use the three equations summarizing the model—the Run Equation, the Euler Equation and the Phillips Curve— to illustrate the trade-off central banks face, when setting the interest rate, between decreasing the likelihood of a run and taming inflation in period 1.

Figure 6 illustrates the equilibrium outcomes both inside and outside the run zone. When interest rates R_2 are sufficiently low, the economy lies outside the run zone. Here, intermediaries' net worth is sufficiently high so that there is no incentive for depositors to coordinate on a run.

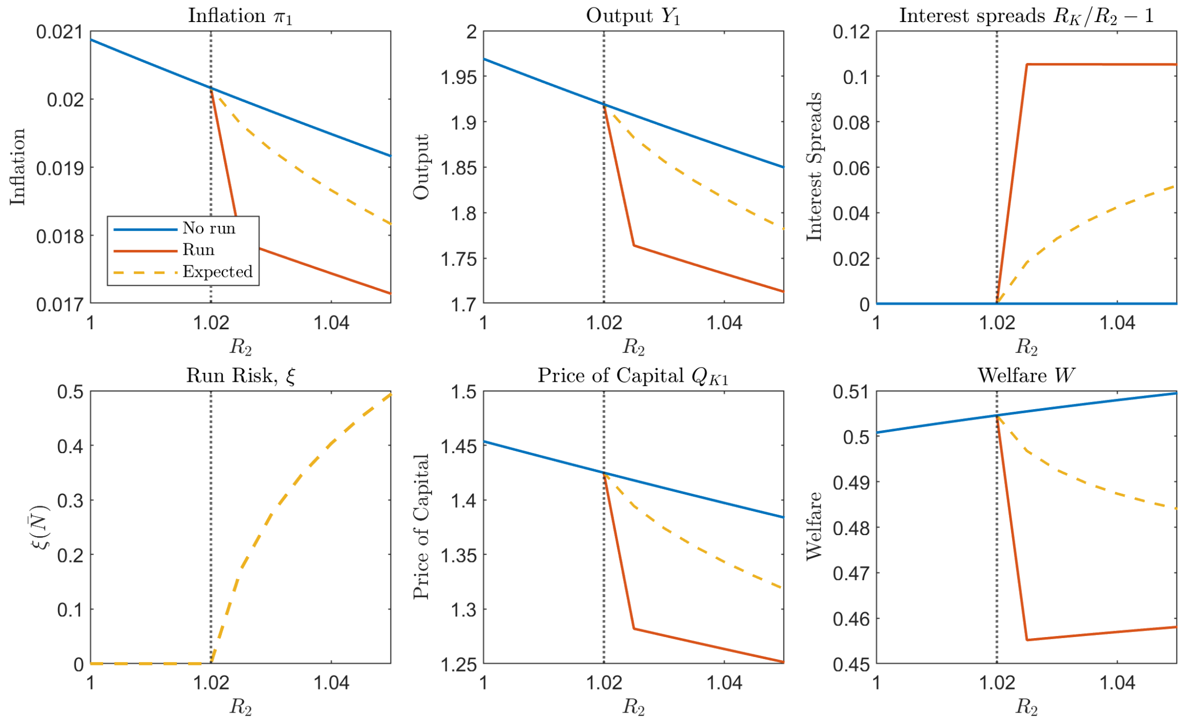


Figure 6: Equilibrium as a function of R_2. Blue line shows the equilibrium where there is no run risk, or a run does not occur. The red line shows the equilibrium where a run occurs. The dashed yellow line is the average of the two equilibria, weighted by the probability of a run. The vertical dashed line shows the interest rate above which there is positive run risk.

The interesting region lies inside the run zone (to the right of the vertical line). Here, there is a positive probability that banks become insolvent if a large enough share of depositors coordinate on withdrawing their deposits. If the run materializes, banks are forced into liquidation and interest spreads $R_K - R_2$ spike significantly. The large efficiency losses in period 2 leads households to increase their savings in period 1. This translates into the large fall in inflation π_1 and output Y_1 seen in period 1.

The dashed yellow line shows the expected outcomes, which is the average of the two run equilibria weighted by the probability of a run. Central banks consider the average outcome when deciding on the optimal policy. Note that this outcome is not observed. For a given interest rate, the equilibrium that materializes is on the blue line if the run does not occur and on the orange line if the run occurs.

5 Optimal Policies in Times of Rising Inflation and Financial Instability

The previous section described the financial-frictions-implied wedges (σ, ξ) and the trade-offs faced by monetary policy. We now characterize the constrained efficient combination of interest rate policy and other tools in times of rising inflation, and analyze how it depends on the costs of the tools (β_K, β_N). We proceed one financial friction at a time, starting with the case where the leverage constraint binds then where the run equilibrium exists. We close by analyzing the extreme case when the run occurs with probability 1.[17]

5.1 Policymakers' Objectives and Instruments

The baseline objective of policymakers is to maximize the expected households' welfare, which is a weighted sum of (log) consumption in the current and future period and inflation subject to the restriction that the allocation is a competitive equilibrium. Policymakers have three instruments $\{K_G, N_G, R_2\}$ which are chosen after the shock is realized, but before the run game happens.

Definition 1 (Constrained Efficient Allocation). *A constrained efficient allocation is a set of quantities $\{Y_1, C_1, Y_2, C_2, \ell_1, \ell_2, K_{F2}, K_{H2}, B_{F2}, B_{H2}\}$, policies $\{K_G, N_G, R_2\}$ and prices and returns $\{W_1, W_2, \pi_1, \pi_2, Q_{k1}, Q_{L1}, r_{K1}, r_{K2}\}$ which solve*

$$W = \max_{C_1, C_2, Y_1, Y_2, \ell_1, \ell_2, K_{F2}, B_{F2}, K_{H2}, B_{H2}, R_2, N_G, K_G} \mathbb{E}\left[(\log C_1 - v(\ell_1)) + \beta(\log C_2 - v(\ell_2)) | \bar{N}_0\right],$$

subject to the constraints that the allocation is a competitive equilibrium of the economy.

The expectation $\mathbb{E}(.)$ captures only the uncertainty about the risk of a run. It is taken under the information set available to policymakers at the beginning of time 1.

[17] These scenario resemble but differ from those considered in the IMF Staff Discussion Note (Bouis et al. (2024)). Specifically, Scenario A (No Stress) corresponds to a case where the leverage constraint does not bind and the run equilibrium does not exist. Scenario B (Modest to Moderate Stress) corresponds to a case where the leverage constraint binds, but the increase in spreads is moderate. Additionally, the run equilibrium does not exist. Scenario C (Heightened Stress) corresponds to a case where banks are severely under-capitalized so that the leverage constraint binds, there is a large increase in spreads, and the run equilibrium exists. Note that in the model, a run equilibrium can exist without the leverage constraint binding. Scenario D (Full Fledged Financial Crisis) corresponds to a case where a systematic run has occurred or the net worth of banks has been fully depleted.

The only variable they can't perfectly predict is N_0. As discussed above, we assume they know the mean $\bar{N}_0$ of the distribution from which it is drawn.

We also consider an alternative objective whereby central banks seek to minimize deviation of inflation from a target, which we denote $\bar{\pi}$ and the rest of the government chooses other tools to maximize welfare. This alternative approach is more in line with a strict inflation targeting mandate that some central banks in the world abide by. Derivations can be found in Appendix C. It turns out that the qualitative results are robust to either objective.

5.2 Outside of the Run and the Constrained Zones

We start by considering the benchmark situation where the economy is in the part of the state space where the risk of run is null, $\xi = 0$, and the balance sheet constraint doesn't bind, $\sigma = 0$. This occurs when the markup shock is small or when banks are well-capitalized, as shown in section 4.2. In that case, the central bank balances the maximization of output with inflation stabilization. The key result is that there is no basis for the government and the central bank to take into account financial frictions in setting interest rates and there is no role for additional tools.

Outside of the run and the constrained zones, we can re-express the social planner's problem described by Definition 1 as a simpler maximization problem subject to the Phillips curve. Recall from section 4.1 that in this part of the state space, the economy is fully characterized by two equations, the Phillips curve and the Euler equation. Given that R_2 and C_1 are related one-to-one through the Euler equation, the social planner can simply choose C_1 and back out the level of R_2 that implements it. Using the market clearing condition to substitute for C_1, assuming that the disutility of labor in period 1 is given by

$$v_1(\ell_1) = \chi \log \ell_1, \tag{31}$$

with $\chi < \alpha$ and using the firm's production technology $Y = \ell^\alpha K^{1-\alpha}$ to substitute for ℓ_1 as a function of Y_1, the problem of the social planner becomes

$$W = \max_{C_1,\pi_1,K_G,N_G} \left\{ \left(1 - \frac{\chi}{\alpha}\right) \log Y_1 + \log\left(1 - \frac{\theta}{2}\pi_1^2\right) + \beta \log C_2 \right\} \quad \text{s.t.} \quad 0 = \mathcal{PC}(Y_1, \pi_1)$$

The optimality condition is given by

$$MRS = \frac{\left(1 - \frac{\chi}{\alpha}\right)\left(1 - \frac{\theta}{2}\pi_1^2\right)}{\theta \pi_1 Y_1} = -\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi} = MRT.$$

Intuitively, the marginal rate of substitution (MRS) between output and inflation $\frac{\left(1-\frac{\chi}{\alpha}\right)\left(1-\frac{\theta}{2}\pi_1^2\right)}{\theta\pi_1 Y_1}$ should be equal to the marginal rate of transformation (MRT) which is also equal to the slope of the Phillips curve $-\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}$. We can then use the Euler equation and the goods market clearing condition to express the implied interest rate:

Lemma 3 (Baseline)**.** *The optimal interest rate is given by*

$$R_2 = \underbrace{-\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}}_{\text{Slope of Phillips curve (MRT)}} \underbrace{\frac{\theta\pi_1}{\left(1 - \frac{\chi}{\alpha}\right)\left(1 - \frac{\theta}{2}\pi_1^2\right)^2}}_{\text{Welfare cost of inflation}} \frac{C_2}{\beta} \tag{32}$$

This formula provides intuition on the determinants of the optimal trade-off between output maximization and inflation stabilization. More specifically, the stronger the size of the cost-push shock $\frac{\epsilon_1}{\epsilon_1 - 1}$, the steeper the Phillips curve $-\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}$ and the higher future consumption C_2, the higher the optimal interest rate. Finally, given that credit policy and recapitalization play no role in this part of the state space, we have $K_G = N_G = 0$.

Strict inflation targeting. In the case where the central bank follows a strict inflation target at $\bar{\pi}$ its policy rate should simply be such that the implied level of output is consistent with this target. Other tools are not useful because there is no inefficiencies in intermediation. Because the central bank is willing to sacrifice output to reach its inflation target, welfare is strictly lower than when the government balances inflation and output objectives. We formalize this idea in Appendix C.

5.3 Inside the Constrained Zone

We now turn to the case where the balance sheet constraint binds. As shown in section 4.2, this happens when the inflation shock requires a bigger rate hike or when banks are less well-capitalized. Inside the constrained zone, the economy is characterized by three equations: the Phillips curve, the Euler equation and the

Balance Sheet Constraint. Like in the benchmark problem analyzed before, the Euler equation is omitted because the social planner can simply choose C_1 and back out the level of R_2. As a result, the social planner's problem is given by:

$$W = \max_{Y_1,\pi_1,K_G,N_G,K_{H2}} \left\{ \left(1 - \frac{\chi}{\alpha}\right) \log Y_1 + \log\left(1 - \frac{\theta}{2}\pi_1^2\right) + \beta C_2(K_{H2}, B_{H2}, K_G, N_G) \right\}$$
$$\text{s.t.} \quad 0 = \mathcal{PC}(Y_1, \pi_1) \quad \text{and} \quad 0 = \mathcal{BSC}(K_{H2}, B_{H2}, K_G, Y_1, N_0, N_G)$$

Note that because we introduce one more constraint ($\mathcal{BSC}$), we will take an additional first order condition with respect to K_{H2}.

When the balance sheet constraint binds, a wedge (σ) opens up between the efficient and the actual allocation. From section 4.2, we know that the spread between the return on capital and on deposits σ is tightly related to $\beta_K \frac{P_2 K_{H2}}{K}$ and is exacerbated by further increases in the policy rate R_2. In appendix A.2 and in the following lemma, we show that it is also closely linked to the shadow cost of the $\mathcal{BSC}$ constraint (i.e. the Lagrange multiplier). Policymakers should deploy other tools such as credit policy or recapitalization to close the wedge and should moderate their policy rate hikes when these tools are costly. The following lemma gives an analytical expression for the optimal interest rate when tools are costly.

Lemma 4 (Constrained - optimal interest rate)**.** *When the other tools are costly $\beta_K, \beta_N > 0$, the optimal interest rate is given by*

$$R_2 = \bar{R}_2 - \sigma \frac{-d\sigma}{dY} \Omega_0 \tag{33}$$

$$\Omega_0 = \frac{K(1 + r_{K2})^2}{\beta_K P_2^2 \left(1 - \frac{\theta}{2}\pi_1^2\right)\left(1 - \frac{\chi}{\alpha}\right)} \tag{34}$$

$$\bar{R}_2 = \frac{\theta \pi_1 C_2}{\beta \left(1 - \frac{\theta}{2}\pi_1^2\right)^2 \left(1 - \frac{\chi}{\alpha}\right)} \frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}$$

$$\frac{-d\sigma}{dY} = \frac{\mathcal{BSC}_Y}{\mathcal{BSC}_{K_{H2}}} \left(1 + \frac{\mathcal{BSC}_\pi}{\mathcal{BSC}_Y} \frac{\mathcal{PC}_Y}{-\mathcal{PC}_\pi}\right) \frac{\beta_K P_2}{K(1 + r_{K2})} \tag{35}$$

The formula reveals an additional term $-\sigma \frac{-d\sigma}{dY} \Omega_0$ that calls for a lowering of the optimal interest rate relative to the policy rate in the absence of financial frictions $\bar{R}_2$. A less aggressive tightening avoids the drop in physical returns tomorrow and asset prices today, which hurts the intermediaries' balance sheets. This deviation is proportional to the shadow utility cost of the balance sheet constraint, which is directly related to the spread between the return on capital and deposits through $\sigma = \frac{\beta_K P_2 K_{H2}}{K(1+r_{K2})}$. It is larger when households hold more of the capital stock (K_{H2}/K) and when these holdings are costly (β_K).

The extent to which this welfare cost should translate into a lower interest rate depends on the sensitivity of the spread to the interest rate which is mathematically closely related to its total derivative to output $-\frac{d\sigma}{dY}$ through the Euler Equation. This itself is the product of the impact of Y on K_{H2} through the balance sheet constraint and the impact of K_{H2} on spreads through the households' first order condition:

$$-\frac{d\sigma}{dY} = \underbrace{\frac{\beta_K P_2}{K(1+r_{K2})}}_{d\sigma/dK_{H2}} \underbrace{\frac{\mathcal{BSC}_Y}{\mathcal{BSC}_{K_{H2}}}\left(1 + \frac{\mathcal{BSC}_\pi}{\mathcal{BSC}_Y}\frac{\mathcal{PC}_Y}{-\mathcal{PC}_\pi}\right)}_{-dK_{H2}/dY}.$$

The bracket includes two additive terms: the direct effect of the interest rate on output and asset prices, and an indirect effect through inflation.

When credit policy are available, they should be used to address the source of financial stress, alleviate the trade-off for interest rate policy and allow monetary policy to focus on price and output stabilization.

Lemma 5 (Constrained - other tools)**.** *If $\beta_G > 0$, the optimal level of credit policy is given by*

$$K_G = \underbrace{\frac{\beta_K}{\beta_G}}_{\text{Relative cost of credit policy}} \underbrace{K_{H2}}_{\text{HH holding}} \underbrace{\frac{\mathcal{BSC}_{K_G}}{\mathcal{BSC}_{K_H}}}_{\text{Relative efficacy of credit policy}}$$

This formula shows that these other tools should be used in proportion to their costs and benefits. Optimal credit policy are higher when the government is efficient at intermediating $1/\beta_G$, when households hold more assets K_{H2}, when households are less efficient at holdings capital β_K and when the government's holdings relax the balance sheet constraint of intermediaries relative to households $\frac{\mathcal{BSC}_{K_G}}{\mathcal{BSC}_{K_H}}$.

More broadly, the optimal mix of policy rate moderation and credit policy and the degree of separation of financial stability objectives depend on the cost of other tools, β_K, β_N. The more costly other tools, the less they should be used and the larger the deviation of interest rate policy from its level outside of the constrained zone. When these other tools are prohibitively costly, or not available, $\beta_K, \beta_N \to +\infty$, policymakers should implement $K_{H2} = N_G = 0$ and the deviations of the interest rate policy are largest to preserve the intermediation capacity of banks. This is a case where separation of financial stability and price stability is impossible.

At the other extreme, when credit policy have no cost $\beta_K = 0$, policymakers can achieve perfect separation of financial stability objectives. By intermediating at no cost the assets that the private intermediate cannot hold, public credit policy can restore the first best allocation. Formally, when credit policy are not costly, we can drop the $\mathcal{BSC}$ constraint. The interest rate is then chosen only to trade-off price stabilization and output maximization while credit policy should address balance sheet constraint. The following lemma formalizes this separation result.

Lemma 6 (Constrained - complete separation)**.** *If $\beta_G = 0$, the optimal interest rate is the same as outside the constrained zone (equation 32) and credit policy are given by*

$$
\begin{aligned}
& K_G > \underline{K}_{G2} \\
\textit{with} \quad & \underline{K}_{G2} = \frac{\phi R_2}{1 + r_{K2}} \Bigg[\bar{R}_1 N_0 - \bar{R}_1 \left(Q_{K0}(K - K_{H1}) + Q_{L0}(B - B_{H1}) \right) + r_{K1}(R_2)(K - K_{H1}) \\
& \qquad + r_B(B - B_{H1}) + \frac{1 + r_B}{R_2} \left(B - B_{H1} - \frac{L}{\phi} \right) + \frac{1 + r_{K2}}{R_2} \left(K - K_{H1} - \frac{K}{\phi} \right) \Bigg]
\end{aligned}
$$

Intuitively, the optimal level of government's investment K_G should be at least $\underline{K}_{G2}$ which is the amount of investment necessary so that private intermediaries can hold all the remaining capital stock $K - K_G$ and bonds L.

Equity injection. We now consider equity injection as an alternative tool policymakers can use to address financial frictions. When they are not costly to use $\beta_N = 0$, equity injection should be used to fully address the source of financial distortions and allow interest rate policy to focus on the trade-off between output and inflation. Using the same reasoning as for credit policy, we use the constraint $\mathcal{BSC}$ to solve for the minimum level of N_G such that households hold no asset in equilibrium $K_{H2} = B_{H2} = 0$. This minimum level is given in the following lemma. When equity injection is costly $\beta_G > 0$, the formula for the optimal rate is the same as for the case where other tools are not available shown before and the optimal choice of other tools is given by the following lemma.

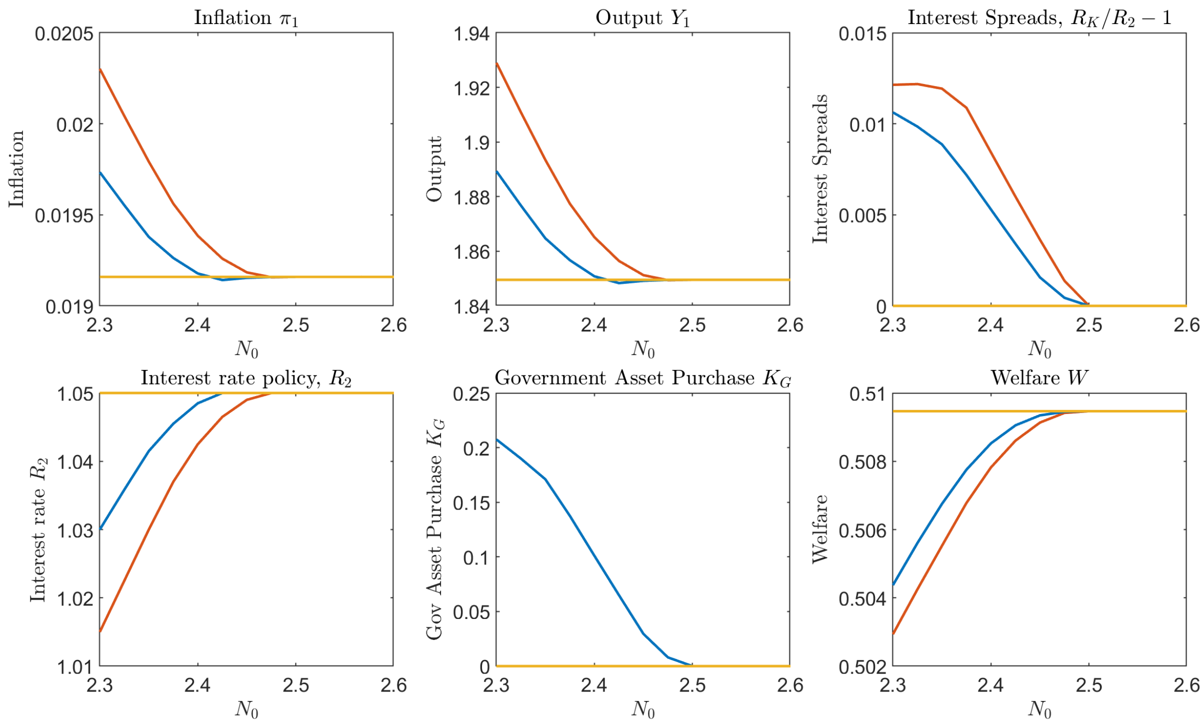


Figure 7: Equilibrium as a function of N_0. Yellow line shows outcomes when there is the leverage constraint does not bind. Orange line shows outcomes given the optimal policy rate R_2 and no alternative policies, when the leverage constraint binds. Blue line shows expected outcomes given the optimal policy rate R_2 and use of alternative policies, when the leverage constraint binds.

Lemma 7 (Constrained - Equity injection - Partial and Complete Separation)**.** *If $\beta_N = 0$, the optimal interest rate is the same as in the baseline (equation 32) and the minimum level of bank equity should be*

$$\underline{N}_G = \Bigg[- \bar{R}_1 N_0 + \bar{R}_1 \left(Q_{K0}(K - K_{H1}) + Q_{L0}(B - B_{H1}) \right) - r_{K1}(R_2)(K - K_{H1}) \\ - r_B(B - B_{H1}) - \frac{1 + r_B}{R_2} \left(B - B_{H1} - \frac{L}{\phi} \right) - \frac{1 + r_{K2}}{R_2} \left(K - K_{H1} - \frac{K}{\phi} \right) \Bigg]$$

If $\beta_N > 0$, the optimal interest rate is the same as in the baseline (equation 32) and the *optimal level of equity injection should be*

$$N_G = \underbrace{\frac{\beta_N}{\beta_G}}_{\text{Relative costs of equity injection}} \underbrace{\frac{K_{H2}}{K}}_{\text{HH holding}} \underbrace{\frac{\mathcal{BSC}_{N_G}}{\mathcal{BSC}_{K_H}}}_{\text{Relative efficacy of equity injection}}$$

Other financial policies. Additional tools could be deployed beyond credit policy and equity injection. Macroprudential policies and capital requirements could be relaxed *ex post* by increasing ϕ^G to raise the intermediaries' overall capacity to invest and close the spreads. However, this would be effective if and only if the macroprudential regulation is binding, i.e. $\phi^G < \phi^P$. Like for equity injections and credit policy, the extent to which it should be used to address the source of the stress depends on its relative costs and benefits. If the binding constraint is the incentives-based one, relaxing macroprudential policies wouldn't have any effect.

Strict inflation targeting. All the previous results still hold, at least qualitatively, when the central bank follows a strict inflation targeting mandate. In particular, as long as other tools are costly to use the central bank should adopt a less aggressive policy stance. We provide a formal proof in Appendix C. Welfare is strictly lower than when interest rate policy is also set to maximize welfare for two reasons: it doesn't balance output and inflation objectives in period 1, and it doesn't internalize its negative consequences on intermediation and output in period 2.

When $\beta_G > 0$, the optimal interest rate is lower in the "constrained" zone than outside and strictly decreasing in K_{H2} and B_{H2}. The reason is that a binding balance sheet constraint depresses future consumption ($C_2 < Y_2 + K$), which depresses current consumption ($C_1 = C_2/(\beta R_2)$). This in turn means the interest rate doesn't need to be as high to control inflation. In that case, credit policy help offset the loss of consumption in period 2.

If other tools are not costly $\beta_G = 0$, governments should use them to address the financial distortions stemming from the financial constraint of intermediaries. In equilibrium, households shouldn't hold any assets, there shouldn't be any spread between the policy rate and asset returns, and $C_2 = Y_2 + K$. This implies that the policy rate should follow the same path as the one outside the "constrained" zone.

5.4 Inside the Run Zone

We now consider the case where the economy is in the "run" zone, i.e. where there is a positive probability of a systemic run $\xi > 0$. As shown in section 3.5, this happens when the inflation shock requires a bigger rate hikes or when banks are less well-capitalized. Given that policymakers need to decide on the policy rate before knowing the outcome of the run, they maximize expected welfare. Abstracting from the balance sheet constraint, the social planner faces two constraints: the Phillips curve in the good and in the run equilibrium. Like before, we omit the Euler equation, choose C_1 and then back out the level of R_2. The problem of policy-makers is given by

$$W = \max_{Y_1, Y_1^*, \pi_1, \pi_1^*, K_{H2}, K_{H2}^*, N_G, K_G} \left\{ (1-\xi) \left(\left(1 - \frac{\chi}{\alpha}\right) \log Y_1 + \log\left(1 - \frac{\theta}{2}\pi_1^2\right) + \beta \log C_2 \right) \right.$$
$$\left. + \xi \left(\left(1 - \frac{\chi}{\alpha}\right) \log Y_1^* + \log\left(1 - \frac{\theta}{2}(\pi_1^*)^2\right) + \beta \log C_2^* \right) \right\}$$
$$\text{s.t.} \quad 0 = \mathcal{PC}(Y_1, \pi_1) = \mathcal{PC}\left(Y_1^*, \pi_1^*\right)$$

where $\xi\left(Y_1^*, \pi_1^*, \frac{C_2^*}{\beta C_1^*}, K_G\right)$ is the probability of a run.

Due to the risk of coordination failures among depositors, an additional wedge—the risk of a run given by ξ—opens up between the efficient and the actual allocation which policies should try to address. When other tools are available, including equity injections, lender of last resort facilities and deposit insurance, they should be used, in proportion to their costs, to decrease the distortions implied by the coordination failure. When there are costly, the central bank should internalize the impact of interest rate hikes on the risk of run, as shown in the following lemma.

Lemma 8 (Run - optimal interest rate policy)**.** *When the other tools are costly $\beta_K, \beta_N > 0$,*

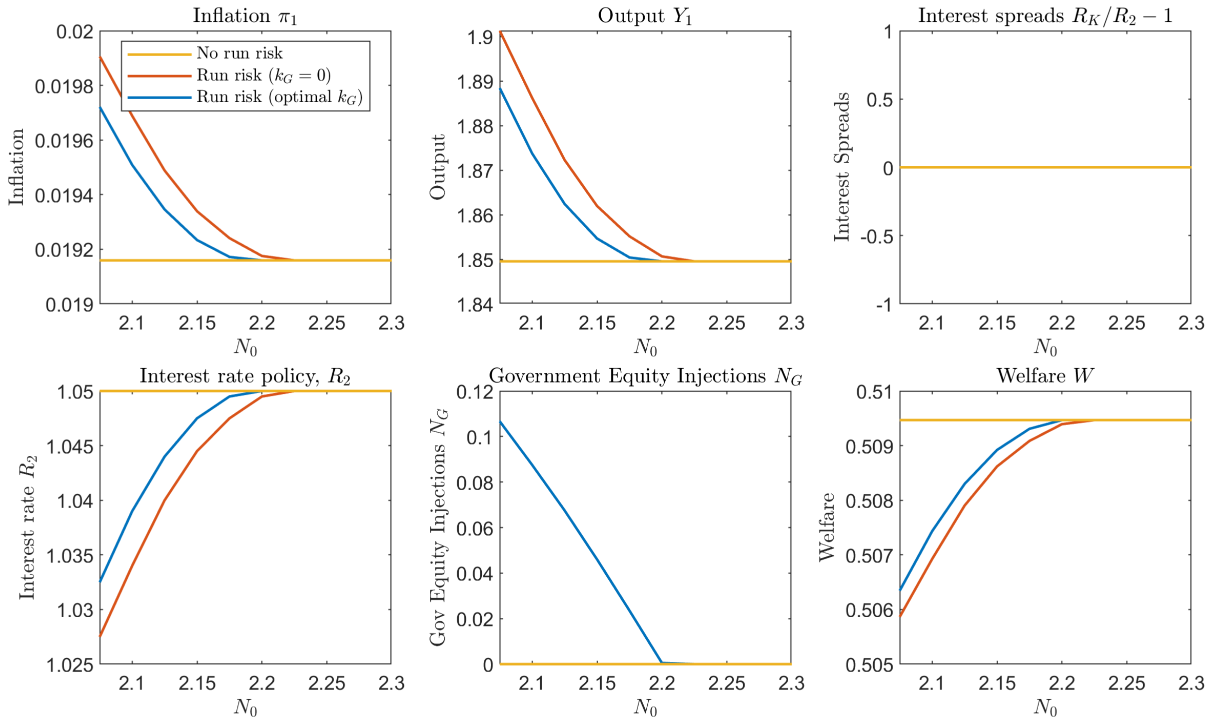


Figure 8: Equilibrium as a function of N_0. Yellow line shows outcomes when there is no run risk. Orange line shows expected outcomes with run risk, at the optimal policy rate R_2 and no alternative policies. Blue line shows expected outcomes when alternative policies are used.

the optimal rate is given by

$$R_2 = \bar{R}_2 - \xi\Omega_0\left(\bar{R}_2 - \underline{R}_2\right) - \xi'\Omega_2 \log\frac{C_2^*}{C_2} \tag{36}$$

$$\textit{with} \quad \bar{R}_2 = \frac{\theta\pi_1 C_2}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi_1)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}$$

$$\underline{R}_2 = \frac{\theta\pi_1^* C_2^*}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi_1^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}$$

$$\Omega_1 = \frac{\frac{\chi}{\alpha}}{1-(1-\frac{\chi}{\alpha})\frac{\underline{R}_2}{\bar{R}_2}}$$

$$\Omega_2 = \frac{\alpha}{\chi}\frac{(1+\beta)}{(1-\frac{\chi}{\alpha})}Y_1^*\Omega_1$$

$$\xi' = \xi_Y + \xi_\pi\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}$$

where ξ' is the total derivative of the probability of a run with respect to Y_1 (through R_2), taking into the account the effect through inflation, $\bar{R}_2, \underline{R}_2$ are the "shadow optimal interest rates" the central bank would like to implement if it could perfectly predict if the run happens or not, respectively.

The formula above makes clear that the optimal interest rate should be below its optimal level without a risk of run $\bar{R}_2$ to take into account the risk of a run. This happens through two separate effects. The first effect $\xi\Omega_1 \left(\bar{R}_2 - \underline{R}_2\right)$ is proportional to the probability of the run and to the gap between the shadow optimal rates in the no run and in the run equilibria $\bar{R}_2 - \underline{R}_2 > 0$. The intuition is as follows: since aggregate demand and inflation drop in case of a run, the central bank should tolerate higher inflation in the good state of the world to avoid a deeper recession in the case of a run. The second term $\xi'\Omega_2 \log \frac{C_2^*}{C_2}$ is linear in the sensitivity of the probability of a run with respect to the interest rate ξ' and should be proportional to the drop in welfare in case of a run $(1+\beta)\log \frac{C_2^*}{C_2}$.[18]

When other instruments are available, they should be used to address the source of financial stress, alleviate the trade-off for interest rate policy and allow monetary policy to focus on price and output stabilization. Equity injection directly helps strengthening banks' balance sheets. Indirectly, they also help improve the allocation by boosting asset prices. Both the direct and indirect channels contribute to lowering the risk of a run.[19]

Lemma 9 (Run - other tools)**.** *If $\beta_G > 0$, the optimal level of equity injection is given by*

$$N_G = \Omega_3 \frac{\xi_{K_G}}{\beta_N} \log C_2^*/C_2$$

$$\Omega_3 = \frac{\beta C_2}{\beta + \left(\frac{C_2}{C_2^*} - 1\right)\left(\xi(1+\beta) + \frac{\xi' Y_1^* (1+\beta) \log \frac{C_2^*}{C_2} - \frac{\chi}{\alpha}\xi}{1 + \frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta \pi_1^* Y_1^*}{\left(1 - \frac{\theta}{2}(\pi^*)^2\right)}\right)}\right)}$$

Equity injections should be used in proportion to their costs and benefits in decreasing the risk of run. The less costly equity injection $1/\beta_N$, the larger the drop in consumption and welfare in case of a run C_2^*/C_2, the higher the efficacy of these

[18]This second effect calls for a lowering of the interest rate since $\xi' < 0$ and $\log \frac{C_2^*}{C_2} < 0$.

[19]credit policy can also help improve the allocation by boosting asset prices, which decrease the probability of run. However they may not be enough.

injections at bringing down the risk of a run ξ'_{N_G}, the more the government should inject.

The optimal mix of policy rate moderation and equity injections and the degree of separation of financial stability objectives depend on their cost, β_N. The less costly equity injections, the more they should be used and the more the policy rate R_2 can focus on inflation (still trading-off output losses) and tighten more aggressively than in the case without these other tool. This is because the equilibrium level of inflation and period-2 consumption in the run equilibrium C_2^*, π_1^* are higher and the run-implied loss C_2^*/C_2 is smaller. The effect through the change in the probability of a run ξ' is ambiguous as it depends on the convexity of the $\xi(.)$ function. In the extreme case where other tools are prohibitively costly, or not available, $\beta_N \to +\infty$, then $N_G = 0$ and separation of financial stability objectives is impossible: the central bank should tighten significantly less to account for its effect on the probability of a run.

By contrast, when other tools have no cost $\beta_N = 0$, policymakers can fully achieve their financial stability goal and separate them from the price/output stability trade-off. The government should inject equity up to the point where intermediaries' balance sheets are repaired and the likelihood of a run is eliminated, $\xi = 0$. Interest rate policy should then focus only on the price-output trade-off.

Lemma 10 (Run with complete separation)**.** *If $\beta_N = 0$, the optimal interest rate is the same as in the baseline (equation 32) and equity injection is given by*

$$N_G > \bar{N} - N_0 = \frac{(\bar{R}_1 - R_{k1}^*)Q_{K0}(K - K_{H1}) + (\bar{R}_1 - R_{L1}^*)Q_{L0}(B - B_{H1})}{\bar{R}_1}$$

Intuitively, the optimal level of government's equity injection N_G should be sufficient to bring the level of intermediaries' equity above $\bar{N}$, the level of net worth such that even if all depositors run, the run is unsuccessful.

Other policies: deposit insurance and lender of last resort. The analysis above has focused on equity injection. It can however be an expensive tool. Two other tools may be better suited to address coordination failures among depositors: deposit insurance and lender of last resort facilities. Through the lens of the model, deposit insurance and lender of last resort facilities are state-contingent versions of equity injection. They are transfers to banks made only if the run happens. Our results on equity injections would thus hold for these two policies too. Arguably they are less costly—since they are activated only in the state of the world where a run happens—so they would correspond to a lower β_N. This would in turn imply a higher degree of separation, more aggressive use of the tools and more aggressive interest rate tightening.

Strict inflation targeting. Since the inflation rate depends on the materialization of a run, it is random. Given that policies are set before the realization of a run, policy-makers can't achieve perfect price stabilization ex post. The best they can do is to achieve price stability on average across states of the world. Accordingly, we assume that the objective of the central bank is to minimize the expected squared deviations from target. In Appendix C, we show that the optimal policy rate is strictly lower than the one outside of the run zone.

To see why the policy rate needs to be below its level outside of the run zone, assume for a moment that it is equal to it. In that case, inflation would be at its target in the no-run equilibrium path but below it in the run path. This is clearly sub-optimal: the central bank could reduce the average deviation by tolerating some inflation in the no-run path, to avoid a drop in inflation in a run period. It will thus set a lower interest rate.

When credit policy, equity injection or deposit insurance are available, they should be used to decrease the risk of runs by boosting asset prices, strengthening intermediaries' balance sheets or directly reassuring depositors, which in turns improve intermediation and raise consumption in period 2. This in turn boosts inflation in the run scenario and leads the central bank to raise its policy rate.

5.5 Large Financial Crisis

We close this section by considering a large financial crisis which we model as a case in which the run happens with probability one, $\xi = 1$. In equilibrium, banks lose all their net worth $N_1 = 0$, and absent government's interventions all the capital is intermediated by households which results in a drop in consumption in period 2, and hence in period 1.

Given that the run, the implied disruption of financial intermediation and the large drop in output occur independently of the stance of monetary policy, the only trade-off the central bank faces is between inflation stabilization and preserving output - the same one it faced outside of the constrained and of the run zones.

Lemma 11 (Large crisis - interest rate policy)**.** *When the other tools are costly $\beta_K, \beta_N > 0$, the optimal rate is given by*

$$R_2 = -\frac{\theta \pi_1^*}{\left(1 - \frac{\theta}{2}(\pi_1^*)^2\right)\left(1 - \frac{\chi}{\alpha}\right)} \frac{\mathcal{PC}_Y^*}{\mathcal{PC}_\pi^*} C_2^*$$

Two effects push the optimal rate in opposite directions. The markup shock pushes the central bank to increase its rate, which is captured by the term $\frac{\mathcal{PC}_Y^*}{\mathcal{PC}_\pi^*}$. The large financial crisis leads to a drop in C_2^*, which pushes the central bank to decrease its rate. If the latter effect is stronger than the former, the central bank should decrease its rate. The drop in C_2^* depends on the use of other tools. In the extreme case where tools are prohibitively costly and are not used, the drop is largest and given by $C_2^* = Y_2 + K - \frac{\beta_K}{2} K - \frac{\beta_B}{2} L$. In this case, the central bank may have to cut rates despite the cost push shock.

When credit policy are available, they should be used to intermediate part of the private assets in the economy and minimize the drop in C_2^*.

Lemma 12 (Large crisis - other tools)**.** *Optimal credit policy and the implied C_2^* are given by*

$$K_G = \frac{\beta_K}{\beta_G + \beta_K} K$$
$$C_2^* = Y_2 + K - \frac{\beta_K \beta_G}{(\beta_K + \beta_G)2} K - \frac{\beta_B}{2} L$$

Full separation: In the case where $\beta_G = 0$, credit policy can fully address financial disruptions:

$$K_G = K$$
$$C_2^* = Y_2 + K$$

Intuitively, the more efficient the government is at intermediating relative to households β_K/β_G the higher the share of private assets it should hold. In the extreme case where the government can intermediate assets without any cost $\beta_G = 0$, then it should hold all assets in the economy $K_G = K$. In addition, if there is no government debt $B = 0$, consumption can be fully stabilized, and the central bank faces only the inflation/output trade-off (and should raise the interest rate). This is a case of full-separation.

Strict inflation targeting. The results are qualitatively similar to those inside the "run" zone: when the central bank uses only the interest rate, the rate that stabilizes inflation is lower than the one prevailing outside of the run and of constrained zones. But quantitatively the rate is lower than in both zones given the larger drop in consumption implied by the large financial crisis, $C_2 = Y_2 + K - \frac{\beta_B}{2}B_{-} \left(\frac{\beta_K}{2}\frac{K-K_G}{K}\right)(K - K_G) - \left(\frac{\beta_G}{2}\frac{K_G}{K}\right)K_G$

It could even be that the loss is so large that the interest rate R_2 that stabilizes inflation when the economy could be below the one that stabilizes inflation without the markup shock and without the run shock. In that case, there wouldn't be a trade-off between output and inflation in period 1 anymore.

When governments use credit policy and $\beta_G = 0$ and $B = 0$ households don't hold any assets, $C_2 = Y_2 + K$ which implies that the policy rate is the same as the one prevailing outside of the constrained and run zones. Otherwise, when $\beta_G > 0$, or $L > 0$, or both, credit policy help offset the drop in consumption in period 2 and the optimal policy rate can be higher than in the case when only interest rate policy are used.

6 Conclusion

The question of whether central banks should consider financial stability concerns has been widely considered, particularly in the aftermath of the Great Financial Crisis. However, these discussions have often taken place in the context of a low inflation environment, where trade-offs between price and financial stability are less pronounced. Recent events, characterized by high inflation and heightened financial stability concerns, raise questions about the applicability of insights gained from the 2008-09 Global Financial Crisis to the current economic landscape.

This paper investigates this issue by presenting a tractable New Keynesian model that incorporates two specific sources of financial frictions relevant to recent events: an occasionally binding leverage constraint on financial intermediaries and the risk of a systemic bank run. Through the lens of this model, we demonstrate that the current economic environment presents policymakers with a more challenging trade-off between price and financial stability objectives.

Our findings show that policymakers can theoretically achieve a full separation of price and financial stability goals through alternative policy tools such as credit policy, equity injections, deposit insurance, and macroprudential measures. However, this separation is feasible only if these tools can be deployed without significant efficiency costs. In the extreme case where these tools are either prohibitively costly or unavailable, interest rate policy must necessarily accommodate financial stability concerns. Under such conditions, policymakers may need to tolerate greater deviations from price stability to ensure financial stability.

The paper focuses on the more realistic scenario where alternative policy tools face implementation challenges but remain viable options. In this case, policymakers should adopt a mixed approach, combining a less aggressive interest rate policy with a sound use of these tools. The degree to which interest rate policy must accommodate financial stability and the extent to which alternative tools should be expanded depend on the severity of financial vulnerabilities and the types of tools available.

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Appendix

A Proof Model

A.1 Outside of the "Constrained" and "Run" Zones

Lemma

Proof. We can rewrite the condition (28) as

$$\begin{aligned}
&\mathcal{LHS}(N_0) > \mathcal{RHS}(R_2) \\
\text{with} \quad &\mathcal{LHS}(N_0) = \bar{R}_1 N_0 - \bar{R}_1 \left(Q_{K0}(K - K_{H1}) + Q_{L0}(B - B_{H1}) \right) \\
&\mathcal{RHS}(R_2) = -r_{K1}(R_2)(K - K_{H1}) - r_B(B - B_{H1}) \\
&\qquad - \frac{1 + r_B}{R_2} \left(B - B_{H1} - \frac{B}{\phi} \right) - \frac{1 + r_{K2}(R_2)}{R_2} \left(K - K_{H1} - \frac{K}{\phi} \right)
\end{aligned}$$

where $r_{K1}(R_2), r_{K2}(R_2)$ are general equilibrium functions.

Assumption 1. *We define the following three regularity conditions:*

- *Households hold positive deposits* $D_1 > 0$.
- *The Phillips curve is upward-sloping and not too steep, i.e. the implicit function* $\pi_1(Y_1)$ *defined by equation (26) is continuous and strictly increasing and* $\theta \pi_1^2 \left(\frac{1}{2} + \epsilon_{\pi_1/Y_1} \right)$ *where* $\epsilon_{\pi_1/Y_1} = \frac{\partial \pi_1}{\partial Y_1} \frac{Y_1}{\pi_1}$ *is the elasticity of inflation to output implied by the Phillips curve.*
- *Households' holdings are not too large relative to the leverage ratio:* $\frac{\phi - 1}{\phi} > \max \left(\frac{K_{H1}}{K}, \frac{B_{H1}}{B} \right)$

We start with showing that the functions $r_{K1}(.), r_{K2}(.)$ is decreasing in R_2 through a decrease in current consumption and the price level in period 1 through the firms' optimal pricing decision. The definition of r_{K1} is given by

$$
\begin{aligned}
r_{K1} &= (1-\alpha)\frac{P_1 Y_1}{K} \\
&= (1-\alpha)\frac{P_0(1+\pi_1(Y_1))Y_1}{K}
\end{aligned}
$$

We first use the Phillips curve to define π_1 implicitly as a function of Y_1, with $\pi_Y > 0$. We then use the market clearing condition for final goods:

$$
Y_1\left(1-\frac{\theta}{2}\pi_1(Y_1)^2\right) = C_1 = \frac{C_2}{\beta R_2}
$$

to show that Y_1 decreases in R_2 under one regularity condition. First note that C_2 is exogenous in the case where the financial constraint of intermediaries doesn't bind. Then the right-hand side of the equation above is decreasing in R_2. The left hand side increases in Y_1 if and only if $\theta\pi_1^2\left(\frac{1}{2}+\epsilon_{\pi/Y}\right)$ where $\epsilon_{\pi/Y}$ is the elasticity of inflation to output implied by the Phillips curve (26). since $\pi_1(Y_1)$ increases in Y_1. We can thus define a continuous and decreasing function $Y_1(R_2)$.

Under these regularity conditions on the Phillips curve, we have that an increase in R_2 leads to a decrease in P_1Y_1. This in turn allows us to define a decreasing function $r_{K1}(R_2)$. Given that r_{K2} in increasing in P_2 and Y_2 is exogenous in the case where the financial constraint of intermediaries doesn't bind, and given the assumption that $P_2 = P_1$ and the result that P_1 is continuous and decreasing in R_2 we have that r_{K2} is continuous and decreasing in R_2.

$\mathcal{LHS}(N_0)$ is increasing and continuous in N_0. In addition, it is strictly negative under the assumption that $D_1 > 0$. Under the assumptions that $\frac{\phi-1}{\phi} > \max\left(\frac{K_{H1}}{K}, \frac{B_{H1}}{B}\right)$, $\mathcal{RHS}(R_2)$ is increasing and continuous in R_2. In addition, it converges to 0 for $R_2 \to +\infty$. By continuity, for all $N_0 > 0$ such that $D_1 > 0$, there exists a unique $\bar{R}_2$ such that $\mathcal{LHS}(N_0) = \mathcal{RHS}(\bar{R}_2)$. We can thus define a new function $\bar{R}_2(N_0)$. This function is strictly increasing in N_0. □

Policies

Proof. We start from

$$
\begin{aligned}
W = \max_{R_2} & \left(1-\frac{\chi}{\alpha}\right)\log Y_1 + \log\left(1-\frac{\theta}{2}\pi_1^2\right) + \beta\log C_2 \\
\text{s.t.} \quad \theta\pi_1(\pi_1+1) &= (\epsilon_1 - 1)\left(\frac{\epsilon_1}{\epsilon_1-1}\frac{W}{(1+\pi_1)P_0\alpha}\left(\frac{Y_1}{K}\right)^{\frac{1-\alpha}{\alpha}} - 1\right) \\
C_1 &= \frac{C_2}{\beta R_2}
\end{aligned}
$$

After substituting C_1 using the second constraint, and given that C_2 is unaffected by the policy rate since Y_2 is exogenous and the households doesn't hold any asset, we can drop C_2 from the definition of welfare. The problem simplifies to what is in the main text. The problem can thus be rewritten as

$$
W = \max_{Y_1,\pi_1}\left(1-\frac{\chi}{\alpha}\right)\log Y_1 + \log\left(1-\frac{\theta}{2}\pi_1^2\right) + \beta\mathbb{E}C_2 \quad \text{s.t.} \quad 0 = \mathcal{PC}(Y_1,\pi_1)
$$

Denoting λ the lagrange multipler, the associated FOCs are

$$
\begin{aligned}
\frac{\left(1-\frac{\chi}{\alpha}\right)}{Y_1} + \lambda\mathcal{PC}_Y = 0 \\
-\frac{\theta\pi_1}{\left(1-\frac{\theta}{2}\pi_1^2\right)} + \lambda\mathcal{PC}_\pi = 0 \Rightarrow \ \lambda = \frac{\theta\pi_1}{\left(1-\frac{\theta}{2}\pi_1^2\right)\mathcal{PC}_\pi}
\end{aligned}
$$

Combining both equations gives

$$
\frac{\left(1-\frac{\chi}{\alpha}\right)}{Y_1} = -\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}\frac{\theta\pi_1}{\left(1-\frac{\theta}{2}\pi_1^2\right)}
$$

Using $C_2 = \beta R_2 C_1$ and the goods market condition we get

$$
R_2 = -\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}\frac{\theta\pi_1}{\left(1-\frac{\chi}{\alpha}\right)\left(1-\frac{\theta}{2}\pi_1^2\right)^2}\frac{C_2}{\beta}.
$$

□

A.2 Inside the "Constrained" Zone

We start by defining the constraint $\mathcal{BSC}$.

$$
\begin{aligned}
\mathcal{BSC}(K_{H2}, K_G, \pi_1, Y_1, R_2, N_0, N_G) &= \mathcal{LHS}(N_0, N_G) - \mathcal{RHS}(K_{H2}, K_G, \pi_1, Y_1, R_2) \\
\text{with} \quad \mathcal{LHS}(N_0, N_G) &= \bar{R}_1 N_0 - \bar{R}_1 \left(Q_{K0}(K - K_{H1}) + Q_{L0}(B - B_{H1})\right) + N_G \\
\mathcal{RHS}(K_{H2}, K_G, \pi_1, Y_1, R_2) &= -r_{K1}(\pi_1, Y_1)(K - K_{H1}) - r_B(B - B_{H1}) \\
&\quad - \frac{1 + r_B - \beta_B \frac{P_2 B_{H2}}{B}}{R_2} \left(B - B_{H1} - \frac{B - B_{H2}}{\phi}\right) \\
&\quad - \frac{1 + r_{K2} - \beta_K \frac{P_2 K_{H2}}{K} - \beta_G \frac{K_G}{K}}{R_2} \left(K - K_{H1} - \frac{K - K_{H2} - K_G}{\phi}\right)
\end{aligned}
$$

where $r_{K1}(\pi_1, Y_1) = (1-\alpha)\frac{P_0(1+\pi_1)Y_1}{K}$. Replacing R_2 by its expression from the Euler equation as a function of C_1 and C_2, the constraint $\mathcal{BSC}$ is no longer a function of R_2:

$$
\begin{aligned}
\mathcal{BSC}(K_{H2}, K_G, \pi_1, Y_1, N_0, N_G) &= \mathcal{LHS}(N_0, N_G) - \mathcal{RHS}(K_{H2}, K_G, \pi_1, Y_1) \\
\text{with} \quad \mathcal{LHS}(N_0, N_G) &= \bar{R}_1 N_0 - \bar{R}_1 \left(Q_{K0}(K - K_{H1}) + Q_{L0}(B - B_{H1})\right) + N_G \\
\mathcal{RHS}(K_{H2}, K_G, \pi_1, Y_1) &= -r_{K1}(\pi_1, Y_1)(K - K_{H1}) - r_B(B - B_{H1}) \\
-\frac{1 + r_B - \beta_B \frac{P_2 B_{H2}}{B}}{C_2(K_{H2}, K_G, N_G)} &\left(B - B_{H1} - \frac{B - B_{H2}}{\phi}\right) \beta Y_1 \left(1 - \frac{\theta}{2}\pi_1^2\right) \\
-\frac{1 + r_{K2} - \beta_K \frac{P_2 K_{H2}}{K}}{C_2(K_{H2}, K_G, N_G)} &\left(K - K_{H1} - \frac{K - K_{H2} - K_G}{\phi}\right) \beta Y_1 \left(1 - \frac{\theta}{2}\pi_1^2\right)
\end{aligned}
$$

Note that $\mathcal{RHS}(K_{H2}, K_G, \pi_1, Y_1)$ is linear and decreasing in Y_1. It is also decreasing in π_1. Hence $\mathcal{BSC}(K_{H2}, K_G, \pi_1, Y_1, N_0, N_G)$ is increasing in Y_1 and in π_1.

Other tools not available.

Proof. The problem of policy-makers is given by

$$
\begin{aligned}
W = \max_{Y_1, \pi_1, K_{H2}} & \left(1 - \frac{\chi}{\alpha}\right) \log Y_1 + \log\left(1 - \frac{\theta}{2}\pi_1^2\right) + \beta \ln C_2\left(K_{H2}, K_G, N_G\right) \\
\text{s.t.} \quad & 0 = \mathcal{PC}\left(Y_1, \pi_1\right) \\
& 0 = \mathcal{BSC}(K_{H2}, K_G, \pi_1, Y_1, N_0, N_G)
\end{aligned}
$$

Denoting λ and μ the lagrange multipliers, the associated FOCs are

$$\frac{\left(1-\frac{\chi}{\alpha}\right)}{Y_1}+\lambda\mathcal{PC}_Y+\mu\mathcal{BSC}_Y=0$$

$$-\frac{\theta\pi_1}{\left(1-\frac{\theta}{2}\pi_1^2\right)}+\lambda\mathcal{PC}_\pi+\mu\mathcal{BSC}_\pi=0\Rightarrow\ \lambda=\frac{\frac{\theta\pi_1}{\left(1-\frac{\theta}{2}\pi_1^2\right)}-\mu\mathcal{BSC}_\pi}{\mathcal{PC}_\pi}$$

$$-\beta\frac{\beta_K K_{H2}}{C_2K}+\mu\mathcal{BSC}_{KH2}=0\Rightarrow\ \mu=\beta\frac{\beta_K K_{H2}}{C_2K\mathcal{BSC}_{KH2}}$$

Substituting for λ and μ in the first equation gives

$$\frac{\left(1-\frac{\chi}{\alpha}\right)}{Y_1}=-\frac{\mathcal{PC}_Y\theta\pi_1}{\left(1-\frac{\theta}{2}\pi_1^2\right)\mathcal{PC}_\pi}-\beta\frac{\beta_K K_{H2}\mathcal{BSC}_Y}{C_2K\mathcal{BSC}_{KH2}}\left(1+\frac{\mathcal{BSC}_\pi}{\mathcal{BSC}_Y}\frac{\mathcal{PC}_Y}{-\mathcal{PC}_\pi}\right)$$

Using $C_1=C_2/(\beta R_2)$ and the goods market condition to substitute for Y_1 gives the result:

$$R_2=\left[\underbrace{-\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}\frac{\theta\pi_1}{\left(1-\frac{\theta}{2}\pi_1^2\right)}\frac{C_2}{\beta}}_{\text{Baseline term}}-\underbrace{\beta_K}_{\text{Cost}}\underbrace{\frac{K_{H2}}{K}}_{\text{HH's holdings}}\underbrace{\frac{\mathcal{BSC}_Y}{\mathcal{BSC}_{K_{H2}}}\left(1+\frac{\mathcal{BSC}_\pi}{\mathcal{BSC}_Y}\frac{\mathcal{PC}_Y}{-\mathcal{PC}_\pi}\right)}_{\text{Sensitivity of balance sheet to }R_2}\right]\frac{1}{\left(1-\frac{\theta}{2}\pi_1^2\right)\left(1-\frac{\chi}{\alpha}\right)}$$

We then show how $\beta_K\frac{K_{H2}}{K}$ related to the wedge σ. From the FOC of households, we directly obtain

$$\sigma=1-\frac{R_2}{R_{k2}}=\frac{\beta_K P_2 K_{H2}}{K(1+r_{K2})}$$

Note that $\frac{\mathcal{BSC}_Y}{\mathcal{BSC}_{K_{H2}}}\left(1+\frac{\mathcal{BSC}_\pi}{\mathcal{BSC}_Y}\frac{\mathcal{PC}_Y}{-\mathcal{PC}_\pi}\right)$ is equal to the total derivative of K_{H2} to Y (total because it incorporates its effect through inflation) implied by the balance sheet constraint, $\frac{dK_{H2}}{dY}$. We re-express this derivative as follows:

$$
\begin{aligned}
\frac{d\sigma}{dY} &= \frac{d\sigma}{dK_{H2}}\frac{dK_{H2}}{dY} \\
\Rightarrow \frac{dK_{H2}}{dY} &= \frac{\frac{d\sigma}{dY}}{\frac{d\sigma}{dK_{H2}}} \\
\frac{\mathcal{BSC}_Y}{\mathcal{BSC}_{K_{H2}}}\left(1+\frac{\mathcal{BSC}_\pi}{\mathcal{BSC}_Y}\frac{\mathcal{PC}_Y}{-\mathcal{PC}_\pi}\right) &= -\frac{K(1+r_{K2})}{\beta_K P_2}\frac{d\sigma}{Y}
\end{aligned}
$$

We can thus rewrite the optimal interest rate as

$$
\begin{aligned}
R_2 &= \bar{R}_2 - \sigma\frac{-d\sigma}{dY}\Omega_0 \\
\Omega_0 &= \frac{K(1+r_{K2})^2}{\beta_K P_2^2\left(1-\frac{\theta}{2}\pi_1^2\right)\left(1-\frac{\chi}{\alpha}\right)}
\end{aligned}
$$

□

credit policy.

Proof. We now consider the choice of credit policy. The FOCs w.r.t to K_G and K_{H2} are given by:

$$
\begin{aligned}
-\beta\frac{\beta_K K_{H2}}{C_2 K} + \mu\mathcal{BSC}_{KH2} = 0 &\Rightarrow\ \mu = \beta\frac{\beta_K K_{H2}}{C_2 K\mathcal{BSC}_{KH2}} \\
-\beta\frac{\beta_G K_G}{C_2 K} + \mu\mathcal{BSC}_{K_G} = 0 &
\end{aligned}
$$

When $\beta_G > 0$, the optimal choice of other tools is given by combining these two FOCs, which gives

$$
K_G = \underbrace{\frac{\beta_K}{\beta_G}}_{\text{Efficiency of CB intermediation}} \underbrace{K_{H2}}_{\text{HH holding}} \frac{\mathcal{BSC}_{K_G}}{\mathcal{BSC}_{K_H}}
$$

When $\beta_G = 0$, the FOC with respect to K_G implies $\mu = 0$, which means that the balance sheet constraint of banks has no welfare cost, or in another words credit policy should intermediate capital up to the point where the banks balance sheet are no longer a constraint on intermediation. This in turn implies $K_{H2} = 0$ through the FOC for K_{H2}. From the FOC for the interest rate analyzed above we obtain the same optimal rate as the one outside the "constrained" zone. Finally, we use the constraint $\mathcal{BSC}$ to solve for the minimum level of K_G such that households hold no asset in equilibrium $K_{H2} = B_{H2} = 0$:

$$\underline{K}_{G2} = \frac{\phi R_2}{1 + r_{K2}} \Bigg[- \bar{R}_1 N_0 + \bar{R}_1 \left(Q_{K0}(K - K_{H1}) + Q_{L0}(B - B_{H1}) \right) - r_{K1}(R_2)(K - K_{H1}) \\ - r_B(B - B_{H1}) - \frac{1 + r_B}{R_2} \left(B - B_{H1} - \frac{L}{\phi} \right) - \frac{1 + r_{K2}}{R_2} \left(K - K_{H1} - \frac{K}{\phi} \right) \Bigg]$$

□

Equity injection When $\beta_G > 0$, the formula for the optimal rate is the same as for the case where other tools are not available shown before and the optimal choice of other tools is given by combining the FOC for K_{H2} and N_G, which gives

$$N_G = \underbrace{\frac{\beta_N}{\beta_G}}_{\text{Efficiency of CB intermediation}} \underbrace{\frac{K_{H2}}{K}}_{\text{HH holding}} \frac{\mathcal{BSC}_{N_G}}{\mathcal{BSC}_{K_H}}$$

Using the same reasoning as for credit policy, we use the constraint $\mathcal{BSC}$ to solve for the minimum level of N_G such that households hold no asset in equilibrium $K_{H2} = B_{H2} = 0$:

$$\underline{N}_G = \Bigg[- \bar{R}_1 N_0 + \bar{R}_1 \left(Q_{K0}(K - K_{H1}) + Q_{L0}(B - B_{H1}) \right) - r_{K1}(R_2)(K - K_{H1}) \\ - r_B(B - B_{H1}) - \frac{1 + r_B}{R_2} \left(B - B_{H1} - \frac{L}{\phi} \right) - \frac{1 + r_{K2}}{R_2} \left(K - K_{H1} - \frac{K}{\phi} \right) \Bigg]$$

A.3 Inside the Run Zone

Using the Euler equation and the equality of R_2 in both states of the world, we obtain an additional constraint: $C_1/C_2 = C_1^*/C_2^*$. The problem of the social planner can be rewritten as

$$
\begin{aligned}
W = \max_{C_1, C_1^*, \pi_1, \pi_1^*, N_G, K_G} & (1-\xi)\left(\log C_1 - \chi \log \ell_1 + \beta \log C_2(K_G, N_G)\right) \\
& + \xi \left(\log C_1^* - \chi \log \ell_1^* + \beta \log C_2^*(K_G, N_G)\right) \\
\text{s.t.} \quad 0 = \mathcal{PC}(Y_1, \pi_1) & = \mathcal{PC}\left(Y_1^*, \pi_1^*\right) \quad \text{and} \quad C_1/C_2 = C_1^*/C_2^*
\end{aligned}
$$

with $\xi = \xi\left(Y_1^*, \pi_1^*, \frac{C_2^*}{\beta C_1^*}, K_G\right)$

Denoting μ the lagrange multiplier associated with the third constraint, the problem can be rewritten as

$$
\begin{aligned}
W = \max_{C_1, C_1^*, \pi_1, \pi_1^*, N_G, K_G} & (1-\xi+\mu)\log C_1 - (1-\xi)\log \ell_1 + [(1-\xi)\beta - \mu]\log C_2(K_G, N_G) \\
& + (\xi - \mu)\log C_1^* - \xi \log \ell_1^* + (\xi\beta + \mu)\log C_2^*(K_G, N_G) \\
\text{s.t.} \quad 0 = \mathcal{PC}(Y_1, \pi_1) & = \mathcal{PC}\left(Y_1^*, \pi_1^*\right)
\end{aligned}
$$

Denoting λ, λ^* the lagrange multipliers associated with the two Phillips curve constraints, the FOCs are

$$
\begin{aligned}
\frac{(1-\xi+\mu)}{Y_1} - \frac{\chi}{\alpha}\frac{(1-\xi)}{Y_1} + \lambda \mathcal{PC}_Y(Y_1, \pi_1) &= 0 \\
(1-\xi+\mu)\frac{-\theta\pi_1}{\left(1 - \frac{\theta}{2}(\pi)^2\right)} + \lambda \mathcal{PC}_\pi(Y_1, \pi_1) &= 0 \\
\frac{(\xi-\mu)}{Y_1^*} - \frac{\chi}{\alpha}\frac{\xi}{Y_1^*} + \lambda^* \mathcal{PC}_{Y^*}(Y_1^*, \pi_1^*) + \xi_{Y^*}\left[\log\frac{C_1^*}{C_1} + \beta \log \frac{C_2^*}{C_2}\right] &= 0 \\
(\xi-\mu)\frac{-\theta\pi_1^*}{\left(1 - \frac{\theta}{2}(\pi^*)^2\right)} + \lambda^* \mathcal{PC}_{\pi^*}(Y_1^*, \pi_1^*) + \xi_{\pi^*}\left[\log\frac{C_1^*}{C_1} + \beta \log \frac{C_2^*}{C_2}\right] &= 0 \\
\xi_{N_G}\beta \log C_2^*/C_2 &= \beta_N N_G \left(\frac{\beta(1-\xi)-\mu}{C_2} + \frac{\xi\beta+\mu}{C_2^*}\right) \\
\xi_{K_G}\beta \log C_2^*/C_2 &= \frac{\beta_G K_G}{K}\left(\frac{\beta(1-\xi)-\mu}{C_2} + \frac{\xi\beta+\mu}{C_2^*}\right) \\
0 &= \mathcal{PC}(Y_1, \pi_1) = \mathcal{PC}\left(Y_1^*, \pi_1^*\right) \\
\log C_1/C_1^* &= \log C_2/C_2^*
\end{aligned}
$$

We use the second and fourth equations we can solve for λ and λ^* to substitute back into the first and third equations. We can also take the ratio of the fifth and sixth equations. This gives

$$
\begin{aligned}
(1-\xi+\mu) - \frac{\chi}{\alpha}(1-\xi) &= (1-\xi+\mu)\frac{-\theta\pi_1 Y_1}{\left(1-\frac{\theta}{2}(\pi)^2\right)}\frac{\mathcal{PC}_Y(Y_1,\pi_1)}{\mathcal{PC}_\pi(Y_1,\pi_1)} \\
\xi - \mu - \frac{\chi}{\alpha}\xi + \xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2} &= \frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left((\xi-\mu)\frac{-\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right) \\
\xi_{N_G}/\xi_{K_G} &= \frac{K\beta_N N_G}{\beta_G K_G} \\
0 &= \mathcal{PC}(Y_1,\pi_1) = \mathcal{PC}\left(Y_1^*,\pi_1^*\right) \\
\xi_{K_G}\beta\log C_2^*/C_2 &= \frac{\beta_G K_G}{K}\left(\frac{\beta(1-\xi)}{C_2}+\frac{\xi\beta}{C_2^*}\right) + \frac{\beta_G K_G}{K}\left(\frac{1}{C_2^*}-\frac{1}{C_2}\right)\mu \\
\log C_1/C_1^* &= \log C_2/C_2^*
\end{aligned}
$$

We now use the second equation of the previous system to solve for $\xi-\mu$ and substitute back into the first equation. We also use this expression to substitute out the Lagrange multiplier μ in the fifth equation.

$$\left(1-\frac{\frac{\chi}{\alpha}\xi-\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^*Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}\right)-\frac{\chi}{\alpha}(1-\xi)=$$

$$\left(1-\frac{\frac{\chi}{\alpha}\xi-\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^*Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}\right)\frac{-\theta\pi_1 Y_1}{\left(1-\frac{\theta}{2}(\pi)^2\right)}\frac{\mathcal{PC}_Y(Y_1,\pi_1)}{\mathcal{PC}_\pi(Y_1,\pi_1)}$$

$$\xi-\mu=\frac{\frac{\chi}{\alpha}\xi-\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^*Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}$$

$$\xi_{K_G}\beta\log C_2^*/C_2=\frac{\beta_G K_G}{K}\left(\frac{\beta(1-\xi)}{C_2}+\frac{\xi\beta}{C_2^*}+\left(\frac{1}{C_2^*}-\frac{1}{C_2}\right)\left(\xi-\frac{\frac{\chi}{\alpha}\xi-\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^*Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}\right)\right)$$

$$\xi_{K_{H2}}/\xi_{K_G}=\frac{\beta_N N_G}{K\beta_G K_G}$$

In the first equation below, we next factorize by the term inside brackets, and then multiply both sides by $\left(1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^*Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)\right)$. In the fourth equation below we factorize by $\left(\frac{1}{C_2^*}-\frac{1}{C_2}\right)$:

$$\left(1+\frac{\theta\pi_1 Y_1}{\left(1-\frac{\theta}{2}(\pi)^2\right)}\frac{\mathcal{PC}_Y(Y_1,\pi_1)}{\mathcal{PC}_\pi(Y_1,\pi_1)}\right)\left(1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)-\frac{\chi}{\alpha}\xi+\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}\right)$$

$$=\left(1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)\right)\frac{\chi}{\alpha}(1-\xi)$$

$$\xi-\mu=\frac{\frac{\chi}{\alpha}\xi-\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}$$

$$\xi_{K_G}\beta\log C_2^*/C_2=\frac{\beta_G K_G}{K}\left(\frac{\beta}{C_2}+\left(\frac{1}{C_2^*}-\frac{1}{C_2}\right)\left(\beta\xi+\xi-\frac{\frac{\chi}{\alpha}\xi-\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}\right)\right)$$

$$\xi_{K_{H2}}/\xi_{K_G}=\frac{\beta_N N_G}{K\beta_G K_G}$$

Dividing both sides of the first equation by $\left(1+\frac{\theta\pi_1 Y_1}{\left(1-\frac{\theta}{2}(\pi)^2\right)}\frac{\mathcal{PC}_Y(Y_1,\pi_1)}{\mathcal{PC}_\pi(Y_1,\pi_1)}\right)$ and $\left(1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)\right)$ gives

$$1+\frac{-\frac{\chi}{\alpha}\xi+\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{\left(1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)\right)}=\frac{\frac{\chi}{\alpha}(1-\xi)}{\left(1+\frac{\theta\pi_1 Y_1}{\left(1-\frac{\theta}{2}(\pi)^2\right)}\frac{\mathcal{PC}_Y(Y_1,\pi_1)}{\mathcal{PC}_\pi(Y_1,\pi_1)}\right)}$$

Gathering all the terms on the right-hand side and using market clearing $Y_1=\frac{C_1}{1-\frac{\theta}{2}\pi_1^2}=\frac{C_2}{\beta R_2\left(1-\frac{\theta}{2}\pi_1^2\right)}$,

$$1=\frac{R_2\frac{\chi}{\alpha}(1-\xi)}{\left(R_2-\frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}\right)}+\frac{R_2\frac{\chi}{\alpha}\xi-\xi'(1+\beta)\frac{C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\log\frac{C_2^*}{C_2}}{\left(R_2-\frac{\theta\pi_1^* C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\right)}$$

Multiplying both sides by $R_2 - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}$

$$R_2 - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi} = R_2\frac{\chi}{\alpha}(1-\xi) + \left(R_2\frac{\chi}{\alpha}\xi - \xi'(1+\beta)\frac{C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\log\frac{C_2^*}{C_2}\right)\frac{R_2 - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}}{\left(R_2 - \frac{\theta\pi_1^* C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\right)}$$

Moving $R_2\frac{\chi}{\alpha}$ from the right to the left hand side and having a common denominator for both terms $R_2\xi\frac{\chi}{\alpha}$:

$$R_2\left(1-\frac{\chi}{\alpha}\right) - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi} = \frac{R_2\frac{\chi}{\alpha}\xi\left(\frac{\theta\pi_1^* C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}} - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}\right) - \xi'(1+\beta)\frac{C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\log\frac{C_2^*}{C_2}}{\left(R_2 - \frac{\theta\pi_1^* C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\right)}$$

Factorizing the entire fraction on the right hand side by $\xi\frac{\chi}{\alpha}$

$$R_2\left(1-\frac{\chi}{\alpha}\right) - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi} = \xi\frac{\chi}{\alpha}\frac{R_2\left(\frac{\theta\pi_1^* C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}} - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}\right) - \frac{\xi'\alpha}{\xi\chi}(1+\beta)\frac{C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\log\frac{C_2^*}{C_2}}{\left(R_2 - \frac{\theta\pi_1^* C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\right)}$$

Dividing both sides by $\left(1-\frac{\chi}{\alpha}\right)$

$$R_2 - \frac{\theta\pi_1 C_2}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi} = \xi\frac{\chi}{\alpha}\frac{R_2\left(\frac{\theta\pi_1^* C_2^*}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}} - \frac{\theta\pi_1 C_2}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}\right) - \frac{\xi'\alpha}{\xi\chi}\frac{(1+\beta)}{(1-\frac{\chi}{\alpha})}\frac{C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\log\frac{C_2^*}{C_2}}{\left(R_2 - \frac{\theta\pi_1^* C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\right)}$$

Denoting the shadow interest rate in the good and in the run equilibrium respectively $\bar{R}_2 = \frac{\theta\pi_1 C_2}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}$ and $\underline{R}_2 = \frac{\theta\pi_1^* C_2^*}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}$ we obtain

$$R_2 - \bar{R}_2 = \xi\frac{\chi}{\alpha}\frac{R_2\left(\underline{R}_2 - \bar{R}_2\right) - \frac{\xi'\alpha}{\xi\chi}\frac{(1+\beta)}{(1-\frac{\chi}{\alpha})}\frac{C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\log\frac{C_2^*}{C_2}}{R_2 - (1-\frac{\chi}{\alpha})\underline{R}_2}$$

Dividing the numerator and the denominator by R_2 gives

$$R_2 = \bar{R}_2 + \xi\frac{\chi}{\alpha}\frac{\underline{R}_2 - \bar{R}_2 - \frac{\xi'\alpha}{\xi\chi}\frac{(1+\beta)}{(1-\frac{\chi}{\alpha})}Y_1^*\log\frac{C_2^*}{C_2}}{1-(1-\frac{\chi}{\alpha})\frac{\underline{R}_2}{R_2}}$$

We can rewrite this as follows:

$$\begin{aligned}
R_2 &= \bar{R}_2 - \xi\Omega_0\left(\bar{R}_2 - \underline{R}_2\right) + \xi'\Omega_2\log\frac{C_2^*}{C_2} \\
\text{with}\quad \bar{R}_2 &= \frac{\theta\pi_1 C_2}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi} \\
\underline{R}_2 &= \frac{\theta\pi_1^* C_2^*}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}} \\
\Omega_2 &= \frac{\alpha}{\chi}\frac{(1+\beta)}{(1-\frac{\chi}{\alpha})}Y_1^*\frac{\frac{\chi}{\alpha}}{1-(1-\frac{\chi}{\alpha})\frac{\underline{R}_2}{R_2}} \\
\Omega_0 &= \frac{\frac{\chi}{\alpha}}{1-(1-\frac{\chi}{\alpha})\frac{\underline{R}_2}{R_2}} \\
\xi' &= \xi_Y + \xi_\pi\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}
\end{aligned}$$

credit policy and equity injection Recall that the FOC for credit policy is $\xi_{K_G}\beta\log C_2^*/C_2 = \frac{\beta_G K_G}{K}\left(\frac{\beta}{C_2} + \left(\frac{1}{C_2^*} - \frac{1}{C_2}\right)\left(\beta\xi + \xi - \frac{\frac{\chi}{\alpha}\xi - \xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}\right)\right)$. Rearranging gives:

$$\frac{K_G}{K} = \Omega_3 \frac{\xi_{K_G}}{\beta_G} \log C_2^*/C_2$$

$$\Omega_3 = \frac{\beta C_2}{\beta + \left(\frac{C_2}{C_2^*} - 1\right)\left(\xi(1+\beta) + \frac{\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2} - \frac{\chi}{\alpha}\xi}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}\right)}$$

Taking the ratio gives the optimal equity injection as a function of the household purchases

$$N_G = \frac{\beta_G}{\beta_N} \frac{K_N}{K} \frac{\xi'_{N_G}}{\xi'_{K_{H2}}}$$

A.4 Large Financial Crisis

Policies Lemma **??** is just a version of Lemma **??** since the problem of the CB is to maximize welfare subject to no constraint. The only difference is that, by assumption, households hold all assets. The definition of C_2^* is also a direct implication of the assumption that households hold all assets.

In Lemma **??**, the optimal interest rate is given by the same first-order condition as in Lemma **??**. The optimal asset purchase stems from taking the derivative of $C_2 = Y_2 + K - \frac{\beta_B}{2}B - \left(\frac{\beta_K}{2}\frac{K-K_G}{K}\right)(K-K_G) - \left(\frac{\beta_G}{2}\frac{K_G}{K}\right)K_G$ with respect to K_G and equalizing it to 0.

B Global Game Microfoundation

The run game happens at the beginning of the period. Consumers enter period 1 with their holdings of capital, long-term bonds and deposits, K_{H1}, B_{H1}, D_1.

Information structure and posterior beliefs. Depositors are uncertain about the structure of the banks' liabilities, how much it owes to depositors and how much it owns. The true equity of the bank N_0 is imperfectly known before the run happens. Agents know that it is drawn from a distribution centered around the end-of-period 0 level of net worth $\bar{N}_0$ and with some dispersion given by σ_N. For simplicity, we will consider the case where net worth is log-normally distributed around $\bar{N}_0$,

$$\log N_0 \sim \mathcal{N}\left(\log \bar{N}_0, \sigma_N\right) \tag{37}$$

. Agents also receive an idiosyncratic signal about the equity of the bank, η. It is common knowledge that it is drawn from a distribution that is centered around the true level of equity N_0 but with some noise, σ_η. We assume that the signals are also log-normally distributed around N_0,

$$\log \eta \sim \mathcal{N}\left(\log N_0, \sigma_\eta\right). \tag{38}$$

For future reference, we denote $F(\eta|N_0)$ the CDF of this distribution.

When deciding whether to run or not, depositors need to form expectations about the likely payoffs which depend on the behaviors (and the signals) of others, given their idiosyncratic signal. When priors and signals are log-normal, the posterior is also log-normal and given by

$$\log N_0 \sim \mathcal{N}\left(\frac{(\sigma_N^2)^{-1}}{(\sigma_N^2)^{-1}+(\sigma_\eta^2)^{-1}} \log \bar{N}_0 + \frac{(\sigma_\eta^2)^{-1}}{(\sigma_N^2)^{-1}+(\sigma_\eta^2)^{-1}} \log \eta, \frac{1}{(\sigma_N^2)^{-1}+(\sigma_\eta^2)^{-1}}\right) \tag{39}$$

Denote $p(N_0|\eta_i)$ the pdf of the posterior belief of agent i about the distribution of N_0 given its signal η_i, $\mu_{N_0}(\eta)$ the mean and σ_{NP}^2 the variance of this distribution given in (39), the density of the posterior is given by

$$p(n|\eta) = \frac{1}{n\sigma_{NP}\sqrt{2\pi}} \exp\left(-\frac{(\ln n - \mu_{N_0}(\eta))^2}{2\sigma_{NP}^2}\right).$$

Conditional on a given level of net worth N_0, the posterior beliefs on the signals received by other depositors is given by (38). And the share of people who receives a signal below η' is thus believed to be, $F(\eta'|N_0, \sigma_\eta)$.

Condition for successful run. There are two outcomes to the run game. Either the run is "successful" in the sense that the banks have to liquidate, or it is not. Denoting $\delta \in [0,1]$ the share of individuals who decide to run, a necessary and sufficient condition for a run to be successful is that the bank doesn't have enough to repay the depositors who run even if it liquidate all its assets:

$$R^*_{k1}Q_{K0}(K-K_{H1})+R^*_{L1}Q_{L0}(B-B_{H1})<\bar{R}_1 D_1\delta \tag{40}$$

$$\text{with}\quad Q^*_{K1}=\frac{1+r^*_{K2}-\beta_K+(\beta_K-\beta_G)\frac{K_G}{K}}{R_2}\quad\text{and}\quad Q^*_{L1}=\frac{1+r_B}{R_2} \tag{41}$$

$$R^*_{k1}=\frac{r_{K1}+Q^*_{K1}}{Q_{K0}}\quad\text{and}\quad R^*_{L1}=\frac{r_B+Q^*_{L1}}{Q_{L0}} \tag{42}$$

where we use the run price to evaluate both the capital stock and the long-term bonds since this is a case where the bank liquidates.

The condition for a successful run is thus that the share of depositors who run is large enough:

$$\delta>\bar{\delta}(N_0)\quad\text{with}\quad\bar{\delta}(N_0)=\frac{R^*_{k1}Q_{K0}(K-K_{H1})+R^*_{L1}Q_{L0}(B-B_{H1})}{\bar{R}_1\left[Q_{K0}(K-K_{H1})+Q_{L0}(B-B_{H1})-N_0\right]} \tag{43}$$

where N_0 is the true level of net worth which is not perfectly known by agents.

For future reference, we define $\bar{N}$ the level of net worth such that even if all depositors run, the run is unsuccessful. It is defined by

$$\bar{N}=\frac{(\bar{R}_1-R^*_{k1})Q_{K0}(K-K_{H1})+(\bar{R}_1-R^*_{L1})Q_{L0}(B-B_{H1})}{\bar{R}_1}$$

Trigger strategy. Each depositor has two strategies: to run or not to run. Following the literature, we guess that the equilibrium strategy is a trigger strategy, where depositors run if and only if the signal they receive is lower than a threshold $\bar{\eta}$, which is common across all depositors and common knowledge. Denoting $\delta^*(N_0)$ the equilibrium mass of depositors who run when the level of equity is n, a direct implication is that the mass of depositors who decide to run is simply given by $\delta^*(N_0)=F(\bar{\eta}|N_0)$, i.e. the depositors who have a signal below the threshold $\bar{\eta}$.

Payoffs of Depositors. Given that there are two aggregate outcomes (successful and not successful) and two strategies, we can consider four different cases, which are shown in Table B. A depositor would prefer to run in the case where the run is successful because in that case it gets a share of the bank liquidation value proportional to its deposits d_1 (which is equal to D_1 in aggregate), $\frac{R^*_{k1}Q_{K0}(K-K_{H1})+R^*_{L1}Q_{L0}(B-B_{H1})}{\bar{R}_1[Q_{K0}(K-K_{H1})+Q_{L0}(B-B_{H1})-N_0]\delta}d_1$

where δ is the share of depositors who run, while it loses its deposits if it doesn't run. A depositor would prefer not to run if the run is not successful because it incurs a small utility cost of running ζ.

To formalize the depositor's problem, we define $U(I)$ the indirect utility of a depositor which receives I from the bank at the end of the run game. I is equal to $R_1 d_1$ in case the run is unsuccessful. If the depositor runs, its utility is thus given by $U(R_1 d_1) - \zeta$. We omit the other holdings (of long term bonds and capital) for clarity. In case the run is successful, and the depositors doesn't run, its indirect utility is denoted $U^*(0)$ where as before $*$ denotes "run equilibrium." If it runs, its gets $U\left(\frac{R^*_{k1} Q_{K0}(K-K_{H1}) + R^*_{L1} Q_{L0}(B-B_{H1})}{\bar{R}_1 [Q_{K0}(K-K_{H1}) + Q_{L0}(B-B_{H1}) - N_0]\delta} d_1\right) - \zeta$.

	Successful	Unsuccessful
Run	$U\left(\frac{R^*_{k1} Q_{K0}(K-K_{H1}) + R^*_{L1} Q_{L0}(B-B_{H1})}{\bar{R}_1 [Q_{K0}(K-K_{H1}) + Q_{L0}(B-B_{H1}) - N_0]\delta} d_1\right) - \zeta$	$U(R_1 d_1) - \zeta$
Don't run	$U(0)$	$U(R_1 d_1)$

Given the guess of a trigger strategy and the posterior beliefs, we can define the expected payoffs in case the depositors decide to run:

$$\int_0^{\max(\bar{N},0)} U\left(\frac{R^*_{k1} Q_{K0}(K-K_{H1}) + R^*_{L1} Q_{L0}(B-B_{H1})}{\bar{R}_1 \left[Q_{K0}(K-K_{H1}) + Q_{L0}(B-B_{H1}) - n\right] F(\bar{\eta}|n)} d_1\right) p(n|\eta_i) dn + U(R_1 d_1) \int_{\max(\bar{N},0)}^{\infty} p(n|\eta_i) dn - \zeta$$

where $\bar{N}$ is the maximum level of net worth above which a run cannot be successful defined above. The expected payoffs in case the depositors decide not to run:

$$\int_0^{\max(\bar{N},0)} U(0)\, p(n|\eta_i) dn + U(R_1 d_1) \int_{\bar{N}}^{\infty} p(n|\eta_i) dn$$

Equilibrium. Assuming that in period 0 all consumers were identical, we have $d_1 = D_1 = Q_{K0}(K-K_{H1}) + Q_{L0}(B-B_{H1}) - \bar{N}_0$. A necessary condition for $\bar{\eta}$ is that a depositor with this signal is indifferent between running and not running:

$$\int_0^{\max(\bar{N},0)} \left[U\left(\frac{R^*_{k1} Q_{K0}(K-K_{H1}) + R^*_{L1} Q_{L0}(B-B_{H1})}{\bar{R}_1 \left[Q_{K0}(K-K_{H1}) + Q_{L0}(B-B_{H1}) - n\right] F(\bar{\eta}|n)} D_1\right) - U(0)\right] p(n|\bar{\eta}) dn = \zeta \tag{44}$$

The ex ante probability of a run ξ is given by:

$$\xi = P\left(\delta^*(N_0)\bar{R}_1 D_1 > R^*_{k1} Q_{K0}(K - K_{H1}) + R^*_{L1} Q_{L0}(B - B_{H1}) | \bar{N}_0\right) \quad (45)$$

$$\text{with} \quad D_1 = Q_{K0}(K - K_{H1}) + Q_{L0}(B - B_{H1}) - N_0 \quad (46)$$

where the equilibrium share of depositors running is given by $\delta^*(N_0) = F(\bar{\eta}|N_0)$.

This probability is a function of $\bar{N}_0$ since this is the only signal policymakers have about the true level of banks equity, of Y^*_1 and π^* through r^*_{k1}, and of K_G and R_2 through Q^*_{K1} hence through R^*_{k1}.

C Strict Inflation targeting

Baseline

Lemma 13. *The optimal interest rate is such that*

$$\mathcal{PC}\left(\frac{C_2}{\beta R_2\left(1 - \frac{\theta}{2}\bar{\pi}^2\right)}, \bar{\pi}\right) = 0 \quad \textit{with} \quad C_2 = Y_2 + K \quad (47)$$

The stronger the cost-push shock $\frac{\epsilon_1}{\epsilon_1 - 1}$, the steeper the Phillips curve $-\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}$ and the higher future consumption C_2, the higher the optimal interest rate.

Inside the constrained zone. We now derive the combination of interest rate policy and other tools when the economy is in the constrained zone. The central bank strictly targets inflation and the other part of the government chooses other tools to maximize households welfare. The following lemma formalizes the result.

Lemma 14. *The optimal interest rate is lower in the "constrained" zone than outside and* *strictly decreasing in K_{H2} and B_{H2}. It is given by*

$$
\begin{aligned}
&\mathcal{PC}\left(\frac{C_2}{\beta R_2\left(1-\frac{\theta}{2}\bar{\pi}^2\right)},\bar{\pi}\right)=0\\
&C_2 = Y_2 + K - \left(\frac{\beta_B}{2}\frac{P_2 B_{H2}}{B}\right)B_{H2} - \left(\frac{\beta_K}{2}\frac{P_2 K_{H2}}{K}\right)K_{H2} - \left(\frac{\beta_G}{2}\frac{K_G}{K}\right)K_G\\
&K_G = \underbrace{\frac{\beta_K}{\beta_G}}_{\text{Efficiency of CB intermediation}} \underbrace{K_{H2}}_{\text{HH holding}} \frac{\mathcal{BSC}_{K_G}}{\mathcal{BSC}_{K_H}}
\end{aligned}
$$

The stronger the cost-push shock $\frac{\epsilon_1}{\epsilon_1-1}$, the steeper the Phillips curve $-\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}$ and the higher future consumption C_2, the higher the optimal interest rate.

Inside the run zone. Given that the inflation rate is random, the central bank cannot achieve perfect stabilization, it can only minimize average deviation from target. The objective is to minimize the expected squared deviations from target

$$\max_{R_2} -(1-\xi)\frac{(\pi_1-\bar{\pi})^2}{2} - \xi\frac{(\pi_1^*-\bar{\pi})^2}{2} \quad s.t. \quad \mathcal{PC}\left(C_1,\pi_1\right)=0 \quad \text{and} \quad \mathcal{PC}\left(C_1\frac{C_2^*}{C_2},\pi_1^*\right)=0$$

The FOCs are given by

$$
\begin{aligned}
(1-\xi)(\pi_1-\bar{\pi}) &= \lambda\mathcal{PC}_\pi\\
\xi(\pi_1^*-\bar{\pi}) &= \lambda^*\mathcal{PC}_\pi^*\\
\lambda\mathcal{PC}_Y + \frac{C_2^*}{C_2}\lambda^*\mathcal{PC}_Y^* &= 0
\end{aligned}
$$

Substituting for the lagrange multipliers gives

$$(1-\xi)\frac{\pi_1-\bar{\pi}}{\mathcal{PC}_\pi}C_2\mathcal{PC}_Y + \xi\frac{\pi_1^*-\bar{\pi}}{\mathcal{PC}_\pi^*}C_2^*\mathcal{PC}_Y^* = 0$$

This FOC implies that in a run period, inflation is below target, and in a no zone situation it is above target. The interest rate must thus be below its level without the risk of a run.

When other tools are available, they can be used to decrease ξ and increase C_2^*, which then allows the central bank to raise its interest rate to stabilize inflation.

D Calibration

This section describes the calibration of the two period model. The parameters that characterize the banking sector are maximum leverage ϕ, bank capital and long-term bond holdings in period 0 (K_{F1}, L_{F1}), and banks' net worth in period 0 N_0, . The price of capital and long-term government bonds in period 0 (Q_{K0}, Q_{L0}) long term government bond coupon r_B as well as the interest rate R_1 are also exogenous in period 1. Leverage is set to five, which is between the values in Gertler and Karadi (2011) and Gertler and Kiyotaki (2015). We assume banks hold all capital and long-term bond holdings in period 0, so $K_{F1} = K$ and $B_{F1} = B$. Interest rate R_1 is set to 1.04 to target a real interest rate of 2 percent, and the long-term government coupon rate is set to 0.04. The remaining parameters Q_{K0}, Q_{L0}, N_0 are chosen so that the leverage constraint does not bind in steady state in the absence of an inflation shock.

The managerial costs of household's holding of capital β_K and long-term government bonds β_B are set to 0.09, which is low enough to ensure that households find it profitable to hold these assets when leverage constraint binds or a bank run occurs. However, it is set high enough to induce an increase in interest spreads when these events occur. For simplicity, we set the government's managerial costs for credit policy β_G and equity injections β_N similar to those of households', although allowing for higher government managerial costs do not significant change the results of the analysis.

The parameters that affect the probability of a run include dispersion of prior beliefs about banks' net worth σ_n, dispersion in private signal about banks' net worth $\beta_e ta$, and the utility costs of running ζ. These parameters are also set so that the probability of a run is zero in steady state. However, they can be eventually calibrated to match the empirical probability of bank runs.

The other macroeconomic parameters are the subjective discount factor β, elasticity of substitution ϵ, the wage level in period 1 $\bar{W}$, labor share α, capital stock K and long-term government bond stock B, coefficient on price adjustment cost θ, labor supply in period 2 $\bar{l}$, and price in period 0 P_0. The price in period 0 P_0 is set equal to the markup $\frac{\epsilon}{\epsilon-1}$ times marginal cost MC is equal to 1 in period 0. The subjective discount factor β, labor share α, and price adjustment cost θ is set following standard values in the literature. Capital stock K, long-term government bond stock B and wage level in period 1, $\bar{W}$, is normalize to 1. Labor supply in period 2 $\bar{l}$ and the elasticity of substitution, ϵ, are chosen to target an inflation rate of 2 percent and interest rate R_2 of 4 percent in steady state.

Variable	Description	Value
ϵ	Markup	10
$\overline{W}$	Wage level in period 1	1
α	Labor share	0.6
K	Capital stock	1
B	Long-term government bond stock	1
θ	Coefficient on price adjustment cost	375
$\bar{l}$	Labor supply in period 2	0.6
β_K	Household managerial cost (capital)	0.09
β_B	Household managerial cost (gov bond)	0.09
β_G	Government managerial cost (capital)	0.09
β_N	Government managerial cost (bank equity)	0.09
β	Household preference	0.96
ϕ	Maximum leverage	5
R_1	Interest rate in period 0	1.04
N_0	Net worth in period 0	2.05
Q_{K0}	Price of capital in period 0	2.7
Q_{L0}	Price of government bond in period 0	2.7
r_B	Long-term government bond coupon	0.04
K_{F1}	Bank capital holdings (period 0)	1
B_{F1}	Bank long-term government bond holdings (period 0)	1
P_0	Price of final output (period 0)	1.5096
χ	Disutility of labor	0.55
σ_n	Dispersion in prior beliefs about banks' net worth	1.5
σ_η	Dispersion in private signal about banks' net worth	1
ζ	Utility cost of running	1

Table 2: Parameter Definitions and Values

E Estimation of supply and demand shocks

To formalize the notion of supply and demand shocks, consider the following system of equations:

$$\text{Aggregate Demand: } \tilde{y}_t = -\alpha\pi_t + d_t$$

$$\text{Aggregate Supply: } \pi_t = \beta\tilde{y}_t + \eta_t$$

where $\tilde{y}_t$ is the output gap and π_t is the inflation rate, and $\alpha, \beta > 0$, and d_t and η_t are aggregate demand and supply shocks. This system can be micro-founded in a standard three-equation New Keynesian model, assuming that structural shocks are zero-mean white noise (Jump and Kohler, 2022). Aggregate demand implies a negative relationship between $(\tilde{y}_t, \pi_t)$. On the other hand, aggregate supply implies a positive relationship between $(\tilde{y}_t, \pi_t)$.

Re-arranging the system of equation yields the following:

$$\tilde{y}_t = \frac{1}{1+\alpha\beta}\left(d_t - \alpha\eta_t\right)$$
$$\pi_t = \frac{1}{1+\alpha\beta}\left(\beta d_t + \eta_t\right)$$

We can see that the demand shock d_t moves output gap and inflation in the same direction, whereas the supply shock η_t moves output gap and inflation in the opposite direction.

To bring this model to data, we first express the system of equation as an SVAR. We proxy the output gap with the change in real GDP Δy_t and inflation with the change in CPI. The SVAR specification is given by

$$Az_t = \sum_{j=1}^{p} A_j z_{t-j} + \epsilon_t \tag{48}$$

where

$$A = \begin{bmatrix} 1 & \alpha \\ -\beta & 1 \end{bmatrix}, z_t = \begin{bmatrix} \Delta y_t \\ \pi_t \end{bmatrix}, \epsilon_t = \begin{bmatrix} d_t \\ \eta_t \end{bmatrix}$$

The relationship between reduced form residuals $v_t = [v_t^y, v_t^\pi]'$ and the structural shocks $\epsilon_t = [d_t, \eta_t]'$ is given by

$$v_t \equiv z_t - E\left[z_t | z_{t-1}, ..., z_{t-p}\right] = A^{-1}\epsilon_t$$

Jump and Kohler (2022) show that the restrictions on the slope of supply and demand curves imply the following restrictions on the signs of the reduced form shocks:

To estimate the demand and supply shocks, we use data on real GDP and CPI

Positive demand shock	$d_t > 0$	$\rightarrow v_t^y > 0, v_t^\pi > 0$
Negative demand shock	$d_t < 0$	$\rightarrow v_t^y < 0, v_t^\pi < 0$
Positive supply shock	$\eta_t > 0$	$\rightarrow v_t^y > 0, v_t^\pi < 0$
Negative supply shock	$\eta_t < 0$	$\rightarrow v_t^y < 0, v_t^\pi > 0$

inflation rate from the Jorda-Schularick-Taylor Macrohistory Database (Jordà et al., 2017), which provides annual data on real and financial sector variables for 18 advanced economies from 1870 to 2020. The countries included in the sample are Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

The reduced form specification is given by

$$\Delta y_t = \delta_i^y + \gamma_{d(t)}^y + \sum_{j=1}^{L} \sum_{\tau}^{d(T)} \beta_{j,\tau}^{yy} y_{t-j} \times 1\{d(t) = \tau\} + \sum_{j=1}^{L} \sum_{\tau}^{d(T)} \beta_{j,\tau}^{y\pi} \pi_{t-j} \times 1\{d(t) = \tau\} + v_t^y \tag{49}$$

$$\pi_t = \delta_i^\pi + \gamma_{d(t)}^\pi + \sum_{j=1}^{L} \sum_{\tau}^{d(T)} \beta_{j,\tau}^{\pi y} y_{t-j} \times 1\{d(t) = \tau\} + \sum_{j=1}^{L} \sum_{\tau}^{d(T)} \beta_{j,\tau}^{\pi\pi} \pi_{t-j} \times 1\{d(t) = \tau\} + v_t^\pi$$

where $(\delta_i^y, \delta_i^\pi)$ are country fixed effects, $(\gamma_{d(t)}^y, \gamma_{d(t)}^\pi)$ are decade fixed effects. By interacting the lagged variables by decade indicators, the specification allows for the auto-regressive coefficients to vary over time. This accommodates structural changes in the relationship between inflation and unemployment over the 151 years observed in the panel. The number of distributive lags in the reduced form VAR is set to 2, based on comparing the Akaike Information Criteria across models with 1 to 10 distributive lags.