Optimal Taxation of Inflation*

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Abstract

This paper analyzes the effectiveness of a tax on inflation policy (TIP), which would require firms to pay a tax proportional to the increase in their prices or wages, in stabilizing inflation. We show that TIP would effectively correct externalities in firms' pricing decisions, tackle excessive inflation and reduce output volatility. While proposals from the 1970s saw TIP as a substitute to MP, we find that they are complementary, with TIP addressing cost-push shocks, and MP addressing demand shocks. In sharp contrast with price controls, TIP doesn't exacerbate price distortions.

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1 Introduction

Monetary Policy (MP) is a powerful tool for managing aggregate demand. But it faces significant challenges when inflation stems from cost-push shocks which break the divine coincidence. For example, persistent wage-price spirals can introduce a trade-off for central banks between accepting higher inflation for some time to preserve employment or triggering a recession to stabilize inflation.

In the perspective of broadening the set of tools to regulate inflation, this paper analyzes the effectiveness of a tax on inflation policy (TIP), which would require firms to pay a tax proportional to the increase in their prices or wages. By giving direct incentives to firms to moderate their price increases without exacerbating relative price distortions, we show that TIP is an effective instrument to control aggregate inflation, especially in the face of cost-push shocks. We do so by embedding a TIP in a workhorse New Keynesian (NK) model that includes several exogenous drivers of inflation and by deriving the optimal combination of MP and TIP in response to these shocks.

Starting with the proposal by Wallich and Weintraub (1971), TIP was widely discussed in the 1970s in the U.S. and in Western Europe, at a time when persistent inflation was the main concern of policymakers. In a few countries, versions of TIP were even briefly implemented. Because TIP has been absent from recent policy discussions and the recent literature, the paper reviews in details these earlier proposals in section A. Building on the ideas developed at the time, we leverage advancements in sticky price models made in the last decades to formalize TIP and characterize the optimal conduct of TIP in a fully microfounded framework. Our analysis yields several contributions.

We first show, in a small-scale NK model, that combining TIP with conventional MP can implement the first best allocation in which inflation is zero and the output gap remains closed under any path of shocks. This is in sharp contrast with a setting where only MP is available, because cost-push shocks cannot be entirely addressed with MP. Then, we show that these two instruments should specialize: MP should track the neutral rate of interest, which varies with aggregate demand and productivity shocks, to keep output at its efficient level, and TIP should rise with cost-push shocks. By introducing a wedge between the private and the social returns to price increases, these shocks create an externality in the firms' pricing decision. By giving direct incentives to moderate price increases, TIP can re-align the private with the social valuations and correct excessive inflation. In contrast with the view of the 1970s which saw TIP as a substitute for MP, we stress that TIP and MP are complementary, each specializing in specific drivers of inflation.

The stabilization properties of TIP continue to hold in a setting in which MP follows a Taylor rule and TIP targets inflation. Based on simulations of the calibrated small-scale NK model, we show that the stabilization gains from using TIP are substantial. Consistent with the results derived in the first-best setting, these gains are especially large for markup shocks: a reasonably calibrated TIP could lower the variance of inflation by 45% and of output by 44%. Welfare gains are smaller for TFP and demand shocks, because the lower inflation volatility is partially mitigated by higher output gap volatility.

We then formalize the equivalence between TIP and production or payroll subsidies—the more traditional tools to address the distortion implied by markups considered in the literature. While both instruments can help implement the first best, we show that subsidies entail large and persistent fiscal costs, which may imply distortionary taxation. In addition, we show that the first-best allocation could be equivalently implemented with other instruments that have a similar flavor as TIP, such as a feebate combining a tax on inflation with a rebate to all firms, and a market for inflation permits on which firms trade rights to increase their prices—a proposal first made by Lerner (1978). The appeal of a feebate is to provide incentives without increasing the average tax burden while the appeal of the market is that it minimizes the involvement of the fiscal authority.

In stark contrast with price controls, we find that TIP does not exacerbate distortions in relative prices. To derive this result, we extend the model to multiple sectors and allow for sector-specific TFP shocks, which requires adjustments in relative prices. We show analytically, that at first order, TIP has no effect on relative prices across sectors if the degree of price stickiness is the same across sectors. We then build a more general version of the model in which sectors differ in their price stickiness and idiosyncratic shocks volatility. We estimate it to match moments of the distribution of price changes at the sector level in the U.S.. Our simulations of the non-linear estimated model confirm that TIP doesn't exacerbate price distortions.

Intuitively, this is because TIP is linear in price changes and gives the same signal to all firms, contrary to price controls which are convex. In the presence of TIP, while firms that are hit by a negative productivity shock moderate their price increases, firms that would otherwise not change their prices are incentivized to decrease them to get a subsidy from TIP. The linearity of TIP keeps relative prices across sectors broadly unchanged.

Finally, we evaluate the stabilization properties of TIP in a richer medium-scale DSGE model à la Smets and Wouters (2007). By including a variety of important features and frictions and by estimating the model's parameters and the structural shocks driving inflation using Bayesian likelihood-based techniques on U.S. data, we provide a more realistic account of the properties of TIP. Simulations confirm that TIP provides substantial inflation stabilization gains. More specifically, TIP attenuates the size of the initial inflation responses by 30 to 50% depending on the shocks. Importantly, TIP also halves the output losses after price and wage markup shocks, confirming the divine coincidence result for these shocks derived in the first-best setting. After all other five non-markup shocks, TIP has only very limited effects on output. In a final exercise we construct a counterfactual time series of the U.S. economy since 1960 if TIP had been used. We show that inflation would have been substantially more stable with TIP, especially during periods of high inflation during the 1970s or the COVID-19 pandemic.

Literature and brief history of TIP. This paper contributes to the early literature that analyzed the microeconomic rationale of TIP (Portes, 1970; Wallich and Weintraub, 1971; Seidman, 1978), its macroeconomic implication (Peel, 1979; Scarth, 1983) and its implementation (Okun and Perry, 1978; Lerner, 1978; Layard, 1982; Dildine and Sunley, 1978). In the context of high inflation due to the combined food and oil price shocks of 1973 and 1979, and persistent wage-price spirals, this literature argued that "Tax-based Incomes Policies" (TIP) could be used to reign in inflation. Lerner (1978) and Lerner and Colander (1980) propose a market-based plan that gives similar incentives to firms to slow wage inflation through the issuance and exchange of permits.

In the 1970s, versions of TIP were implemented. From 1974 to 1977, the French governments implemented the "prélèvement conjoncturel", which covered the largest 1500 firms, representing 60% of the economy, and was based on the excess increase in value-added in nominal terms relative to an announced threshold, with an adjustments for fast-growing firms. Other versions of TIP were implemented in Mexico, Belgium, Italy, as mentioned in Paci (1988), and in the Netherlands as explained in OECD (1975). TIP came close to be implemented in the U.S. in 1978 when the Carter administration proposed to Congress the "real wage insurance" to supplement the wage-price guidelines. This program meant to give incentives to workers to enforce the guidelines: a worker belonging to an employee group whose earnings increased by less than 7% in a year would receive a tax-credit proportional to the difference between the realized inflation rate and 7% (Colander, 1981).

In the context of the transition of formerly Soviet countries to market economies, versions of TIP were implemented in Bulgaria, Poland and Romania. Koford et al. (1993) put forward an anti-inflation plan and incentive policies to stabilize prices and output in transition economies. A few papers analyze the design, implementation and enforcement of these policies in Bulgaria, Poland and Romania and concludes that they helped stabilize output and prices, at least in Bulgaria and Romania (Bogetic and Fox, 1993; Enev and Koford, 2000; Crombrugghe and de Walque, 2011). These examples suggest that TIP is implementable, effective, and not too costly to administer (Paci, 1988).

Our key contribution is to re-assess the effectiveness of TIP in a micro-founded conventional NK model. We share with the earlier literature the conclusion that TIP is a powerful tool to address excessive inflation, but while they saw TIP as a substitute for MP, we find that MP and TIP are complementary instruments, each specializing in their area of comparative advantage. TIP should focus on cost-push shocks, and MP should focus on demand shocks. In addition, our analysis highlights that TIP should be state-contingent and vary with the level of inflation. It should increase when inflation rises, and decrease as inflation reaches its long-term target.

Our paper also contributes to the rich literature on tax policies in New Keynesian models. Correia et al. (2008) shows that a sufficiently rich set of fiscal and monetary instruments, including labor taxes and subsidies, can implement the first-best allocation. When monetary policy is constrained by the ZLB, papers have found a welfare-enhancing role for tax increases aimed at restricting supply (Eggertsson and Woodford, 2006), tax cuts aimed by stimulating demand (Eggertsson, 2011), temporary government spending (Woodford, 2011), cuts in marginal labour tax rates that boost confidence (Mertens and Ravn, 2014), and well-designed paths of consumption tax and payroll subsidies, import and export tariffs (Correia et al., 2013; Farhi et al., 2014). We contribute to this literature by generalizing the set of fiscal instruments, and by showing that, like payroll subsidies, TIP helps implement the first best. Importantly, we argue that TIP is much less costly for the government budget than payroll subsidies.

The remainder of this paper is organized as follows. Section 2 introduces a conventional sticky price model augmented with a TIP. Section 3 analyzes the optimal design of TIP and MP and the macro-implications of an inflation-targeting TIP. Section 4 discusses the equivalence with other fiscal tools. Section 5 analyzes the implications of TIP for relative price distortion. Section 6 analyzes the stabilization properties of TIP in a medium-scale New Keynesian model. Section 7 concludes.

2 A Model with Sticky Prices and a Tax on Inflation

We start by introducing a tax on inflation policy in an otherwise conventional small-scale New Keynesian model. After describing the households' problem, we present the one of firms maximizing their discounted sum of profits subject to Rotemberg (1982)-adjustment costs and a tax on price increases.

2.1 Households

There is a continuum of mass one of identical infinitely-lived households, indexed by h. Households preferences are given by

$$\mathcal{U}(B_{t-1h}) = \max_{C_{th},N_{th},B_{th}} \left\{ \frac{C_{th}^{1-\sigma}}{1-\sigma} - \frac{N_{th}^{1+\psi}}{1+\psi} + \beta_t E_t \mathcal{U}(B_t) \right\} \tag{1}$$

where C_{th}, N_{th} and B_{th} denote consumption, labor supply and nominal wealth in period t, $\beta \in (0,1)$ the time discount factor, σ and ψ the inverse elasticity of intertemporal substitution of consumption and labor respectively.

Households choose consumption of good $C_{th} > 0$, labor $N_{th} > 0$ and one-period bonds in the next period B_{th} subject to the following budget constraint:

$$P_t C_{th} + Q_t B_{th} = B_{t-1h} + W_{th} N_{th} + T_t \tag{2}$$

where Q_t is the price of one-period bonds, W_{th} denotes nominal wages and T_t includes transfers from the government as well as the firms' profits. We also assume that households are subject to a solvency condition which rules out Ponzi schemes $\lim_{T\to+\infty} \Pi_{j=0}^{T} Q_j B_{Th} \geq 0$.

Households are differentiated by their idiosyncratic labor type. When choosing their labor supply, households take into account the firms' demand:

$$N_{th} = \left(\frac{W_{th}}{W_t}\right)^{-\epsilon_{Nt}} N_t. \tag{3}$$

The problem of the households is to maximize (1) subject to their budget constraint (2) and the no-ponzi condition, the labor demand of firms (3) and taking prices as given. The optimality conditions are given by

$$\frac{W_{th}}{P_t} = \mathcal{M}_t^w C_{th}^\sigma N_{th}^\psi \tag{4}$$

$$Q_t = E_t \left[\beta_t \left(\frac{C_{t+1h}}{C_{th}}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right] \tag{5}$$

for all $t = 0, 1, 2....$ The first equation determines the optimal labor supply given consumption C_{th} and the real wage W_{th}. The optimal markup on wages by households, $\mathcal{M}_t^w = \frac{\epsilon_{Nt}}{\epsilon_{Nt}-1}$, is allowed to vary over time and it captures attempts by workers to increase their real wage for a given supply of labor. The second equation is the traditional Euler equation which determines the optimal path of consumption given real returns on bonds. The discount factor β_t is also allowed to vary to capture demand shocks.

2.2 Final Good Competitive Firms

The final good is produced competitively by a continuum of firms. The production technology uses a continuum of varieties of intermediate goods supplied by monopolistic firms described in the next section, which we index by $i \in [0, 1]$

$$Y_t = \left(\int_0^1 Y_{ti}^{1-1/\epsilon_t} di\right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{6}$$

where ϵ_t is the elasticity of substitution across varieties. Taking the price of the final good P_t and the prices of inputs $\{P_{ti}\}_i$ as given, final good firms maximize profits $\max_{Y_{ti}} P_t Y_t - \int_0^1 P_{ti} Y_{ti} di$ subject to the technology (6). The optimality condition for variety i is given by

$$Y_{ti} = \left(\frac{P_{ti}}{P_t} \right)^{-\epsilon_t} Y_t \tag{7}$$

In equilibrium, the free entry of firms implies a no-profit condition $\int_0^1 P_{ti} Y_{ti} di = P_t Y_t$, which in turn, and after substituting out for Y_{it} using the demand from final goods firm, gives the following expression for the consumption price index $P_t = \left(\int_0^1 P_{ti}^{1-\epsilon_t} di \right)^{\frac{1}{1-\epsilon_t}}$.

2.3 Intermediate Good Monopolistic Firms

2.3.1 Technology and Market Structure

There is a continuum of mass one of firms indexed by $i \in [0,1]$. Each firm specializes in the production of a single variety which they sell to the final good firms. The technology to produce goods displays decreasing marginal returns:

$$Y_{ti} = A_t N_{ti}^{1-\alpha} \tag{8}$$

where A_t denotes total factor productivity, which is common across all firms, and $1 - \alpha$ is the elasticity of output to labor. Productivity shocks capture supply-chain disruptions and technological progress, but also changes in the prices of intermediate goods and energy. For example, increases in energy prices would translate into negative TFP shocks.

Firms are in monopolistic competition and choose the price P_{ti} at which they sell their good taking into account the final good firms' demand given by (7). They also face adjustment costs to price changes, described by a function $\mathcal{C}(P_{t-1i}, P_{ti})$ which is differentiable, strictly convex and equal to 0 when prices are unchanged, $\mathcal{C}(x,x) = 0$ for all $x > 0$. A firm's i gross profit is given by

$$P_{ti} Y_{ti} - W_t N_{ti} - \mathcal{C}_t \left(P_{t-1i}, P_{ti} \right).$$

Following Rotemberg (1982), we assume that adjustment costs are quadratic:

$$\mathcal{C}_t\left(P_{t-1i}, P_{ti}\right) = \frac{\theta}{2}\left(\frac{P_{ti}}{P_{t-1i}} - 1\right)^2 P_t Y_t \tag{9}$$

where the assumption that the adjustment costs scale with the nominal level of output $P_t Y_t$, is made for tractability and captures in a reduced-form way the notion that firms need to buy the final good to change their prices.

An alternative microfoundation of price rigidities is the time-dependent Calvo (1983) frictions whereby firms are allowed only occassionally to reset their price. While both microfoundations are used in the literature, Rotemberg adjustment costs turn out to be more tractable in our setting. In addition, it is appealing to introduce TIP in a framework with adjustment costs to price changes because it will allow us to shed light on how TIP resembles but also differs from the technological adjustment costs. We show in Appendix E that our results hold with time-dependent Calvo-type frictions.

2.3.2 The Firm's Problem with a TIP

The key novelty of our framework is that firms pay a tax proportional to the increase in their price $\tau_t(P_{ti} - P_{t-1i})$ where $\tau_t \in \mathbb{R}$ is the tax rate which is allowed to vary over time. In addition, we specify that a firm pays the tax on the price increase of each unit of goods sold so that a firm's total tax payment scales with its output, Y_{ti}. Hence profits net of taxes are given by

$$\Pi(P_{t-1i}, P_{ti}) = P_{ti}Y_{ti} - W_t N_{ti} - \tau_t(P_{ti} - P_{t-1i})Y_{ti} - \mathcal{C}_t\left(P_{t-1i}, P_{ti}\right). \tag{10}$$

where labor demand N_{it} is given by the technology (8) and output Y_{ti} by (7). Because the adjustment costs and the tax payment depend on a firm's current and past prices, the firm's problem is dynamic. In recursive form, it is given by

$$V(P_{t-1i}) = \max_{P_{ti}} \Pi(P_{ti}, P_{t-1i}) + E_t\left[Q_t V(P_{ti})\right] \tag{11}$$

where profits are given by (10). After substituting for labor using the production function, and for output using the demand schedule, one can take the first order condition for the optimal price. After imposing that all firms are identical in equilibrium, $P_{ti} = P_t$, and defining the rate of inflation $\pi_t = P_t/P_{t-1} - 1$, the optimality condition is given by

$$(\epsilon_t - 1)(\mathcal{M}_t MC_t - 1) + E_t\left[Q_t \frac{Y_{t+1}}{Y_t}(\tau_{t+1} + \theta(\pi_{t+1}+1)\pi_{t+1})\right] = \tau_t\left(1 - \epsilon_t \frac{\pi_t}{1+\pi_t}\right) + \theta\pi_t(\pi_t + 1) \quad (12)$$

where the marginal cost is given by $MC_t = \frac{W_t}{P_t(1-\alpha)} \frac{Y_t^{\frac{\alpha}{1-\alpha}}}{A_t^{\frac{1}{1-\alpha}}}$ and the ideal markup in the flexible price equilibrium is given by $\mathcal{M}_t = \frac{\epsilon_t}{\epsilon_t - 1}$. The markup is allowed to vary over time and it is the second cost-push shock in the economy: it captures attempts by firms to increase their prices for a given marginal cost, as in Clarida et al. (1999). Together with the time-varying wage markup, it is a reduced-form way to model conflicting aspirations of workers and firms over relative wages and prices, as in Werning and Lorenzoni (2023).

2.3.3 The Mechanics of TIP at the Micro-level

To see how TIP works at the micro-economic level, we look at the version of the first-order condition (12) linearized around a zero inflation and zero TIP steady-state:

$$(\epsilon - 1)\hat{mc}_t + u_t + \beta E_t[\tau_{t+1} + \theta\pi_{t+1}] = \tau_t + \theta\pi_t \quad (13)$$

where u_t is the firm's markup shock. To get more intuition on how TIP operates, we consider a simple case in which TIP is positive at the initial period and zero forever after—$\tau_0 > 0$, and $\tau_t = 0$ for all $t > 0$. In addition, we assume that there is no inflation in the future $\pi_{t+1} = 0$ and no change in the marginal cost so that $\hat{mc}_t = 0$.[1] Using equation (13) we obtain

$$p_0 = p_{-1} + \frac{u_0 - \tau_0}{\theta} \tag{14}$$

which shows that the optimal reaction of firms to an increase in TIP τ_0 is to decrease their (log) price p_0. This reaction is very intuitive: with a tax on price increases, firms have an incentive to moderate their price increases. If there is a positive markup shock u_0, TIP can thus be used to give firms incentives to attenuate the pass-through of these shocks to prices. Consistent with this intuition, in the absence of shocks, $u_0 = 0$, a positive TIP would lead firms to lower their price to receive a subsidy, and hence to deflation. Interestingly, the smaller the adjustment cost θ, the stronger the disinflationary effect of a given level of TIP, τ_0.

2.4 Equilibrium

Definition and linearized equilibrium. The market clearing conditions and the definition of equilibrium are given in Appendix B.1. In Appendix B.2, we derive the log-linear approximation of the model which we will use to simulate the economy when policies follow targeting rules in section 3.3. We denote $\tilde{y}_t = y_t - y_t^n$ the log-deviation of the output gap from its flexible price level and u_t the deviation of markup from its steady-state.

Lemma 1. *Around a steady-state with zero inflation and no TIP, the linearized Euler equation and the Phillips curve are given by:*

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^n\right) \tag{15}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t + \frac{1}{\theta}\left[\beta E_t \tau_{t+1} - \tau_t + \tilde{u}_t\right] \tag{16}$$

where $\kappa = \frac{\epsilon - 1}{\theta}\left(\sigma + \frac{\psi + \alpha}{1 - \alpha}\right)$, $r_t^n = -\log \beta_t + \sigma E_t \left[\frac{1+\psi}{(1-\alpha)\sigma + \psi + \alpha}(a_{t+1} - a_t)\right]$ *and* $\tilde{u}_t = u_t + u_t^w$.

[1] These assumptions are consistent with an equilibrium of the model where there is no shock other than u_t and where monetary policy sets the real rate equal to the neutral rate.

In Appendix B.2, we provide a generalized expression of the Phillips curve around a non-zero steady-state level of TIP. In Appendix E, we show that Calvo-type frictions deliver the same first-order approximation and macroeconomic dynamics. This implies that the results on optimal TIP derived in the linearized model in the next sections are robust to using these frictions instead.[2]

3 Optimal Tax on Inflation

In this section, we analyze how TIP can address inefficient macroeconomic fluctuations. In general, TIP is useful to re-align private with social valuations of price increases and to correct excessive aggregate inflation. We first characterize the combination of MP and TIP that implements the first-best allocation. We show that TIP should increase with cost-push shocks, as they introduce an externality in the firms' pricing decision. We then analyze the stabilization properties of TIP in a second best environment in which both monetary policy and TIP follow targeting rules. We show that in the face of cost-push shocks there is a divine coincidence, but TIP faces a trade-off in the face of demand shocks.

3.1 First Best: Complete Stabilization and Specialization

Social Planner's First Best Allocation. The social planner seeks to maximize the household utility given by (1) subject to the resource constraints (36), the technology (8), and (37). The optimal condition is that at any time t the marginal rate of substitution between labor and consumption is equal to the marginal product of labor: $(C_t^e)^\sigma (N_t^e)^\psi = (1-\alpha) A_t (N_t^e)^{-\alpha}$. After

[2]The equivalence holds for a first-order linearization around a zero inflation steady-state and no indexing. The same Phillips curve also arises as a linear approximation of the model of staggered price contracts by Taylor (1979, 1980) and the state-dependent pricing model with Ss foundations by Gertler and Leahy (2008). The exact equivalence breaks when inflation is strictly positive in steady-state or there is incomplete indexing (Ascari and Sbordone, 2014).

substituting N_t^e using the technology one obtains the optimal consumption and output:

$$C_t^e = (1-\alpha)^{\frac{1-\alpha}{(1-\alpha)\sigma+\psi+\alpha}} A_t^{\frac{1+\psi}{(1-\alpha)\sigma+\psi+\alpha}} \quad and \quad Y_t^e = C_t^e \tag{17}$$

Externalities of Price Changes and Limitations of Monetary Policy. If the steady state is efficient and if there are no cost-push shocks, it is well-known that in the small-scale NK model monetary policy alone can implement the first best by tracking the neutral rate of interest

$$i_t = (Q_t^e)^{-1} - 1 \quad with \quad Q_t^e = E_t\left[\beta_t \left(\frac{A_{t+1}}{A_t}\right)^{-\frac{\sigma(1+\psi)}{(1-\alpha)\sigma+\psi+\alpha}}\right]. \tag{18}$$

This policy closes the output gap $Y_t = Y_t^e$ and maintain price stability $P_t = P_{t-1}$ at all times.

However, when firms and workers would like to increase their markup, $\mathcal{M}_t$ ($\mathcal{M}_t^w$), at the expense of consumers, the firms' returns to increasing prices are larger than their social returns. Cost-push shocks open a wedge between the private and the social returns to increasing prices and thus create an externality in the pricing decision of firms. This wedge can be seen by taking the difference between the private—the left-hand side of the first-order condition of firms (12)–and the social planner's returns, equal to 0, of price increases:

$$\text{Private} - \text{Social} = (\epsilon_t - 1)\left(\mathcal{M}_t\mathcal{M}_t^w - 1\right) + \theta E_t\left[Q_t\frac{Y_{t+1}}{Y_t}(\pi_{t+1}+1)\pi_{t+1}\right] \tag{19}$$

where we set N_t and C_t equal to their first-best value given by (17) for simplicity. The wedge (19) is clearly positive for $\mathcal{M}_t > 1, \mathcal{M}_t^w > 1$ or $E_t\pi_{t+1} > 0$ or any combination of the three.

Monetary policy alone cannot address these externalities and as a result cannot implement the first best and fully stabilize inflation and the output gap. This limitation is commonly known in the literature as the trade-off between inflation and output in the face of cost-push shocks. To correct these externalities giving rise to excessive inflation, policymakers need an instrument that directly affects the price-setting behavior of firms.

First-Best Implementation with TIP. By giving direct incentives to moderate price increases when these shocks hit, TIP can re-align the private with the social valuations of price increases and therefore correct excessive aggregate inflation. Using the Euler equation (5) and the Phillips curve (12), the proposition below derives necessary conditions on MP and TIP such that the decentralized equilibrium coincides with the first best allocation with no inflation and no output gap.

Proposition 1 (First Best Policies)**.** *For any paths of $\mathcal{M}_t$ and $\mathcal{M}_t^w$, aggregate demand β_t and TFP shocks A_t, there exists a combination of TIP and MP that can implement the first best. The nominal policy interest rate is given by* (62) *and TIP is given by*

$$\tau_t = (\epsilon_t - 1)\left(\mathcal{M}_t \mathcal{M}_t^w - 1\right) + E_t Q_t^e \left(\frac{A_{t+1}}{A_t}\right)^{\frac{(1+\psi)}{(1-\alpha)\sigma+\psi+\alpha}} \tau_{t+1} \tag{20}$$

This proposition highlights two important results. First it says that with TIP and MP, policy makers can completely stabilize the output gap and inflation. This is in sharp contrast with an economy where only MP is available. Second, it says that MP should specialize in shocks that affect the neutral rate of interest—aggregate demand β_t, and TFP shocks A_{t+1}/A_t, and that TIP should specialize in markup shocks $\mathcal{M}_t, \mathcal{M}_t^w$.[3]

It is also clear from equation (20) that TIP is most effective at mitigating current markup shocks when it is expected to decrease in the future. When these shocks hit, the government should temporarily raise TIP and announce it will decrease it later (if no new shocks occur). This makes it very appealing

[3]The combination of TIP and MP that implements the first-best allocation in a setting with time-dependent Calvo frictions is given in Appendix E.

for firms to postpone their price adjustments to avoid the tax in the current period. If firms expected the policy to be permanent, these incentives would be significantly weaker and the short-term pass-through of the shock to inflation would be stronger.

An Efficient Steady-state with TIP. In steady state, the market power of firms and workers introduce two wedges between the rate of substitution of consumption and labor and the marginal product of labor which distort the allocation and lower output and consumption. While these distortions are usually addressed with a production or payroll subsidy a_w such that $\mathcal{M}\mathcal{M}^w(1-a_w) = 1$, proposition 1 shows that TIP can also correct it with

$$\tau^{ss} = \frac{\epsilon - 1}{1-\beta}\left(\mathcal{M}\mathcal{M}^w - 1\right). \tag{21}$$

The mechanism behind this result is intuitive. Start from a steady state in which TIP is zero and firms markup $\mathcal{M} > 0$ is optimal, and suppose that the government increases TIP forever. Firms now face an incentive to decrease their price, expand production and hire more labor. In general equilibrium, this leads to an increase in production and in the real wage. But the firms' original incentive to decrease prices is now exactly counterbalanced by their lower-than-optimal markup, which implies that the economy settles in a new steady state with higher output.

First-Best Allocation with TIP in the Linearized Model. To build intuition, we now give a simpler expression, using the linearized model, for the combination of TIP and MP that implements the first best allocation with no output gap and no inflation. Let's denote x_t the log-deviation of output from its efficient level and $\hat{\tau}_t$ the deviation of TIP from its steady-state level.

Corollary 1. *In the linearized model, the first best allocation with zero inflation and zero output gap $x_t = \pi_t = 0$ requires the following path of MP and TIP*

$$i_t = r_t^e \quad \textit{and} \quad \hat{\tau}_t = \beta E_t \hat{\tau}_{t+1} + u_t + u_t^w.$$

In addition, for the allocation to be first best the steady-state needs to be efficient. This requires either a production subsidy or a TIP given by equation 21.[4] Corollary 1 makes it clear that TIP and MP should fully specialize and that TIP should rise with current and future markup shocks. To be more concrete, suppose both shocks u_t, u_t^w are AR(1) processes with identical auto-regressive coefficient $\rho_u < 1$ and with disturbances distributed jointly normal with mean zero.

Example 1. *The TIP that fully stabilizes the economy is given by*

$$\hat{\tau}_t = \frac{u_t + u_t^w}{1 - \beta\rho_u}. \tag{22}$$

It increases with the current level $(u_t + u_t^w)$ *and the persistence* (ρ_u) *of the shock.*

The more persistent the shocks the stronger the reaction to give sufficient incentives to postpone raising prices into the future. On the contrary, if the shocks are i.i.d. over time, TIP should simply match the current ones $\tau_t = u_t + u_t^w$.

The set of policies described in corollary 1, although necessary, may not be sufficient to implement the first-best allocation. These policies may be consistent with several equilibria that are not all first best. To ensure uniqueness, we add the requirement that the central bank reacts strongly to inflation, which is the usual condition for determinacy in this class of model: $i_t = r_t^e + \phi_\pi \pi_t$, with $\phi_\pi > 1$. With this rule, inflation is always zero in equilibrium, $\pi_t = 0$, and the interest rate tracks the neutral rate of interest, $i_t = r_t^e$.[5]

[4] Recall that a second-order approximation of the loss function of the households' utility function around an efficient steady state is given by $\mathcal{L} = \sum_{t=0}^{\infty} E_0 \beta^t \left[\pi_t^2 + \eta_y (x_t)^2\right]$ for a positive constant $\eta_y > 0$. The allocation with zero inflation and no output gap thus achieves the first best.

[5] In a setting with TIP, the Taylor principle may not be the only way to ensure determinacy. We investigate whether TIP can provide new ways to ensure uniqueness in section 3.3.

3.2 Remarks

Time-consistency of TIP. An appealing property of using TIP is that the combination of MP and TIP is time-consistent. The best MP and TIP under discretion are identical to the best policies under commitment, and therefore also deliver full stabilization of the output gap and inflation. This is in sharp contrast with a setting without TIP, where discretionary policies deliver higher inflation, larger output gap and lower welfare (Clarida et al., 1999). To see why, recall that committed policies are more effective than discretionary ones only when commitment about future actions can help alleviate current trade-offs. But given that TIP eliminates the trade-off between output and inflation in the short-run, discretionary policies become as good as committed ones.

Policy Coordination. Although we solved jointly for the optimal MP and TIP, coordination is not required to implement the first best. In other words, the policies described in proposition (1) are also a Nash equilibrium of a game between two hypothetical authorities controlling MP and TIP separately.[6]

Second-best and TFP Shocks. According to the previous analysis, there is no role for TIP when inflation stems from TFP shocks. But if one were to incorporate additional realistic frictions, such labor market imperfections, or financial stability concerns, the clear specialization result we obtained previously may need to be amended and TIP could supplement MP even in the face of demand and TFP shocks. If for example the central bank couldn't increase its interest rate enough to implement the first best after a negative energy price shock due to financial stability concerns, it would arguably be valuable to use TIP besides MP to make firms internalize the social gains of a temporary price moderation and thereby limit an excessive inflation.

[6] An important underlying assumption for this result is that both authorities share the same objective of maximizing the households' welfare.

3.3 Second Best with Targeting Rules

So far in this section we have assumed that policymakers perfectly observe the underlying shocks driving inflation. We now relax this assumption and suppose instead that both TIP and MP follow rules targeting inflation and the output gap relative to steady state, $\hat{y}_t$:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t \tag{23}$$

$$\tau_t = \varphi_\pi \pi_t + \varphi_y \hat{y}_t. \tag{24}$$

In this second best environment, we assess the effectiveness of TIP in reducing the volatility of inflation and the output gap. We start by analyzing how TIP changes the impulse response of the economy to three types of shocks: markup, monetary and productivity shocks.[7] We then evaluate the implications of TIP for the volatility of the economy that is stochastically hit by these shocks, one type of shocks at a time. Our simulations use the four-equation linearized model, calibrated as in Galí (2015) (see Appendix B.3 for details). Unless otherwise mentioned, monetary policy follows a standard Taylor rule (23) with $\phi_\pi = 1.5$ and $\phi_y = 0.125$ and TIP only targets inflation, i.e. $\varphi_y = 0$. We explain at the end of the section that setting this parameter to zero is a reasonable assumption.

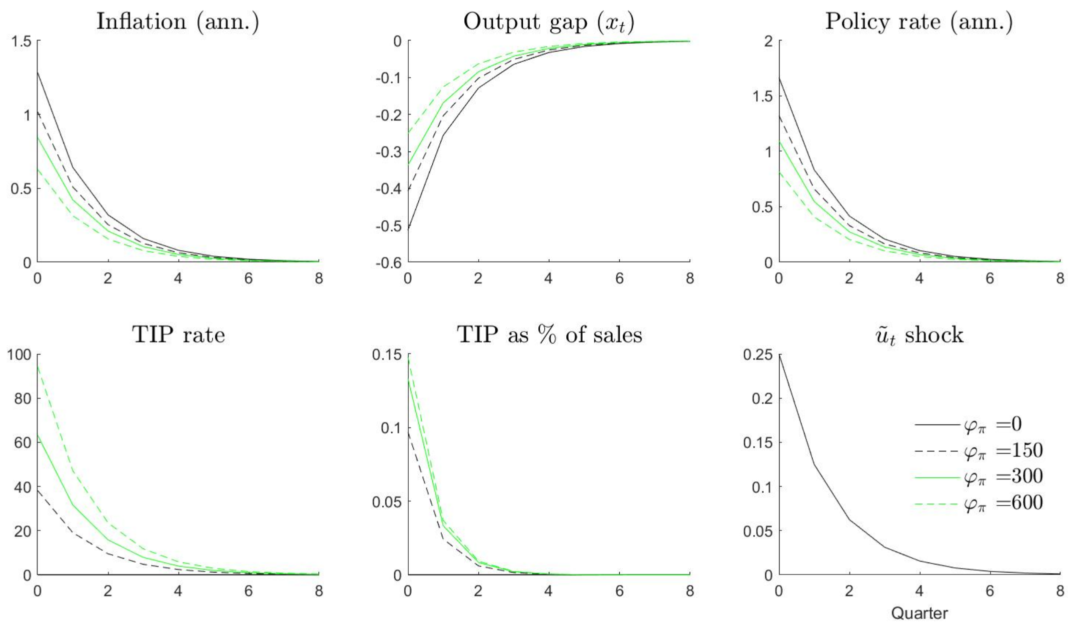
3.3.1 Divine Coincidence with Markup Shocks

Unlike monetary policy, TIP can very effectively reduce both inflation and the output gap after cost-push shocks. This result is analogous to the "divine coincidence" for monetary policy after a demand shock.

Impulse Response Functions. Figure 1 shows the IRFs for different values of φ_π, following a markup shock. In the absence of TIP, inflation initially rises to 1.3% and output drops by slightly over 0.51%. A moderate TIP with

[7] We report the IRF of of (annualized) inflation, the output gap, the (annualized) policy rate, the TIP rate and TIP as a percent of total sales. We report the TIP tax normalized by sales because it is a better measure of the effective tax burden on firms than the tax rate.

$\varphi_\pi = 150$ reduces the initial inflation response to 1.0% and the decline in output to -0.40% by imposing a 38% tax on price changes in the first quarter. A stronger TIP with $\varphi_\pi = 300$ brings down the initial inflation response to 0.84% and the output gap to -0.34% by imposing a 63% tax on price changes.



Notes: The initial markup shock is 0.25pp. Inflation and the policy rate are annualized. Output gap refers to the deviation of output from its efficient level, x_t.

Figure 1: Effects of TIP following a markup ($\tilde{u}$) shock

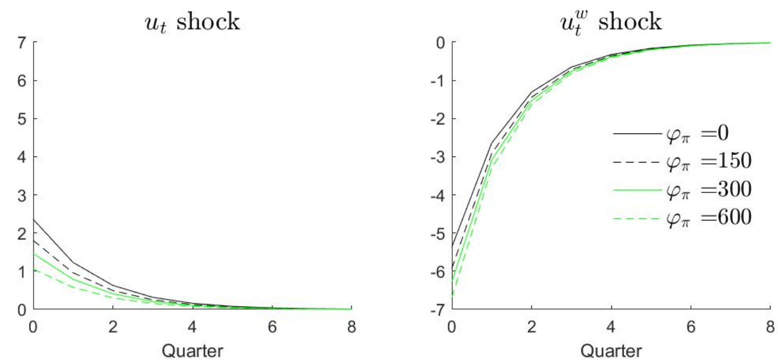
The mechanism behind this divine coincidence is intuitive. The impact of TIP on inflation reduces the need to increase the policy rate, which mitigates the negative impact on output. Relative to a baseline situation with no TIP where the policy rate increases by 170 basis points on impact, a strong TIP with $\varphi_\pi = 300$ reduces the increase in the policy rate to 110 basis points.

While the tax rates may seem high, even if temporary, the actual tax burden on firms is extremely low as a percent of sales because the tax only applies to price changes. For example, the tax burden of the strong TIP represents 0.13% of sales in the first quarter (or 0.04% in the first year). Interestingly, the tax burden relative to sales does not increase linearly as φ_π

increases. This can be seen by looking at expression for the tax burden

$$\frac{\tau_t(P_{ti} - P_{t-1i})Y_{ti}}{P_{ti}Y_{ti}} = \frac{\tau_t \pi_t}{1+\pi_t} \approx \varphi_\pi \pi_t^2,$$

and its expectation, $E\pi_t\tau_t = \varphi_\pi V(\pi_t)$. As φ_π quadruples from 150 to 600, both π_t^2 and $V(\pi_t)$ decrease, yielding a small increase in taxes relative to sales.[8]



Notes: The figure plots the deviation of corporate profits from steady state as a percentage of the steady-state output, following a price (left-hand side) and a wage (right-hand side) markup shock.

Figure 2: Profits following markup shocks

TIP affects corporate profits not only through its direct effect on the tax burden, but also through the general equilibrium changes in inflation and aggregate demand. Figure 2 displays the deviation of corporate profits from steady-state as a percentage of steady-state output, $\left(\frac{\Pi_t}{P_t} - \frac{\Pi}{P}\right) Y^{-1}$, after the firms' and workers' markup shock.[9] When inflation is driven by the firms' desire to increase markups, firms obtain higher profits as a share of output in equilibrium as shown in the left panel, and TIP effectively moderates the increase in profits. When the rise of inflation is driven by an increase in wage

[8]While these issues are outside of the scope of the paper, the result that the tax burden of TIP is very small has an important implication: it should lead firms to comply with the tax, because the costs and legal risks of tax evasion are likely to outweigh the reduction in the tax burden.

[9]An approximation of the share of profit in output is derived in Appendix B.2.

markups, profits decrease due to the increased labor cost as shown in the right panel, and TIP further worsens firms' profitability.

Sensitivity to the Degree of Price Flexibility. Contrary to monetary policy, we find that the effectiveness of TIP is increasing in the degree of price flexibility. To see this, we decrease the Rotemberg parameter θ such that it implies the same slope of the Phillips curve as a decrease in the Calvo parameter $\bar{\phi}$ from 0.75 to 0.5. We also re-scale the $\tilde{u}_t$-shock so that the inflation response in the baseline without TIP is the same as in Figure 1. Figure C3 reports the results.

The intuition behind this result is that when the adjustment cost parameter θ is lower, firms find it less costly to adjust prices, which implies that TIP becomes a relatively stronger obstacle to price changes. Conversely, if prices changes are already very costly for technological reasons, TIP is relatively less important in the decisions of firms to adjust prices.

	No TIP	Moderate TIP	Strong TIP	Extreme TIP
φ_π	0	150	300	600
$\sigma(\pi_t^{ann})$	0.74	0.59	0.49	0.36
$\sigma(x_t)$	0.59	0.47	0.39	0.29
$\sigma(i_t^{ann})$	0.96	0.76	0.63	0.47
$\sigma(\tau_t)$	0.00	44.06	73.02	108.77
$E(\pi_t \tau_t)$	0.00	0.13	0.18	0.20
$\mathcal{L}^* \times 10^4$	1.22	0.77	0.53	0.29

Notes: Simulations with markup shocks only. Inflation π_t and the interest rate i_t are annualized. The welfare-relevant output gap x_t is the deviation of output from its efficient level. The standard deviation of $\frac{u_t}{\theta}$ is 0.25pp, as in Figure 1. $\mathcal{L}^*$ is defined in equation (39) using quarterly variables (i.e., not annualized). All standard deviations and $E(\pi_t \tau_t)$ have been multiplied by 100. The welfare loss function $\mathcal{L}^*$ and its calibration are given in Appendix B.3

Table 1: Evaluation of rules under markup shocks

Stochastic Simulations. In Table 1 we report the results of a stochastic simulation with markup shocks. Consistent with the IRFs just discussed, the standard deviations of inflation and the output gap uniformly decrease as φ_π increases. With a standard Taylor rule and a strong TIP ($\varphi_\pi = 300$), the variance of inflation and the output gap shrinks from 2.52 and 1.26 to 1.87 and .94, respectively. As a result, the welfare losses ($\mathcal{L}$) shrink by 47% from 7.91 when there is no TIP to 4.38. Based on unreported simulations, we confirm that this holds across many Taylor rules.

3.3.2 Trade-off with Demand and Productivity Shocks

TIP mitigates the inflationary effect of a demand or productivity shock to the same extent it does for markup shocks, but it also amplifies the fluctuations of output.

Impulse Response Functions. Figure C1 in Appendix reports the IRFs following a aggregate demand and a productivity shock. The size of the shock has been chosen such that the inflation response in the baseline without TIP is the same as in Figure 1.

We see that TIP mitigates the inflationary effect of the shock to the same extent it does for markup shocks. But because TIP also reduces the need to increase the policy rate, it leads to a widening of the output gap. This is a key difference with the markup shocks: TIP faces a trade-off between inflation and output under aggregate demand and productivity shocks (just as monetary policy faces a trade-off under markup shocks). As discussed in the section on first-best policies, ideally, aggregate demand and productivity shocks should be addressed with monetary policy. When both TIP and MP follow targeting rules, TIP can still complement MP in stabilizing inflation, but it faces a trade-off.[10]

[10]Figure C2 reports the IRFs corresponding to a monetary policy shock. TIP has a qualitatively similar effect on inflation and on the gap between output and its efficient level in the case of a monetary and productivity shock. This suggests that when monetary policy needs to deviate from the standard targeting rule (23) to address financial markets disruptions, or to manage the exchange rate, TIP can greatly mitigate the cost of these interventions.

Stochastic Simulations. Table C2 reports the results of a stochastic simulation with aggregate demand or productivity shocks. It confirms that TIP faces a trade-off between inflation and the output gap when these shocks hit the economy. Increasing φ_π to 300 lowers the variance of inflation from 2.52 to 1.87 but raises the variance of the output gap from 0.94 to 1.27. Because inflation is more volatile than output in the baseline without TIP, the reduction in the variance of inflation outweighs the increase in the variance of the output gap, resulting in higher welfare (lower $\mathcal{L}$).

3.3.3 Determinacy with TIP

To ensure uniqueness of the equilibrium path, it is well known that in the small-scale New Keynesian model, MP should implement the Taylor principle according to which the policy rate reacts strongly to inflation. Without determinacy, the economy is subject to coordination failures: for example, if all firms expect high inflation and high output gap, the economy could shift to a self-fulfilling equilibrium with excessive inflation. These coordination failures are a potential source of excessive inflation.

A natural question is whether TIP could guarantee the uniqueness of the equilibrium path. We show in Appendix B.5 that theoretically it is possible to obtain determinacy if TIP reacts strongly to the output gap, and MP reacts passively to inflation. However, we also find that the necessary strength of the reaction is beyond what could be realistically implemented. This leads us to the conclusion that the Taylor principle remains the only way to ensure determinacy in practice.

4 Equivalence with Alternative Fiscal Instruments

Production subsidies are the traditional tools employed in the literature to address the distortion caused by the firms' markup. In this section we formalize the equivalence between these instruments and TIP. While both instruments can help implement the first best, subsidies entail large and persistent fiscal costs, which may imply distortionary taxation. We then show that a feebate on inflation, and a market for inflation permits could also implement the first best allocation.

4.1 Production and Payroll Subsidies

Traditionally, the literature has emphasized the role of production or payroll subsidies in implementing the first-best allocation.[11] Denoting a_w the rate of payroll subsidies,

$$\Pi(P_{t-1i}, P_{ti}, Y_{ti}) = P_{ti}Y_{ti} - (1 - a_t^w)W_tN_{ti} - \mathcal{C}\left(P_{t-1i}, P_{ti}\right). \tag{25}$$

the following proposition formalizes the equivalence with TIP:

Proposition 2. *Given a path of exogenous shocks, the equilibrium paths of outputs Y_t, employment N_t, wages W_t, profits Π_t and prices P_t are identical in the economy with a payroll subsidy and in the economy with TIP if and only if the path of subsidies follows*

$$a_t^w = \frac{\tau_t - E_tQ_t^e \left(\frac{A_{t+1}}{A_t}\right)^{\frac{(1+\psi)}{(1-\alpha)\sigma+\psi+\alpha}} \tau_{t+1}}{(\epsilon_t - 1)\mathcal{M}_t\mathcal{M}_t^w}$$

This proposition directly stems from the comparison of the first-order condition (20) with the one with a payroll subsidy:

$$(\epsilon_t - 1)\left(\mathcal{M}_t\mathcal{M}_t^w(1 - a_w) - 1\right) + E_tQ_t^e\left(\frac{A_{t+1}}{A_t}\right)^{\frac{(1+\psi)}{(1-\alpha)\sigma+\psi+\alpha}} \theta u_t^E = 0. \tag{26}$$

The advantages of payroll and production subsidies is that they are more conventional and easier to communicate. However, they imply very

[11] Beyond the traditional optimal payroll subsidy to correct the distortion implied by the firms' markup, Correia et al. (2013) and Farhi et al. (2014) show that a payroll subsidy can help achieve the first best in combination with other conventional fiscal instruments when monetary policy is limited by the ZLB or a fixed exchange rate in an economy subject to demand shocks.

large and persistent fiscal costs for the government's budget. Using the calibrated version of the model of section 3.3, we show in Appendix B.4 that the fiscal costs associated with a payroll subsidy that achieves the same macroeconomic path as TIP following a markup shock are very large, and an order of magnitude higher than the fiscal revenues—the tax burden for firms—implied by TIP. In our calibration, to reduce inflation by half a percentage point, payroll subsidies amount to 3.5% of output in the first quarter after the shock, while the tax revenues generated by TIP amount to 0.15% of output. In addition, subsidies, which are proportional to π_t, need to be much more persistent than the cost of TIP, which is proportional to π_t^2.

It is worth nothing that while subsidies are financed with non distortionary lump-sum taxes in the model, in a richer and more realistic setting, taxation is distortionary. The large fiscal cost of subsidies (relative to the small tax burden of TIP) would thus have potentially large economic costs. In addition, these figures represent the additional temporary costs or tax burden resulting from the rise in inflation. They add to any steady-state costs, if there was a production subsidy in steady state.[12]

4.2 Feebate on Inflation Policy (FIP)

TIP increases the tax burden on firms in periods of high inflation. Although we find in the calibrated version of the model in section 3.3 that the tax burden on firms implied by TIP is small, we now show that it is possible to design a system that is budget-neutral on average, in which firms whose prices increase less than the average firm would receive a subsidy and firms whose prices increase more would pay a tax. Such combination of a tax on inflation with a well-designed rebate, which we call a feebate on inflation policy (FIP), would preserve firms' profits on average.

We now formally show that FIP can implement the same macroeconomic outcomes as TIP. Denoting F_t the lump-sum rebate, firms' profits are given

[12] Political acceptability may be an additional barrier to the implementation of these subsidies beyond their economic costs. Policy-makers may find it hard to justify transferring resources to firms that are already benefiting from monopolistic rents.

by:

$$\Pi(P_{t-1i}, P_{ti}, Y_{ti}) = P_{ti}Y_{ti} - W_t N_{ti} - \tau_t(P_{ti} - P_{t-1i})Y_{ti} - \mathcal{C}\left(P_{t-1i}, P_{ti}\right) + F_t \tag{27}$$

$$\text{with} \quad F_t = \tau_t(P_t - P_{t-1})Y_t. \tag{28}$$

It is easy to see that firms whose prices increase less than the average firm would receive a subsidy and firms whose prices increase more would pay a tax. In addition, the FIP is by construction budget neutral, i.e. all the receipts from the TIP are given back to firms, and households' transfers are net of the rebate:

$$T_t = \int_0^1 \Pi_{it} + \tau_t(P_{ti} - P_{t-1i})Y_{ti} - F_t di = \int_0^1 \Pi_{it} di \tag{29}$$

The definition of an equilibrium is the same as before except that firms now maximize (11) subject to the new definition of profits (27) and the definition of transfers to households is given by (29). The following proposition establishes that the allocation in the economy with FIP is exactly the same as in an economy with TIP, except for profits. In the proposition, we denote x_t^{TIP} the value of variable x at time t in the economy with a TIP.

Proposition 3. *Given a path of shocks and of TIP τ_t, the equilibrium paths of output Y_t, employment N_t, wages W_t and prices P_t are identical in the economy with a FIP and a TIP, i.e. $Y_t^{TIP} = Y_t^{FIP}$, $N_t^{TIP} = N_t^{FIP}$, $W_t^{TIP} = W_t^{FIP}$ and $P_t^{TIP} = P_t^{FIP}$. In addition, the path of profits in FIP is higher than in TIP by $F_t : \Pi_t^{FIP} = \Pi_t^{TIP} + F_t$.*

To understand this proposition, it is important to see that the rebate F doesn't affect the firms' behaviors since it is lump-sum and that it doesn't affect the households' income since the lower receipts from the tax on inflation are exactly offset by the higher profits they receive from firms. As a result, no behavior is changed and the equilibrium allocations are the same in both settings. However, profits are higher, which is exactly why FIP is appealing.

4.3 Market for Inflation Permits (MIP)

An alternative instrument, initially proposed by Lerner (1978), is the market for inflation permits (MIP), where firms would issue and trade rights to increase their prices. With a MIP, the quantity of permits is controlled by the government and the price for a firm to change its price is an endogenous clearing price instead of an exogenous tax rate. Relative to a tax on inflation, a MIP could provide more certainty on the level of inflation and would not require approval from the fiscal authority, allowing for quicker reaction when inflation rises. It turns out that, like the FIP, a well-designed MIP may achieve exactly the same macroeconomic outcomes as a TIP.

Let's denote q_t the price of one permit and H_t the quantity of such permits. We assume that H_t is issued every period by the government and that firms can't accumulate permits over time.[13] Under a MIP, profits net of taxes are

$$\Pi(P_{t-1i}, P_{ti}, Y_{ti}) = P_{ti}Y_{ti} - W_t N_{ti} - q_t(P_{ti} - P_{t-1i})Y_{ti} - \mathcal{C}\left(P_{t-1i}, P_{ti}\right). \quad (30)$$

In addition, the market for permits should clear

$$\int_0^1 (P_{ti} - P_{t-1i})Y_{ti} di = H_t \quad (31)$$

and the receipts from the sale of permits are given back to households $T_t = \int_0^1 \Pi_{it} di + q_t H_t$. The definition of an equilibrium is the same as before except that firms now maximize their discounted sum of profits (11) subject to the definition of profits (30). All markets clear including the MIP (31). The following proposition establishes that the allocation in the economy with a MIP is exactly the same as in an economy with TIP, provided that the path of permits issued by the government is appropriate.

Proposition 4. *Given a path of shocks, the equilibrium paths of outputs Y_t, employment N_t, wages W_t, profits Π_t and prices P_t are identical in the economy with a*

[13]The allocation of permits at the beginning of each period across firms and between the government and firms doesn't affect the equilibrium level of output and inflation, it only affects the distribution of profits across firms.

MIP and in the economy with TIP if and only if the path of permits follows

$$H_t = \frac{\pi_t^{TIP}}{1+\pi_t^{TIP}} P_t^{TIP} Y_t^{TIP}$$

To understand this proposition, first observe that the definition of profits in MIP (30) is the same as its definition in TIP (10) if and only if $\tau_t = q_t$. In turn, this equality is true if and only if the supply of permits in the economy with MIP H_t is equal to the total units of price changes in the economy with a TIP, which is equal to

$$H_t = \int_0^1 (P_{ti}^{TIP} - P_{t-1i}^{TIP}) Y_{ti}^{TIP} di = \int_0^1 \frac{\pi_{it}^{TIP}}{1+\pi_{it}^{TIP}} P_{it}^{TIP} Y_{it}^{TIP} di = \frac{\pi_t^{TIP}}{1+\pi_t^{TIP}} P_t^{TIP} Y_t^{TIP}$$

where the last equality used the fact that in equilibrium all firms are identical. This shows that the allocation with TIP can be replicated by issuing the value of permits equal to the increase in nominal output in the economy with TIP.

5 TIP and Relative Price Distortions

One concern with TIP is that it could impede the adjustment of relative prices leading to a misallocation of resources. To assess these effects, we extend the model to include a large number of sectors facing specific TFP shocks that require relative price adjustments. In contrast with price controls, we find that TIP doesn't exacerbate relative price distortions across sectors. We summarize our main results below and provide details in Appendix D.

5.1 An Extended Model with Multiple Sectors

The economy is made of a continuum of sectors, indexed by $s \in [0, 1]$. The production technology for final goods combines sector goods, with an unitary elasticity of substitution, $Y_t = \exp\left(\int_0^1 \gamma_s \ln Y_{ts} ds\right)$ with $\int_0^1 \gamma_s ds = 1$. Each sector is populated by a continuum of firms in monopolistic competition and sector goods combine varieties produced in their sector with a CES production function $Y_{ts} = \left(\int y_{tis}^{1-1/\epsilon_t} di\right)^{\frac{\epsilon_t}{\epsilon_t - 1}}$.

Firms face sector-specific TFP shocks. The production technology of a monopolist producing variety i is sector s is given by, $Y_{tis} = A_{ts} N_{tis}^{1-\alpha}$, where A_{ts} is sector-specific and stochastic. Sector prices change over time, because of aggregate and sector shocks and for future reference, we denote the (log) relative productivity of sector s, $\tilde{a}_{st} = \log(A_{st}/A_t)$ where A_t is the average productivity. Relative to the setting in section 2, the equilibrium now features a non-degenerate distribution of relative prices across sectors, which we denote, in log, $\tilde{p}_{st} = p_{st} - p_t$.[14]

Firms would ideally like to pass through variations in productivity to prices but they face quadratic adjustment costs. This implies that, even without TIP, relative prices depart from their values in the flexible price equilibrium because nominal frictions slow down their adjustment. Furthermore, and consistent with empirical evidence, we allow pricing frictions to differ across sectors and we denote θ_s the sector-specific degree of price stickiness. In Appendix D.1, we give more details on the model environment and derive the equilibrium conditions.

Distortions in relative prices implied by nominal frictions misallocate sector outputs which decreases welfare. In Appendix D.2, we show that in this multi-sector economy, the second-order approximation of the household welfare loss also depends on the average deviation of relative prices *across* sectors from their efficient levels, $E\left((\hat{\tilde{p}}_{st})^2\right)$ where $\hat{\tilde{p}}_{st} = \tilde{p}_{st} - \tilde{p}^e_{st}$. To investigate whether TIP amplifies these price distortions, we proceed in two steps. First we uncover conditions such that TIP has exactly no effect on relative prices and the economy behaves like in the previous sections. Second, we numerically simulate a calibrated model to assess this result quantitatively.

[14]While in our setting "relative prices" unambiguously refer to relative prices across sectors, in a setting with Calvo frictions, there are also distributions of relative prices within sectors across firms. We discuss the robustness of our findings to Calvo frictions in Appendix.

5.2 Independence of Relative Prices: Analytical Insights

We start with evaluating TIP in the first-order approximation of the economy. Under some conditions, TIP affects aggregate inflation and the output gap without changing relative prices.

Proposition 5. *Assume the adjustment cost is common across sectors, $\theta_s = \theta$.*

1. *The distribution of relative prices is independent of deviations of TIP from its steady-state, $\hat{\tau}_t$. It depends on the adjustment cost θ, other parameters $(\epsilon, \alpha, \beta)$, on the sector-specific productivity shocks process $(\tilde{a}_{st})$ and on the steady-state level of TIP, τ^{ss}. If $\tau^{ss} = 0$, it is independent of TIP, τ_t.*

2. *The responses of the output gap x_t and inflation, π_t, to aggregate shocks and to deviations of TIP from steady-state, $\hat{\tau}_t$, are the same as in the linearized economy of Section 3.*

To understand this independence result, it is key to remember that TIP is linear and that it gives the same signal to all firms, irrespective of the sectoral productivity shock they face. On the one hand, TIP gives incentives to firms in sectors that face a negative TFP shock, and would like to increase their price, to moderate these price increases. On the other hand, it gives incentives to firms in sectors that would not change their price without TIP, to lower them and benefit from the subsidy. In general, nothing guarantees that relatives prices across sectors remain exactly unchanged. But it turns out that, at first order, they do when all sectors face the same adjustment cost parameter $\theta_s = \theta$.[15]

To get more intuition, consider the sector-s Phillips curve in the simpler case where $\beta = 0$ (the general proof can be found in Appendix D.3):

$$\pi_{st}\left(1 - \epsilon\frac{\tau^{ss}}{\theta_s}\right) = \frac{\epsilon - 1}{\theta_s}\left[\hat{mc}_t - \left(\frac{1}{1-\alpha}\right)\hat{\tilde{p}}_{st}\right] + \frac{1}{\theta_s}\left[u_t - \hat{\tau}_t\right]. \tag{32}$$

[15] If we had initially allowed for heterogeneous elasticities of substitution, ϵ_{st}, and hence markups, they would have had to be equal across sectors for the independence result to hold.

It is clear from this equation that a condition for deviations of TIP $(\hat{\tau}_t)$ and aggregate shocks $(\hat{mc}_t, u_t)$ to leave relative prices unchanged is that $\theta_s = \theta$. This ensures that deviations of TIP, $\hat{\tau}_t$, and aggregate shocks $(\hat{mc}_t, u_t)$ drop out when taking the difference between the sector-specific and the aggregate Phillips curves:

$$\left(1 - \epsilon \frac{\tau^{ss}}{\theta}\right)(\tilde{p}_{st} - \tilde{p}_{st-1}) = -\frac{\epsilon - 1}{\theta} \frac{1}{1-\alpha}(\tilde{p}_{st} + \tilde{a}_{st}). \tag{33}$$

This linear difference equation in $\tilde{p}_{st}$ is independent of $\hat{\tau}_t$ and depends only on $\theta, \epsilon, \beta, \alpha$, the stochastic process for $\tilde{a}_{st}$ and τ^{ss}. If in addition, $\tau^{ss} = 0$ then the difference equation—hence the distribution of relative prices—is completely independent of TIP.

This independence result stands in sharp contrast with price controls, which distort relative prices. This is because price controls are akin to a convex cost, which one can think of as an additional Rotemberg cost with parameter ϑ. Very strict price controls would imply $\vartheta \to +\infty$. To compare the distortion implied by TIP and price controls, consider two sectors s, s', both starting from an efficient relative price, $\tilde{p}_{st-1} = -\tilde{a}_{st-1}$ and $\tilde{p}_{s't-1} = -\tilde{a}_{s't-1}$. Sector s is hit by a negative TFP shock $\tilde{a}_{st} = \tilde{a}_{st-1} - \Delta$ and sector s' is not. Using (33), it is easy to see that the distortions of relative prices, measured by $(\tilde{p}_{st} + \tilde{a}_{st})^2 + (\tilde{p}_{s't} + \tilde{a}_{s't})^2$, are largest with price controls, smallest with a positive steady-state TIP, and at an intermediate level without TIP: $\left(\frac{\Delta}{1+\frac{\epsilon-1}{1-\alpha}\frac{1}{\theta+\vartheta}}\right)^2 \geq \left(\frac{\Delta}{1+\frac{\epsilon-1}{1-\alpha}\frac{1}{\theta}}\right)^2 \geq \left(\frac{\Delta}{1+\frac{\epsilon-1}{1-\alpha}\frac{1}{(\theta-\epsilon\tau^{ss})}}\right)^2$. In the case of very strict price controls $(\vartheta \to +\infty)$, relative prices between sector s and s' are fixed, so price distortions are maximal and given by Δ^2. This striking difference with price controls highlights the importance of the linearity of TIP. If TIP were convex, or if it applied only to positive price changes and didn't subsidize price decreases, it would distort relative prices.[16]

[16] A steady-state level of TIP τ^{ss} shapes the distribution of relative prices, but not necessarily in a distortionary way. The previous paragraph already showed this result in a simple setting. In Appendix D.4, we solve analytically for the stationary distribution of relative prices and distortions $\tilde{p}_{st} + \tilde{a}_{st}$in the general version of the difference equation (33)

The second point in proposition 5—that the aggregate economy behaves like in the linearized model of the previous sections—implies that the TIP given in Corollary 1 perfectly stabilizes inflation and the output gap in response to aggregate markup shocks. Similarly, all results derived in section 3.3 remain quantitatively exactly the same.

5.3 Independence Result: Numerical Simulations

Although Proposition 5 is general since it holds at and outside of the steady-state, in deterministic and in stochastic settings, for temporary and secular changes in sectoral TFP, it relies on a first-order approximation, and on the homogeneity of price stickiness across sectors. To assess its robustness, we now turn to numerical simulations in the non-linear model with heterogeneous price stickiness. The model is calibrated to match moments of the distribution of price changes in the U.S.

Calibration. We start by specifying in each sector an AR(1) process for TFP: $\log(A_{ts}) = (1-\rho)\log A_s + \rho \log(A_{t-1s}) + \nu_{st}$ where ν_{st} are i.i.d. across time and sectors with standard deviation σ_s which is allowed to be sector-specific. There are thus two sets of sector-specific parameters: θ_s and σ_s.

For the sector-specific pairs of parameters (θ_s, σ_s), our calibration strategy consists in matching the standard deviation and autocorrelation coefficient

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for a non-zero β. Using the same calibration used in section 3.3 and described in Appendix B.3, simulations show that an increase in the steady-state TIP from 0 to 100% only has a marginal effect on price distortions. If anything it slightly decreases price distortions by lowering the persistence of relative prices and by increasing the response of relative prices to sector- specific TFP shocks. To understand why the average distortion can even decrease, recall that TIP influences the pricing decision through two channels which can be seen in the term $\tau_t\left(1-\epsilon_t\frac{\pi_t}{1+\pi_t}\right)$ in the optimal pricing condition (12): a linear first-order channel—raising one's price increases taxes owed—and a second-order channel—increasing one's price reduces a firm's demand, which lowers the tax bill. Since the latter effect is proportional to the firm's price increase, firms that want to increase their price more see a stronger reduction in their tax bill as the demand for their good decreases. This gives them more, not less, incentives to adjust prices, which facilitates the overall relative price adjustments process and lowers price distortions. This result is robust for a range of β and adjustment cost parameter θ.

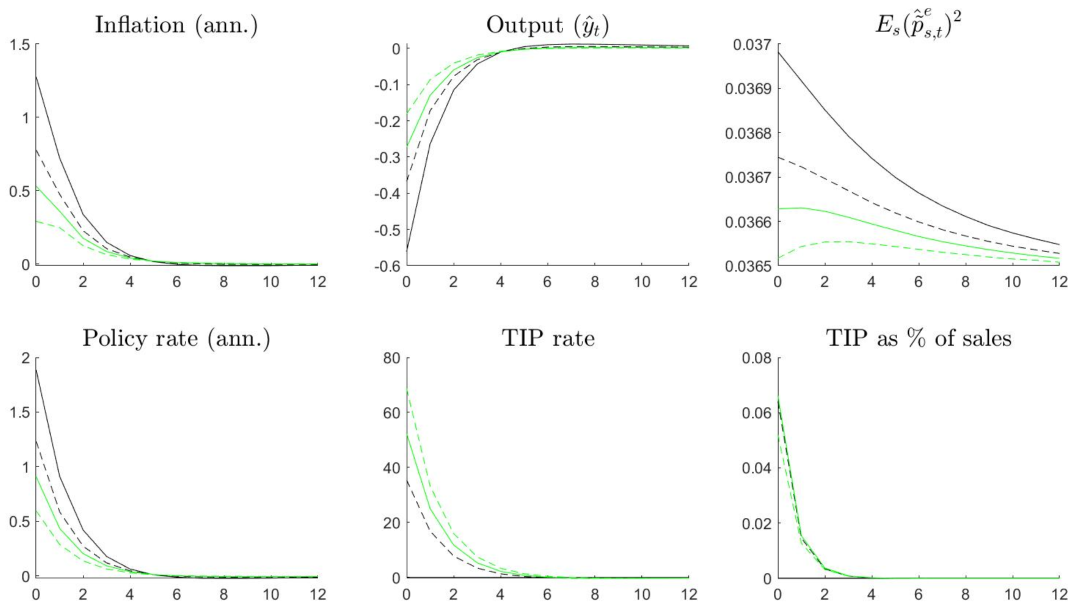
of sectoral price changes in the data as in Ruge-Murcia and Wolman (2022). Intuitively, the former pins down σ_s while the latter informs θ_s. We construct the set of empirical moments using quarterly data from the Bureau of Economic Analysis over the period 1960Q1-2019Q4, excluding the COVID-19 pandemic. To achieve a reasonable degree of sectoral heterogeneity, we group sectors into 13 categories of the core PCE price index. In the model, we split sectors into 13 segments, in proportion to their expenditure weights.

We estimate the 26 parameters using a simulated method of moments. In Appendix D.5 we give more details on the construction of moments, and report the empirical targets, model-implied moments, sector weights and the resulting parameters in Table D4. In addition, we set the steady-state TIP to zero $\tau^{ss} = 0$. For parameters that are common across sectors, we closely follow the calibration in section 3.3 (Table B1) with the exception of ρ. We set ρ to .43 so that the weighted mean of adjustment costs θ_s is comparable to the value we used in the one-sector model in section 2.

We then use the calibrated model to simulate the impulse response of the non-linear economy to unexpected persistent aggregate markup, productivity and demand shocks. Shocks are calibrated such that the initial inflation response is the same as in Figure 1. To solve for the exact transitional dynamics and address the computational difficulties implied by the rich heterogeneity across sectors, we use a quasi-Newton method and the sequence-space Jacobian (Auclert et al., 2021).[17] We then compare the impulse responses of inflation, output and relative price dispersion for different strength of the TIP responses to inflation (φ_π).

Results. Figure 3 shows the aggregate response of the economy, including the model-implied measure of relative price distortion. We find that the average distortion of relative prices across sectors, $E\left(\left(\hat{\hat{p}}_{st}\right)^2\right)$, remains broadly unaffected by the strength of the response of TIP, φ_π, and if anything, decreases very slightly with it. This confirms Proposition 5: because it is a linear tax, TIP doesn't exacerbate price distortions across sectors, in contrast

[17]See Appendix D.6 for details.



Notes: The initial markup shock is calibrated to match the initial inflation response in Figure 1. Inflation and the policy rate are annualized. Output refers to the deviation of output from its steady-state level, $\hat{y}_t$.

Figure 3: Effects of TIP following a markup shock in the multi-sector economy

with price controls. Consistent with the findings in the one-sector economy (Figure 1), a stronger response of TIP leads to lower aggregate inflation and a smaller output gap. Similarly, in the case of demand and productivity shocks, TIP doesn't exacerbate relative price distortions and the aggregate responses are similar to the one-sector economy (see Appendix D.7).

6 TIP in a Medium-Scale DSGE Model

Finally, we evaluate the stabilization properties of TIP in a richer medium-scale DSGE model à la Smets and Wouters (2007). By including a variety of important features and frictions and by estimating the model's parameters and the structural shocks driving fluctuations using likelihood-based techniques, this approach provides a more realistic account of the properties of TIP. Given the results of section 5, we abstract from heterogeneity across sectors and simulate a one-sector economy.

6.1 The Model with TIP and Estimation

We extend the small-scale New Keynesian model considered in section 3.3 by adding habits in consumption, investment in physical capital subject to adjustment costs, sticky nominal wage and indexation of prices and wages, variable capital utilization and fixed costs in production, following closely Smets and Wouters (2007). We allow for multiple shocks driving inflation: a price markup, wage markup, risk premium, investment-specific technology, monetary policy, and exogenous spending shocks.

In this medium-scale model with TIP, we derive the linearized Phillips curve around a trend inflation and zero TIP (see Appendix F). It includes the usual terms—the backward- and forward-looking inflation rates and the marginal cost—and TIP enters in a way that is analogous to the simpler Phillips curve (16) in the small-scale model. Using the same notation as Smets and Wouters (2007) to facilitate comparison, it is given by

$$\hat{\pi}_t = \pi_1 \hat{\pi}_{t-1} + \pi_2 E_t \hat{\pi}_{t+1} + \pi_3 \hat{mc}_t - \pi_4 \left[\tau_t - \bar{\beta}\gamma E_t \tau_{t+1}\right] + \tilde{u}_t \tag{34}$$

with $\pi_1 = \frac{\iota_p}{1+\bar{\beta}\gamma\iota_p}$, $\pi_2 = \frac{\bar{\beta}\gamma}{1+\bar{\beta}\gamma\iota_p}$, $\pi_3 = \frac{(1-\xi_p\bar{\beta}\gamma)(1-\xi_p)}{\xi_p(1+\bar{\beta}\gamma\iota_p)(1+\lambda_p\epsilon)}$ and $\pi_4 = \frac{\pi_3}{\epsilon-1}$. We close the model by assuming that TIP follows an inflation-targeting rule

$$\tau_t = \varphi_\pi(\pi_t - \bar{\pi}) \tag{35}$$

where $\bar{\pi}$ is the estimated trend inflation rate and φ_π a positive parameter.

We bring this model to the U.S. data and estimate it using Bayesian likelihood-based techniques. Our sample starts in 1960Q1 and ends in 2024Q3. When estimating the parameters, we set TIP to zero in all periods ($\varphi_\pi = 0$) and restrict the sample to 1960Q1-2019Q4 to exclude the COVID pandemic whose extreme recession and rebound could bias our parameters' estimates. We then recover the structural shocks for the full sample period. During the periods where the Federal Funds rate is at the zero-lower bound and the Fed deployed unconventional tools, we replace the Federal Funds rate with the Wu and Xia (2016) shadow rate, as in Anderson et al. (2017), Wu and Zhang (2019), Avdjiev et al. (2020).[18] Table F5 in Appendix F reports our priors and posteriors.

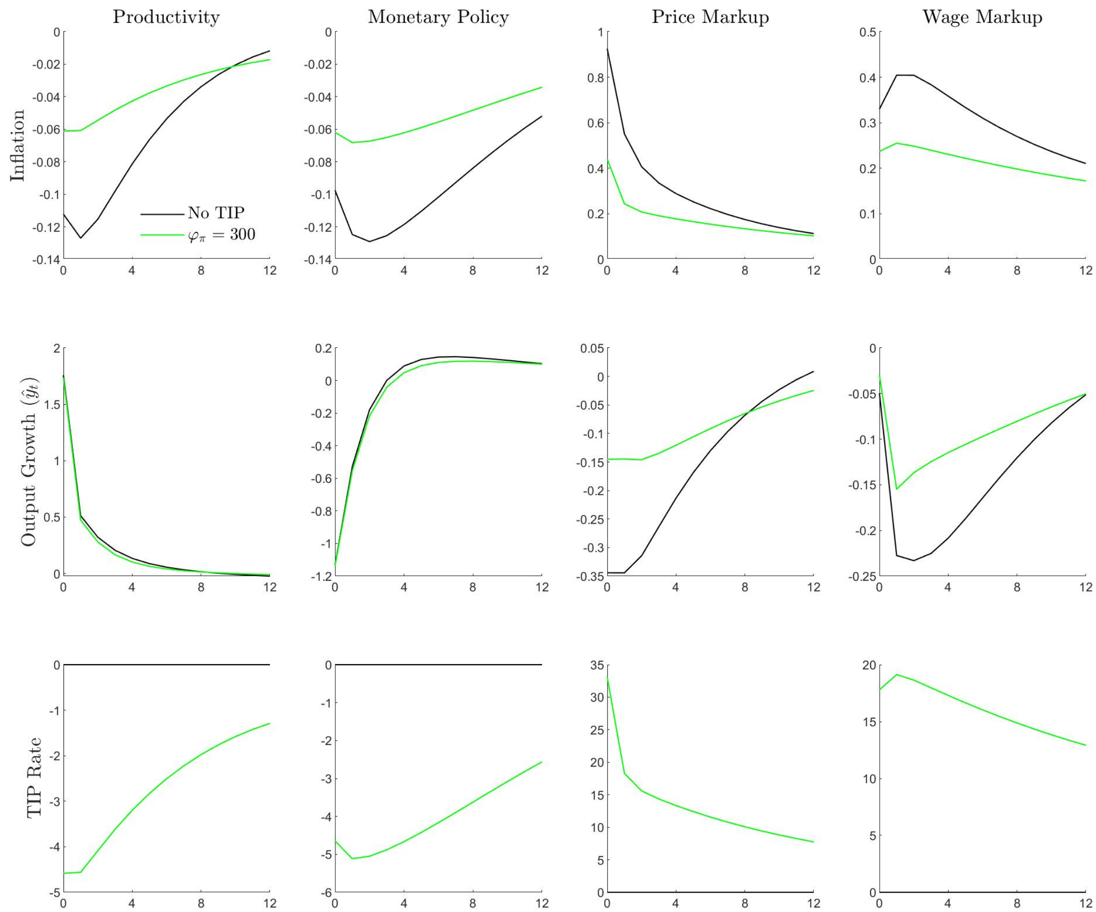
6.2 Impulse Response Functions

To assess the stabilization properties of TIP, we first look at the responses of the economy to each structural shock, when TIP follows the inflation-targeting rule (35) with $\varphi_\pi = 300$. Figure 4 reports the IRF of inflation, output and the TIP rate to productivity, monetary policy, price markup and wage markup shocks. Other variables and other shocks are displayed in Appendix Figure F10.

We find that TIP could provide substantial inflation stabilization gains. More specifically, TIP attenuates the size of the initial inflation responses by 30 to 50% depending on the shocks. Importantly, TIP also halves the output losses after price and wage markup shocks, confirming the divine coincidence result for these shocks derived in section 3.3. After all other five non-markup shocks, TIP has only very limited effects on output which means that the trade-off discussed in section 3.3 is quantitatively negligible.

The result that the effectiveness of TIP increases with the degree of price flexibility continues to hold in the medium-scale model. Recalibrating the Calvo probability from 0.7747 to 0.5247 as in Section 3.3, and keeping all other parameters unchanged, we find that TIP provides even stronger stabilization gains (see Appendix Figure F11). The effects are quantitatively large: TIP lowers the response of inflation by 60 to 85% depending on the shock.

[18] We also try alternative measures for robustness, including the new shadow rate by Jones et al. (2021), the 3-year Treasury yields, and keeping the Federal Funds rate unchanged. Despite all the differences, the effectiveness of TIP is virtually unchanged. Results are available upon request.

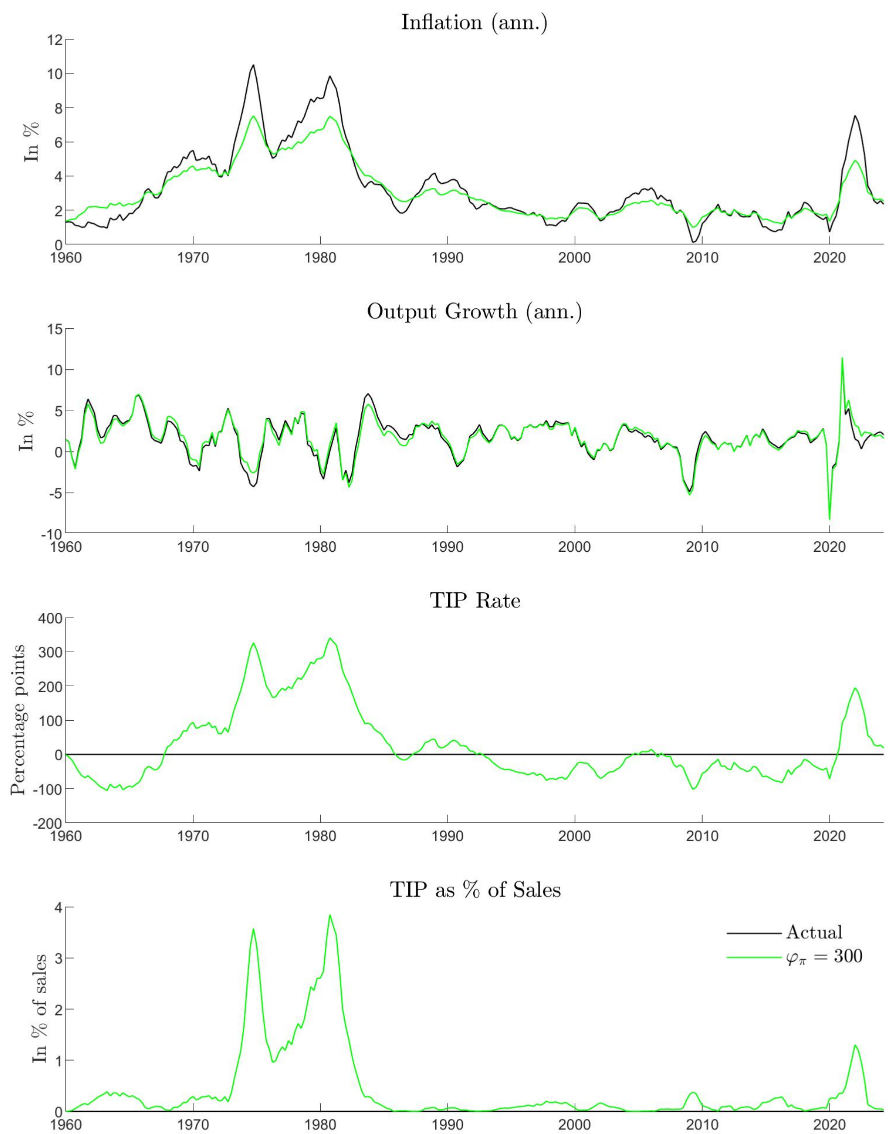


Notes: The initial shock size is one standard deviation. Inflation and the output growth rate are annualized. Output refers to the deviation of output from its steady-state level, $\hat{y}_t$.

Figure 4: Effects of TIP following selected shocks in the medium-scale DSGE model

6.3 Historical Counterfactual

Our final exercise simulates the counterfactual time series of inflation in the U.S. since 1960 if TIP had been used. Using the estimated parameters and structural shocks, we trace out the counterfactual historical path of all variables from 1960 to 2024 assuming that TIP was used and followed the inflation-targeting rule (35) with $\varphi_\pi = 300$.



Notes: All series are 4-quarter moving averages. Inflation and the output growth rate are annualized, while TIP rate and TIP as % of sales are not.

Figure 5: Effects of TIP on the historical path of inflation and output

Figure 5 shows that inflation would have been substantially more stable with TIP. It is especially true in periods where markup shocks are large, which according to the model was the case during the 1970s and the COVID-19 pandemic. When inflation peaked in 1974Q4 and 1980Q4 at around 10% per annum, TIP would have cut it by almost 3 p.p.. During the recent COVID-19 pandemic, TIP would have lowered the peak inflation in 2022Q1 by 2.6 p.p.. During lower inflation periods, TIP is also highly effective at reducing the volatility of inflation. Finally, despite its high effectiveness on inflation, TIP barely affects output growth.

7 Conclusion

In this paper, we put forward a tax on inflation policy (TIP), which would require firms to pay a tax proportional to the increase in their prices or wages. By giving direct incentives to firms to moderate their price increases without exacerbating relative prices distortions, we find that TIP is an effective instrument to control inflation. We show that combining TIP with MP can significantly lower the volatility of inflation and output relative to an economy where only MP is available and that TIP should specialize in addressing cost-push shocks, and MP in addressing demand shocks.

Our paper opens avenues for future research. First, our analysis has focused on how TIP can complement MP in the face of markups shocks, but there are other challenges faced by MP that TIP could help address. For example, a negative TIP could help avoid a deflationary spiral at the ZLB. Second, a few implementation issues deserve a more in-depth quantitative inquiry. Tax avoidance, for example, warrants more attention. The main risk is that firms relabel old products as seemingly new ones, or that they shrink their quality. The paper has argued that these risks are small given the remarkably low tax burden implied by TIP. Quantifying the effects of alternative designs of TIP in a framework with endogenous product creation, information asymmetries about product quality and costly monitoring by the tax administration is an important next step. Third, delving into the political economy of TIP is another important avenue for future research.

What are the risks that TIP be used for objectives other than macroeconomic stabilization? Could it lead to less independent monetary policy in countries with weak institutional frameworks?

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For Online Publication - Appendix

A Brief History of TIP and Literature

Early Proposals of TIP during the Great Inflation. In the context of high and accelerating inflation due to the combined food and oil price shocks of 1973 and 1979, and persistent wage-price spirals, Wallich and Weintraub (1971) formulated the first proposal for a permanent tax on wage increases. At this time, TIP stood for "Tax-based Incomes Policies" and according to their proposal it would be levied on wage increases in excess of a pre-announced target and it would be paid by employers.

This proposal started a literature analyzing the theoretical rationale for a TIP. Kotowitz and Portes (1974) build a microeconomic model in which a union sets wages and firms set prices and find that imposing such a tax does reduce the rate of change of wages. They show that this result is robust to assuming that the union is myopic or forward-looking. Latham and Peel (1977) show that the tax on wage increases is less effective when the firm is a monopsony. From a normative perspective, Seidman (1978) argues that when increasing their price, firms don't take into account their economy-wide inflationary effects. Like a tax on pollution, TIP would signal to firms the social costs of their actions and would make them internalize the externalities of their price-setting behaviors.

A stream of papers studied other macroeconomic implications of TIP. In a macroeconomic model, Peel (1979) shows how the tax could reduce the likelihood of business cycles. Scarth (1983) finds that an employer TIP based on price increases, and an employee TIP based on wage increases are destabilizing, while an employer TIP based on wage increases is stabilizing. Oswald (1984) discusses the conditions for an inflation tax to be equivalent to an employment subsidy and to result in higher employment. Jackman and Layard (1989) show how TIP could reduce the NAIRU and increase welfare even when the tax has an effect on workers' effort.

Contributions to the Literature on TIP. It was acknowledged at the time that a better understanding of price-setting behaviors and their macro implications was required to design TIP effectively (Koford and Miller, 1992). Our first contribution is to leverage improvements made in sticky price models since then to re-assess the effectiveness of TIP.

We share with the earlier literature the conclusion that TIP is an effective tool to control inflation, but while the earlier literature saw TIP as a substitute for MP, we find that MP and TIP are complementary instruments, each specializing in their area of comparative advantage. TIP should focus on markup shocks, and MP should focus on demand shocks. We also confirm the idea that a TIP in steady-state increases welfare by decreasing markup and increasing employment, but we find that TIP should vary over time. It should increase when inflation rises, and decrease as inflation reaches its long-term target.

On implementation, we contribute by analyzing the robustness of TIP to several alternative instruments (feebate and a market for inflation permits), to the use of targeting rules for MP and TIP and to alternative tax bases (tax on wage increases, or tax on large firms), which could be easier to administer. We discuss in conclusion a few other important issues of implementation that deserve more attention in future research.

Optimal Tax Policies. Our paper contributes to the rich literature on tax policies in New Keynesian models. Correia et al. (2008) shows that a sufficiently rich set of fiscal and monetary instruments, including labor taxes and subsidies, can implement the first-best allocation. When monetary policy is constrained by the ZLB, papers have found a welfare-enhancing role for tax increases aimed at restricting supply (Eggertsson and Woodford, 2006), tax cuts aimed by stimulating demand (Eggertsson, 2011), temporary government spending (Woodford, 2011), cuts in marginal labour tax rates that boost confidence (Mertens and Ravn, 2014), and well-designed paths of consumption tax and payroll subsidies, import and export tariffs (Correia et al., 2013; Farhi et al., 2014). We generalize the set of fiscal instruments, showing that, like labor subsidies, TIP helps implement the first best. Importantly, we argue that TIP is much less costly for the government budget than payroll subsidies.

TIP in Practice. In the 1970s, versions of TIP were implemented. From 1974 to 1977, the French governments implemented the "prélèvement conjoncturel", which covered the largest 1500 firms, representing 60% of the economy, and was based on the excess increase in value-added in nominal terms relative to an announced threshold, with an adjustments for fast-growing firms. Other versions of TIP were implemented in Mexico, Belgium, Italy, as mentioned in Paci (1988), and in the Netherlands as explained in OECD (1975). More research is needed to analyze their institutional details and impacts.

TIP came close to be implemented in the U.S. as well. In 1978, the Carter administration proposed to Congress the "real wage insurance" to supplement the wage-price guidelines which included voluntary limits on nominal wage and price increases of 7% and 5.75% respectively. This program meant to give incentives to workers to enforce the guidelines: a worker belonging to an employee group whose earnings increased by less than 7% in a year would receive a tax-credit proportional to the difference between the realized inflation rate and 7% (Colander, 1981).

Why have discussions around TIP stopped in the U.S.? We see three causes. First, afraid of its uncertain costs for the Federal budget (the proposal was more a feebate than a tax), Congress didn't support the plan. Second, the plan combined TIP with price controls which were unpopular, especially since their use by Nixon in the early 1970s. Finally, and most importantly, with Volcker's successful anti-inflation policy in the 1980s, MP has emerged as the sole legitimate instrument for achieving price stability up until the present day. While these may explain why TIP was not further pursued at the time, this paper reassesses its potential role as an additional stabilization tool.

In the context of the transition of formerly Soviet countries to market economies, TIP came back to the forefront of policy discussions and versions of TIP were implemented in Bulgaria, Poland and Romania. Koford et al. (1993) put forward an anti-inflation plan and incentive policies to stabilize prices and output in transition economies. Bogetic and Fox (1993) analyze the design, implementation and enforcement of these policies in Bulgaria and Romania and concludes that they helped stabilize output and prices. Enev and Koford (2000) find a fairly substantial inflation-reducing effect from the Bulgarian policy but no significant results from the Polish policy. Another analysis of the effects of the tax on inflation and on the employment behavior of Polish firms can be found in Crombrugghe and de Walque (2011). These examples suggest that TIP is implementable, effective, and not too costly to administer (Paci, 1988).

B Baseline Model with TIP

B.1 Market Clearing and Equilibrium Definition

Market Clearing We now show the market clearing conditions. In equilibrium, the markets for each intermediate good Y_{ti} and for the final good should clear

$$Y_t = C_t + \int_0^1 \frac{\theta}{2} \left(\frac{P_{ti}}{P_{t-1i}} - 1 \right)^2 Y_t di \tag{36}$$

The sum of labor hired in all firms should be equal to the supply of labor by households:

$$\left(\int N_{th}^{1-1/\sigma_N} dh \right)^{\frac{\sigma_N}{\sigma_N - 1}} = N_t = \int_0^1 N_{ti} di \tag{37}$$

With no government debt, holdings of bonds by households are zero: $B_t = 0$. Finally, transfers received by households include profits and tax receipts:

$$T_t = \int_0^1 \Pi_{it} + \tau_t (P_{ti} - P_{t-1i}) Y_{ti} di. \tag{38}$$

Equilibrium. An equilibrium is a path of output, labor, bonds, wages, price level, bond prices and TIP $\{\{C_{th}, N_{th}, B_{th}, W_{th}\}_h, \{Y_{ti}, N_{ti}, P_{ti}\}_i, W_t, P_t, Q_t, \tau_t\}_{t=0,1,2..}$, such that

- Taking TIP as given, intermediate firms maximize their discounted sum of profits (11) subject to the definition of profits (10), the technology (8) and the demand schedule (7).
- Taking prices as given, final good firms maximize their profits subject to the technology (6).
- Taking prices and transfers (38) as given, households maximize their utility (1) subject to their budget constraint (2), no-ponzi condition (**??**) and demand for labor (3).
- The markets for final good (36), intermediate goods, labor (37) and bonds clear.

B.2 Linearization

Firms' First-Order Condition The first-order conditions associated with the firm's problem are:

$$(1-\epsilon_t)\left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t} Y_t - (1-\epsilon_t)\left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t} Y_t \tau_t - \tau_t \epsilon_t \frac{P_{t-1i}}{P_{ti}} Y_t \left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t}$$

$$+ \frac{\epsilon_t}{1-\alpha} \frac{W_t}{P_{ti}} \left[\left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t} \frac{Y_t}{A_t}\right]^{\frac{1}{1-\alpha}} - \frac{\theta}{P_{t-1i}} \left(\frac{P_{ti}}{P_{t-1i}} - 1\right) P_t Y_t + E Q_t V'(P_{ti}) = 0$$

and

$$V'(P_{t-1i}) = \tau_t Y \left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t} + \frac{P_{ti}}{P_{t-1i}^2}\theta\left(\frac{P_{ti}}{P_{t-1i}} - 1\right) P_t Y_t.$$

Assuming symmetry gives

$$\begin{aligned}
&\left((1-\epsilon_t)Y_t - (1-\epsilon_t)Y_t\tau_t - \tau_t\epsilon_t\frac{P_{t-1}}{P_t}Y_t + \frac{\epsilon_t}{1-\alpha}\frac{W_t}{P_t}\left[\frac{Y_t}{A_t}\right]^{\frac{1}{1-\alpha}} - \frac{\theta}{P_{t-1}}\pi_t P_t Y_t\right) \\
&\qquad + E_t Q_t\left[\tau_{t+1}Y_{t+1} + (\pi_{t+1}+1)^2\theta\pi_{t+1}Y_{t+1}\right] = 0 \\
\iff &\left(1-\epsilon_t - \tau_t\left(1-\epsilon_t\frac{\pi_t}{1+\pi_t}\right) + \epsilon_t MC_t - \theta\pi_t(\pi_t+1)\right) \\
&\qquad + E_t Q_t\left[\tau_{t+1}\frac{Y_{t+1}}{Y_t} + (\pi_{t+1}+1)^2\theta\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] = 0 \\
\iff &\left((1-\epsilon_t)\left(1-\frac{\epsilon_t}{\epsilon_t-1}MC_t\right) - \tau_t\left(1-\epsilon_t\frac{\pi_t}{1+\pi_t}\right) - \theta\pi_t(\pi_t+1)\right) \\
&\qquad + E_t Q_t\left[\tau_{t+1}\frac{Y_{t+1}}{Y_t} + (\pi_{t+1}+1)^2\theta\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] = 0 \\
\iff &\frac{1}{\theta}\left((1-\epsilon_t)\left(1-\mathcal{M}_t MC_t\right) - \tau_t\left(1-\epsilon_t\frac{\pi_t}{1+\pi_t}\right)\right) - \pi_t(\pi_t+1) \\
&\qquad + E_t Q_t\left[\frac{\tau_{t+1}}{\theta}\frac{Y_{t+1}}{Y_t} + (\pi_{t+1}+1)^2\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] = 0
\end{aligned}$$

$$\text{with} \quad MC_t = \frac{W_t}{P_t(1-\alpha)}\frac{Y_t^{\frac{\alpha}{1-\alpha}}}{A_t^{\frac{1}{1-\alpha}}} \quad \text{and} \quad \mathcal{M}_t = \frac{\epsilon_t}{\epsilon_t - 1}.$$

We denote the steady-state markup $\bar{\mathcal{M}}$. The next step is to linearize this optimality condition around a steady-state with no inflation, constant output, a *non-zero tax* on price changes, a price markup, $\pi = 0, \tau = \tau^{ss}, Y = Y', MC_t\mathcal{M} = 1$. Denoting *mc* the log of the real marginal cost *MC* and μ the log of $\mathcal{M}$, we obtain:

$$
\begin{aligned}
&\frac{1}{\theta}\left[(1-\epsilon)\left(1-*(1+mc_t-mc+\mu_t-\mu)\right)-\hat{\tau}_t-\tau^{ss}+\tau^{ss}\epsilon\pi_t\right]\\
&+\beta\frac{\tau^{ss}}{\theta}\left[1+\hat{q}_t+\frac{\hat{\tau}_{t+1}}{\tau^{ss}}+\hat{y}_{t+1}-\hat{y}_t\right]+\beta\pi_{t+1}=\pi_t\\
&\frac{1}{\theta}\left[(1-\epsilon)\left(1-1*(1+mc_t-mc+\mu_t-\mu)\right)-\hat{\tau}_t-\tau^{ss}+\tau^{ss}\epsilon\pi_t\right]\\
&+\beta\frac{\tau^{ss}}{\theta}\left[1-\pi_{t+1}+\frac{\hat{\tau}_{t+1}}{\tau^{ss}}+(1-\sigma)(\hat{c}_{t+1}-\hat{c}_t)\right]+\beta\pi_{t+1}=\pi_t\\
&\frac{1}{\theta}\left[(1-\epsilon)\left(-1*(\hat{mc}_t+\hat{\mu}_t)\right)-\hat{\tau}_t+\tau^{ss}\epsilon\pi_t\right]\\
&+\beta\frac{\tau^{ss}}{\theta}\left[-\pi_{t+1}+\frac{\hat{\tau}_{t+1}}{\tau^{ss}}+(1-\sigma)(\hat{c}_{t+1}-\hat{c}_t)\right]+\beta\pi_{t+1}=\pi_t
\end{aligned}
$$

$$
\begin{aligned}
\pi_t\left(1-\epsilon\frac{\tau^{ss}}{\theta}\right)=\beta E_t\pi_{t+1}\left(1-\frac{\tau^{ss}}{\theta}\right)+\frac{(\epsilon-1)}{\theta}(\hat{mc}_t+\hat{\mu}_t)+\frac{1}{\theta}\left[\beta E_t\hat{\tau}_{t+1}-\hat{\tau}_t\right]\\
+\beta\frac{\tau^{ss}}{\theta}(1-\sigma)(\hat{c}_{t+1}-\hat{c}_t)
\end{aligned}
$$

where $\hat{mc}_t = mc_t - mc$ is the gap between the effective marginal cost and the desired marginal cost under flexible prices in steady-state. Later we will define the cost push shock $u_t = (\epsilon - 1)(\mu_t - \mu)$ is the markup shock.

From this we see why $\tau_t = \beta E_t \tau_{t+1} + u_t + u_t^w$ is the path of TIP that can stabilize inflation as shown in corollary 1.

With log consumption σ, the expression simplifies slightly:

$$
(\epsilon-1)(\hat{mc}_t+\hat{\mu}_t)-\hat{\tau}_t+\beta E_t\left[\hat{\tau}_{t+1}+\pi_{t+1}\left(\theta-\tau^{ss}\right)\right]=\pi_t\left(\theta-\epsilon\tau^{ss}\right)
$$

and dividing by θ on both sides

$$
\pi_t\left(1-\epsilon\frac{\tau^{ss}}{\theta}\right)=\beta E_t\pi_{t+1}\left(1-\frac{\tau^{ss}}{\theta}\right)+\frac{(\epsilon-1)}{\theta}(\hat{mc}_t+\hat{\mu}_t)+\frac{1}{\theta}\left[\beta E_t\hat{\tau}_{t+1}-\hat{\tau}_t+u_t\right]
$$

Alternatively, if $\tau^{ss} = 0$ then the expression simplifies for an arbitrary σ to

$$\begin{aligned}\pi_t &= \frac{1}{\theta}\left((\epsilon-1)\hat{mc}_t - \tau_t + u_t\right) + \beta E_t\left[\frac{\tau_{t+1}}{\theta} + \pi_{t+1}\right] \\ &= \beta E_t \pi_{t+1} + \frac{\epsilon-1}{\theta}\hat{mc}_t + \frac{1}{\theta}\left[\beta E_t \tau_{t+1} - \tau_t + u_t\right]\end{aligned}$$

Equilibrium From the labor market clearing condition $N_t = \int_0^1 N_{ti} di$, we get

$$\begin{aligned}N_t &= \int_0^1 \left(\frac{Y_{ti}}{A_t}\right)^{\frac{1}{1-\alpha}} di \\ &= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}\end{aligned}$$

where we used the fact that all firms are ex post identical $Y_{ti} = Y_t$.

Phillips Curve and Euler Equation Since all firms are idential in equilibrium, the market clearing condition for the final goods market is given by

$$Y_t\left(1 - \frac{\theta}{2}\pi_t^2\right) = C_t.$$

Taking logs

$$y_t + \log\left(1 - \frac{\theta}{2}\pi_t^2\right) = c_t.$$

and approximating around the efficient equilibrium with zero inflation:

$$y_t^e + x_t + 0 = c_t^e + (c_t - c_t^e) \Rightarrow c_t = y_t.$$

We now turn to the first order conditions of the households given by

$$
\begin{aligned}
w_t - p_t &= \sigma c_t + \psi n_t \\
c_t &= E_t c_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - \rho\right)
\end{aligned}
$$

with $i_t = -\log Q_t$. Combining them with the market clearing condition for the final goods gives

$$
\begin{aligned}
w_t - p_t &= \left(\sigma + \frac{\psi}{1-\alpha}\right) y_t - \frac{\psi}{1-\alpha}\log a_t \\
y_t &= E_t y_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - \rho\right)
\end{aligned}
$$

From the definition of markup, we obtain

$$
\begin{aligned}
mc_t &= w_t - p_t + \frac{\alpha}{1-\alpha} y_t - \frac{1}{1-\alpha} a_t - \log(1-\alpha) \\
&= \left(\sigma + \frac{\psi}{1-\alpha}\right) y_t - \frac{\psi}{1-\alpha}\log a_t + \frac{\alpha}{1-\alpha} y_t - \frac{1}{1-\alpha} a_t - \log(1-\alpha) \\
&= \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) y_t - \frac{1+\psi}{1-\alpha} a_t - \log(1-\alpha)
\end{aligned}
$$

where the second line uses the first order condition of the households and the market clearing condition for labor.

Hence the deviation of the markup from steady-state is proportional to the deviation of output from its flexible price level

$$
\hat{mc}_t = \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)\left(y_t - y_t^n\right)
$$

and its deviation from its efficient level is proportional to the deviation of output from its efficient level

$$
mc_t - mc_t^e = \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)\left(y_t - y_t^e\right)
$$

where the flexible price and efficient (no markup) level of output are defined as follows

$$y_t^e = \frac{mc_t^e + \log(1-\alpha)}{\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} + \frac{1+\psi}{(1-\alpha)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} a_t$$
$$mc_t^e = mc$$

From the Euler equation, we obtain the flex-price rate of interest:

$$r_t^n = \rho + \sigma E_t (y_{t+1}^n - y_t^n)$$
$$r_t^n = \rho + \sigma E_t \left[\frac{1+\psi}{(1-\alpha)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} (a_{t+1} - a_t) \right]$$

Combining everything gives

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma} \left(i_t - E_t \pi_{t+1} - r_t^n \right)$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t + \frac{1}{\theta} \left[\beta E_t \tau_{t+1} - \tau_t \right]$$

where $\tilde{y}_t$ denotes the deviation of output from its flex-price level. In addition, we have

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} \left(i_t - E_t \pi_{t+1} - r_t^e \right)$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \frac{1}{\theta} \left[\beta E_t \tau_{t+1} - \tau_t + u_t \right]$$

where x_t is the deviation of output from its efficient level, with $\kappa = \frac{\epsilon-1}{\theta}\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)$.

For future reference we also define the natural rate of output and the neutral interest rate, which are simply equal to their value in flexible price equilibrium:

$$
\begin{aligned}
y_t^n &= \frac{mc^n + \log(1-\alpha)}{\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} + \frac{1+\psi}{(1-\alpha)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} a_t \\
r_t^n &= \rho + \sigma E_t (y_{t+1}^n - y_t^n) \\
r_t^n &= \rho + \sigma E_t \left[\frac{mc_{t+1} - mc_t}{\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} + \frac{1+\psi}{(1-\alpha)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} (a_{t+1} - a_t) \right] \\
&= \rho + \sigma E_t \left[\frac{u_{t+1} - u_t}{(\epsilon - 1)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} + \frac{1+\psi}{(1-\alpha)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} (a_{t+1} - a_t) \right] \\
mc_t^n &= -\mu_t
\end{aligned}
$$

Firms' profits. Real profits are given by

$$
\frac{\Pi(P_{t-1i}, P_{ti})}{P_t} = \frac{1}{P_t} \left[P_{ti} Y_{ti} - W_t N_{ti} - \tau_t (P_{ti} - P_{t-1i}) Y_{ti} - \mathcal{C}_t \left(P_{t-1i}, P_{ti} \right) \right].
$$

In a symmetric equilibrium, $P_t = P_{ti}$, and using the household's first-order condition $\frac{W_t}{P_t} = \mathcal{M}_t^w Y_t^{\left(\sigma + \frac{\psi}{1-\alpha}\right)} A_t^{-\frac{\psi}{1-\alpha}}$ gives:

$$
\begin{aligned}
\frac{\Pi(P_{t-1}, P_t)}{P_t} &= Y_t - \frac{W_t}{P_t} N_t - \tau_t \frac{\pi_t}{1+\pi_t} Y_t - \frac{\theta}{2} \pi_t^2 Y_t \\
&= Y_t - \mathcal{M}_t^w Y_t^\sigma \left(\frac{Y_t}{A_t} \right)^{\frac{\psi}{1-\alpha}} N_t - \tau_t \frac{\pi_t}{1+\pi_t} Y_t - \frac{\theta}{2} \pi_t^2 Y_t \\
&= Y_t - \mathcal{M}_t^w Y_t^\sigma \left(\frac{Y_t}{A_t} \right)^{\frac{\psi+1}{1-\alpha}} - \tau_t \frac{\pi_t}{1+\pi_t} Y_t - \frac{\theta}{2} \pi_t^2 Y_t \\
&= Y_t \left[1 - \mathcal{M}_t^w Y_t^{\sigma-1} \left(\frac{Y_t}{A_t} \right)^{\frac{\psi+1}{1-\alpha}} - \tau_t \frac{\pi_t}{1+\pi_t} - \frac{\theta}{2} \pi_t^2 \right]
\end{aligned}
$$

In the steady state, the efficient level of output is given by

$$
\begin{aligned}
y_t^e &= \frac{mc + \log(1-\alpha)}{\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} + \frac{1+\psi}{(1-\alpha)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} a_t \\
&= \frac{1}{\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} \left[mc + \log(1-\alpha) + \frac{1+\psi}{(1-\alpha)} a_t \right],
\end{aligned}
$$

which solves the steady-state labor share

$$
\begin{aligned}
\mathcal{M}^w (Y_t^e)^{\sigma-1} \left(\frac{Y_t^e}{A_t}\right)^{\frac{\psi+1}{1-\alpha}} &= \frac{\epsilon_N}{\epsilon_N - 1} e^{\left(\sigma - 1 + \frac{\psi+1}{1-\alpha}\right) y_t^e - \frac{1+\psi}{(1-\alpha)} a_t} \\
&= \frac{\epsilon_N}{\epsilon_N - 1} e^{\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) y_t^e - \frac{1+\psi}{(1-\alpha)} a_t} \\
&= \frac{\epsilon_N}{\epsilon_N - 1} e^{mc + \log(1-\alpha)} \\
&= \frac{\epsilon_N}{\epsilon_N - 1} \frac{\epsilon - 1}{\epsilon} (1-\alpha).
\end{aligned}
$$

We can now derive the steady-state profit share:

$$
\frac{\Pi}{PY} = 1 - \frac{\epsilon_N}{\epsilon_N - 1} \frac{\epsilon - 1}{\epsilon} (1-\alpha)
$$

It is natural to simplify the profit share $\frac{\Pi(P_{t-1}, P_t)}{P_t Y_t}$ by log-linearizing the labor share. Although TIP and the adjustment costs are negligible in the first-order approximation around the zero-inflation steady state, we keep them in the expression in order to clarify the different mechanisms at play

$$\begin{aligned}
\frac{\Pi_t}{P_t Y_t} &= 1 - \mathcal{M}_t^w Y_t^{\sigma-1}\left(\frac{Y_t}{A_t}\right)^{\frac{\psi+1}{1-\alpha}} - \tau_t \frac{\pi_t}{1+\pi_t} - \frac{\theta}{2}\pi_t^2 \\
&= 1 - \frac{\mathcal{M}_t^w}{\mathcal{M}^w}\left(\frac{Y_t}{Y_t^e}\right)^{\sigma-1+\frac{\psi+1}{1-\alpha}}\left[\mathcal{M}^w (Y_t^e)^{\sigma-1}\left(\frac{Y_t^e}{A_t}\right)^{\frac{\psi+1}{1-\alpha}}\right] - \tau_t \frac{\pi_t}{1+\pi_t} - \frac{\theta}{2}\pi_t^2 \\
&= 1 - \frac{\mathcal{M}_t^w}{\mathcal{M}^w}\left(\frac{Y_t}{Y_t^e}\right)^{\sigma-1+\frac{\psi+1}{1-\alpha}}\left[\frac{\epsilon_N}{\epsilon_N-1}\frac{\epsilon-1}{\epsilon}(1-\alpha)\right] - \tau_t\pi_t - \frac{\theta}{2}\pi_t^2 \\
&= 1 - \left[\frac{\epsilon_N}{\epsilon_N-1}\frac{\epsilon-1}{\epsilon}(1-\alpha)\right] e^{\left(\sigma-1+\frac{\psi+1}{1-\alpha}\right)x_t + \frac{u_t^w}{\epsilon-1}} - \tau_t\pi_t - \frac{\theta}{2}\pi_t^2 \\
&\approx 1 - \left[\frac{\epsilon_N}{\epsilon_N-1}\frac{\epsilon-1}{\epsilon}(1-\alpha)\right]\left[1 + \left(\sigma-1+\frac{\psi+1}{1-\alpha}\right)x_t + \frac{u_t^w}{\epsilon-1}\right] - \tau_t\pi_t - \frac{\theta}{2}\pi_t^2
\end{aligned}$$

The fourth equation uses the relationship between m_t^w ($\ln \frac{\mathcal{M}_t^w}{\mathcal{M}^w}$) and u_t^w: $u_t^w = (\epsilon-1)m_t^w$. The approximated profit share allows us to analyze both the deviation of the profit share,

$$\frac{\Pi_t}{P_t Y_t} - \frac{\Pi}{PY} \approx -\left[\frac{\epsilon_N}{\epsilon_N-1}\frac{\epsilon-1}{\epsilon}(1-\alpha)\right]\left[\left(\sigma-1+\frac{\psi+1}{1-\alpha}\right)x_t + \frac{u_t^w}{\epsilon-1}\right] - \tau_t\pi_t - \frac{\theta}{2}\pi_t^2$$

and the deviation of the real profit level normalized by steady-state output

$$\left(\frac{\Pi_t}{P_t} - \frac{\Pi}{P}\right)Y^{-1} = \frac{\Pi_t}{P_t Y_t}\frac{Y_t}{Y} - \frac{\Pi}{PY} \approx \frac{\Pi_t}{P_t Y_t}(1+x_t) - \frac{\Pi}{PY}.$$

B.3 Calibration of the three equation model.

Welfare is evaluated using the following *ad hoc* welfare function:

$$\mathcal{L} = \sum_{t=0}^{\infty} E_0 \beta^t \left[\pi_t^2 + \eta_y (x_t)^2 + \eta_i i_t^2\right] \tag{39}$$

for some $\eta_y, \eta_i \geq 0$. This welfare function is *ad hoc* because in the calibration, we will choose $\eta_i > 0$ so that the optimal MP implied by equation (39) is consistent with the standard Taylor rule observed empirically. Note however that a second-order approximation of the household welfare loss around the efficient steady-state would lead to a similar expression of this objective function for $\eta_i = 0$ as shown in Rotemberg and Woodford (1999). One could assume that there is a steady-state production subsidy in the background.

In our calibration, a period is a quarter. The list of parameters is in Table B1. We follow Galí (2015) to calibrate the elasticity of output to labor $(1-\alpha)$, the discount factor β, the elasticity of intertemporal substitution σ, the Frisch elasticity ψ and the elasticity of substitution ϵ.

Parameters	Description	Value
α	One minus the elasticity of output to labor	0.25
β	Time discount factor	0.99
σ	Elasticity of intertemporal substitution	1
ψ	Inverse Frish elasticity of labor	5
ϵ	Elasticity of substitution across varieties	9
ϵ_N	Elasticity of substitution across labor types	∞
θ	Adjustment cost	372.8
ρ_a	Autocorrelation of productivity shock	0.5
ρ_u	Autocorrelation of markup shock	0.5
ρ_{mp}	Autocorrelation of monetary shock	0.5
η_y	Preference for output stability	0.113
η_i	Preference for interest rate stability	0.687

Table B1: Model parameters

We choose the adjustment cost parameter θ such that the slope of the linearized Phillips curve, κ, in our model is equal to the slope in Galí (2015). If $\bar{\phi}$ denotes the Calvo parameter, the slope with Calvo pricing is given by $\kappa = \frac{(1-\bar{\phi})(1-\bar{\phi}\beta)}{\bar{\phi}} \frac{1-\alpha}{1-\alpha+\alpha\epsilon}\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)$. In our model, the slope of the Phillips curve is instead given by $\kappa = \frac{\epsilon-1}{\theta}\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)$. Equating the two gives:

$$\theta = (\epsilon - 1)\frac{\bar{\phi}}{(1-\bar{\phi})(1-\bar{\phi}\beta)}\frac{1-\alpha+\alpha\epsilon}{1-\alpha} \tag{40}$$

A Calvo parameter of 0.75 which corresponds to an average price duration of one year implies $\kappa = 0.17$, and a Rotemberg parameter of 372.8.

To calibrate the parameters in the welfare loss function η_y, η_i, we choose η_y and η_i such that the Taylor rule with coefficients $\phi_y = .125$ and $\phi_\pi = 1.5$ minimizes welfare losses (39) in the absence of TIP. Both η_y, η_i depend on the persistence of the shocks, and for an interior solution to exist, the persistence of the shocks cannot be too high (Giannoni and Woodford, 2003; Giannoni, 2014). Therefore, we set all persistence parameters to 0.5.[19]

B.4 Fiscal cost of subsidies

We first log-linearize equation 26,

$$\frac{\epsilon - 1}{\theta}(mc_t - mc + \mu_t - \mu - a_t^w) + E_t Q_t \pi_{t+1} = \pi_t$$

$$\frac{\epsilon - 1}{\theta}(\hat{mc}_t - a_t^w) + \frac{u_t}{\theta} + \beta E_t \pi_{t+1} = \pi_t$$

To derive the fiscal cost of a payroll subsidy, we assume that its rate, a_t^w, follows a rule targeting inflation, similar to equation 23:

$$a_t^w = \varphi_\pi^w \pi_t.$$

We choose φ_π^w so that the impulse response functions of inflation and the output gap match the ones in Figure 1 after a markup shock.

Figures C4, C5, and C6 report the effects of payroll subsidies following a markup shock, a productivity shock, and a monetary policy shock. To achieve the same macroeconomic outcome after a markup shock as a strong TIP ($\varphi_\pi = 300$) does in Figure 1, payroll subsidies amount to 4% of total payrolls or 3.5% of output in the first period. In sharp contrast, the revenues collected with TIP amount to about 0.15% of output in the same period. The persistence of the fiscal cost also differs across the two instruments. The fiscal cost of payroll subsidies, which are proportional to π_t, decreases much

[19]We have checked that the impulse response functions and welfare implications of TIP are similar when ρ is larger, even when there is no interior solution for the optimal MP.

slower than the revenues from TIP, because they are proportional to π_t^2 instead.

Finally, as we increase the strength of the instrument, payroll subsidies grow linearly while the fiscal revenues implied by TIP increase at a decreasing speed. This is because a stronger TIP can significantly reduce its tax base by moderating price changes, while a higher payroll subsidy rate marginally raises its base by increasing the output gap.

B.5 Determinacy

In this Appendix we analyze the conditions ensuring the uniqueness of the equilibrium path. When there are multiple equilibria, the economy is subject to coordination failures. For example, if all firms expect high inflation and high output gap, the economy could shift to a self-fulfilling equilibrium with excessive inflation. These coordination failures are a potential source of excessive inflation.

To ensure determinacy, it is well-known that in the baseline New Keynesian model, MP should implement the Taylor principle according to which the policy rate reacts strongly to inflation. A natural question is whether TIP could guarantee the uniqueness of the equilibrium path.

Proposition 6. *Assume $\tau^{ss} = 0$. The equilibrium path is unique if one of the following conditions holds*

- $\phi_\pi > 1$ *and* $\varphi_y < \min\left(\frac{\theta\kappa}{1-\beta}, \frac{\theta\phi_\pi\kappa + \theta(1-\beta)(\sigma(1-\beta)+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)}{\phi_\pi - \beta}\right)$
- $\beta < \phi_\pi < 1$ *and* $\frac{\theta\kappa}{1-\beta} < \varphi_y < \frac{\theta\phi_\pi\kappa + \theta(1-\beta)(\sigma(1-\beta)+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)}{\phi_\pi - \beta}$
- $\phi_\pi < \beta$ *and* $\frac{\theta\kappa}{1-\beta} < \varphi_y$

The first bullet point corresponds to the traditional Taylor principle according to which the reaction of monetary policy to inflation is strong enough so that the path of inflation is always unique. In this case, the reaction of TIP to the output gap can't be too strong. The second and third cases correspond to a new principle according to which the reaction of the monetary policy to inflation is weak but the reaction of TIP to the output gap is strong.

This finding that a high enough φ_y can ensure determinacy is intuitive and the mechanism is analogous to the Taylor principle. Assume that agents expect the economy to jump to a situation of high inflation and high output gap. Following the TIP rule, policymakers set a very high tax on inflation which leads to deflation, which in turn triggers a recession. This outcome contradicts the initial expectation. This ensures that the economy always stays on a unique equilibrium path.

However, in practice, only the Taylor principle can realistically be implemented for any reasonable calibration of the model's parameters. Following the calibration laid out in the quantitative section, the value of the coefficient φ_y necessary to ensure determinacy, $\frac{\theta\kappa}{1-\beta}$, is slightly above 6300. This coefficient implies a reaction function that is too strong to be realistically implemented. For this reason, in the rest of the paper, we restrict our attention to targeting rules where the Taylor principle applies and the reaction of TIP to the output gap is not too strong.

We now provide a formal proof for the theorem. Substituting out the Taylor rule (23) into the Euler equation (15) gives

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(\rho + \phi_\pi \pi_t + \phi_y \hat{y}_t - E_t \pi_{t+1} - r_t^e\right),$$
$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(\rho + \phi_\pi \pi_t + \phi_y x_t - E_t \pi_{t+1} + \phi_y (y_t^e - y^{ss}) - r_t^e\right),$$

and substituting out the rule for TIP (24) into the Phillips curve (16) gives

$$\left(1 - \epsilon\frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right)\pi_t = \beta\left(1 - \frac{\tau^{ss}}{\theta} + \frac{\varphi_\pi}{\theta}\right)E_t\pi_{t+1} + \left(\kappa - \frac{\varphi_y}{\theta}\right)x_t + \frac{\varphi_y}{\theta}\beta E_t x_{t+1} + \frac{1}{\theta}u_t.$$

To analyze the uniqueness of the solution to this system of two difference equations, we compute its eigenvalues. This system with two non predetermined variables is determinate if and only if both eigenvalues are inside the unit circle (Blanchard and Kahn, 1980). We derive the necessary and sufficient conditions for this to hold in appendix, and show sufficient conditions in the following proposition which turn out to be more intuitive.

We first rewrite this system in matrix form

$$A\begin{pmatrix}\pi_t\\ x_t\end{pmatrix}=B\begin{pmatrix}E_t\pi_{t+1}\\ E_tx_{t+1}\end{pmatrix}+C\begin{pmatrix}\tilde{u}_t\\ r^e_t-\rho-\phi_y(y^e_t-y^{ss})\end{pmatrix}$$

with $A=\begin{pmatrix}\phi_\pi & \sigma+\phi_y\\ 1-\epsilon\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta} & \frac{\varphi_y}{\theta}-\kappa\end{pmatrix}$, $B=\begin{pmatrix}1 & \sigma\\ \beta\left(1-\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right) & \frac{\varphi_y}{\theta}\beta\end{pmatrix}$ and $C=\begin{pmatrix}1 & 0\\ 0 & \frac{1}{\theta}\end{pmatrix}$

We now solve for the eigenvalues of this system. We first invert A

$$A^{-1}=\frac{1}{\phi_\pi\left(\frac{\varphi_y}{\theta}-\kappa\right)-(\sigma+\phi_y)\left(1-\epsilon\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right)}\begin{pmatrix}\frac{\varphi_y}{\theta}-\kappa & -\sigma-\phi_y\\ \epsilon\frac{\tau^{ss}}{\theta}-1-\frac{\varphi_\pi}{\theta} & \phi_\pi\end{pmatrix}$$

and then multiply it by B:

$$\begin{aligned}A^{-1}B&=\frac{1}{\phi_\pi\left(\frac{\varphi_y}{\theta}-\kappa\right)-(\sigma+\phi_y)\left(1-\epsilon\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right)}\begin{pmatrix}\frac{\varphi_y}{\theta}-\kappa & -\sigma-\phi_y\\ \epsilon\frac{\tau^{ss}}{\theta}-1-\frac{\varphi_\pi}{\theta} & \phi_\pi\end{pmatrix}\begin{pmatrix}1 & \sigma\\ \beta\left(1-\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right) & \frac{\varphi_y}{\theta}\beta\end{pmatrix}\\
&=\Omega\begin{pmatrix}\frac{\varphi_y}{\theta}-\kappa-(\sigma+\phi_y)\beta\left(1-\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right) & \sigma\left(\frac{\varphi_y}{\theta}-\kappa\right)-(\sigma+\phi_y)\beta\frac{\varphi_y}{\theta}\\ \left(1+\frac{\varphi_\pi}{\theta}\right)[\beta\phi_\pi-1]+\frac{\tau^{ss}}{\theta}(\epsilon-\phi_\pi\beta) & -\sigma\left(1-\epsilon\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right)+\phi_\pi\frac{\varphi_y}{\theta}\beta\end{pmatrix}\\
&=-\Omega\begin{pmatrix}-\frac{\varphi_y}{\theta}+\kappa+(\sigma+\phi_y)\beta\left(1-\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right) & \sigma\left(\kappa-\frac{\varphi_y}{\theta}\right)+(\sigma+\phi_y)\beta\frac{\varphi_y}{\theta}\\ \left(1+\frac{\varphi_\pi}{\theta}\right)[1-\beta\phi_\pi]+\frac{\tau^{ss}}{\theta}(\phi_\pi\beta-\epsilon) & \sigma\left(1-\epsilon\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right)-\phi_\pi\frac{\varphi_y}{\theta}\beta\end{pmatrix}\end{aligned}$$

with $\Omega=\frac{1}{\phi_\pi\left(\frac{\varphi_y}{\theta}-\kappa\right)-(\sigma+\phi_y)\left(1-\epsilon\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right)}$. We denote this matrix A', and we now compute its trace and determinant.

$$\begin{aligned}TrA'&=-\Omega\left[-\frac{\varphi_y}{\theta}+\kappa+(\sigma+\phi_y)\beta\left(1-\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right)+\sigma\left(1-\epsilon\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right)-\phi_\pi\frac{\varphi_y}{\theta}\beta\right]\\
&=\frac{1}{(\sigma+\phi_y)\left(1-\epsilon\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)}\\
&\qquad\left[-\frac{\varphi_y}{\theta}(1+\beta\phi_\pi)+\kappa+(\sigma(\beta+1)+\phi_y\beta)\left(1+\frac{\varphi_\pi}{\theta}\right)-\frac{\tau^{ss}}{\theta}((\sigma+\phi_y)\beta+\epsilon\sigma)\right]\end{aligned}$$

$$
\begin{aligned}
detA' = &\left(\frac{1}{(\sigma+\phi_y)\left(1-\epsilon\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)}\right)^2 \\
&\times\left[\left[-\frac{\varphi_y}{\theta}+\kappa+(\sigma+\phi_y)\beta\left(1-\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right)\right]\left[\sigma\left(1-\epsilon\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right)-\phi_\pi\frac{\varphi_y}{\theta}\beta\right]\right. \\
&\quad\left.-\left[\sigma\left(\kappa-\frac{\varphi_y}{\theta}\right)+(\sigma+\phi_y)\beta\frac{\varphi_y}{\theta}\right]\left[\left(1+\frac{\varphi_\pi}{\theta}\right)[1-\beta\phi_\pi]+\frac{\tau^{ss}}{\theta}(\phi_\pi\beta-\epsilon)\right]\right] \\
= &\left(\frac{1}{(\sigma+\phi_y)\left(1-\epsilon\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)}\right)^2 \\
&\times\left(\left[\left[-\frac{\varphi_y}{\theta}+\kappa+(\sigma+\phi_y)\beta\left(1+(\epsilon-1)\frac{\tau^{ss}}{\theta}+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)\right]\left[\sigma\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)-\phi_\pi\frac{\varphi_y}{\theta}\beta\right]\right.\right. \\
&\quad\left.\left.-\left[\sigma\left(\kappa-\frac{\varphi_y}{\theta}\right)+(\sigma+\phi_y)\beta\frac{\varphi_y}{\theta}\right]\left[\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)[1-\beta\phi_\pi]+\frac{\tau^{ss}}{\theta}\beta\phi_\pi(1-\epsilon)\right]\right]\right) \\
= &\left(\frac{1}{(\sigma+\phi_y)\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)}\right)^2 \\
&\times\left(\beta\left[(\sigma+\phi_y)\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)\right]\left[\sigma\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)-\frac{\varphi_y}{\theta}\right]\right. \\
&\left.+\frac{\tau^{ss}}{\theta}(\epsilon-1)\left((\sigma+\phi_y)\beta\left[\sigma\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)-\phi_\pi\frac{\varphi_y}{\theta}\beta\right]+\beta\phi_\pi\left[\sigma\left(\kappa-\frac{\varphi_y}{\theta}\right)+(\sigma+\phi_y)\beta\frac{\varphi_y}{\theta}\right]\right)\right) \\
= &\left(\frac{1}{(\sigma+\phi_y)\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)}\right)^2 \\
&\times\beta\left(\left[(\sigma+\phi_y)\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)\right]\left[\sigma\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)-\frac{\varphi_y}{\theta}\right]\right. \\
&\left.+\frac{\tau^{ss}}{\theta}(\epsilon-1)\sigma\left((\sigma+\phi_y)\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
detA' &= \left(\frac{1}{(\sigma+\phi_y)\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)}\right)^2 \\
&\times\beta\left[(\sigma+\phi_y)\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)\right]\left[\sigma\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)-\frac{\varphi_y}{\theta}+\frac{\tau^{ss}}{\theta}(\epsilon-1)\sigma\right] \\
&= \left(\frac{1}{(\sigma+\phi_y)\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)}\right)\beta\left[\sigma\left(1+\frac{\varphi_\pi-\tau^{ss}}{\theta}\right)-\frac{\varphi_y}{\theta}\right]
\end{aligned}
$$

Let's assume $\varphi_y, \varphi_\pi, \phi_\pi, \phi_y \geq 0$ and restrict our analysis to the case where the determinant is positive (both eigenvalues have the same sign and are non-imaginary), i.e. assume that $\left(\sigma\left(1+\frac{\varphi_\pi-\tau^{ss}}{\theta}\right)-\frac{\varphi_y}{\theta}\right)\left((\sigma+\phi_y)\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)+\phi_\pi\kappa-\phi_\pi\frac{\varphi_y}{\theta}\right) > 0$. These restrictions are consistent with any empirically reasonable parametrization.

This system with two non predetermined variables is determinate if and only if both eigenvalues are within the unit circle (Blanchard and Kahn, 1980). There are then two sufficient and necessary conditions for both eigenvalues to be within the unit circle: $detA' < 1$ and $TrA < 1 + detA'$. The latter condition can be derived from the condition that both eigenvalues are strictly below 1 $(1-\lambda_1)(1-\lambda_2) > 0$ when both are positive, or both strictly above -1 $(-1-\lambda_1)(-1-\lambda_2) > 0$ when both are negative. The condition $detA' < 1$ gives:

$$
\frac{\tau^{ss}}{\theta}((\sigma+\phi_y)\epsilon+\beta\sigma)+\frac{\varphi_y}{\theta}(\phi_\pi-\beta) < \phi_\pi\kappa+(\sigma(1-\beta)+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)
$$

This condition is that φ_y is small enough:

$$
\varphi_y < \frac{\theta\phi_\pi\kappa+\theta(\sigma(1-\beta)+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)-\tau^{ss}((\sigma+\phi_y)\epsilon+\beta\sigma)}{\phi_\pi-\beta}
$$

The second necessary and sufficient condition is that $TrA' < 1 + detA'$ which gives

$$\frac{\left[-\frac{\varphi_y}{\theta}(1+\beta\phi_\pi)+\kappa+(\sigma(\beta+1)+\phi_y\beta)\left(1+\frac{\varphi_\pi}{\theta}\right)-\frac{\tau^{ss}}{\theta}((\sigma+\phi_y)\beta+\epsilon\sigma)\right]}{(\sigma+\phi_y)\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)} < 1+\frac{\beta\left[\sigma\left(1+\frac{\varphi_\pi-\tau^{ss}}{\theta}\right)-\frac{\varphi_y}{\theta}\right]}{(\sigma+\phi_y)\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)}$$

$$-\frac{\varphi_y}{\theta}(1+\beta\phi_\pi)+\kappa+(\sigma(\beta+1)+\phi_y\beta)\left(1+\frac{\varphi_\pi}{\theta}\right)-\frac{\tau^{ss}}{\theta}((\sigma+\phi_y)\beta+\epsilon\sigma) < (\sigma+\phi_y)\left(1+\frac{\varphi_\pi-\epsilon\tau^{ss}}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)+\beta\left[\sigma\left(1+\frac{\varphi_\pi-\tau^{ss}}{\theta}\right)-\frac{\varphi_y}{\theta}\right]$$

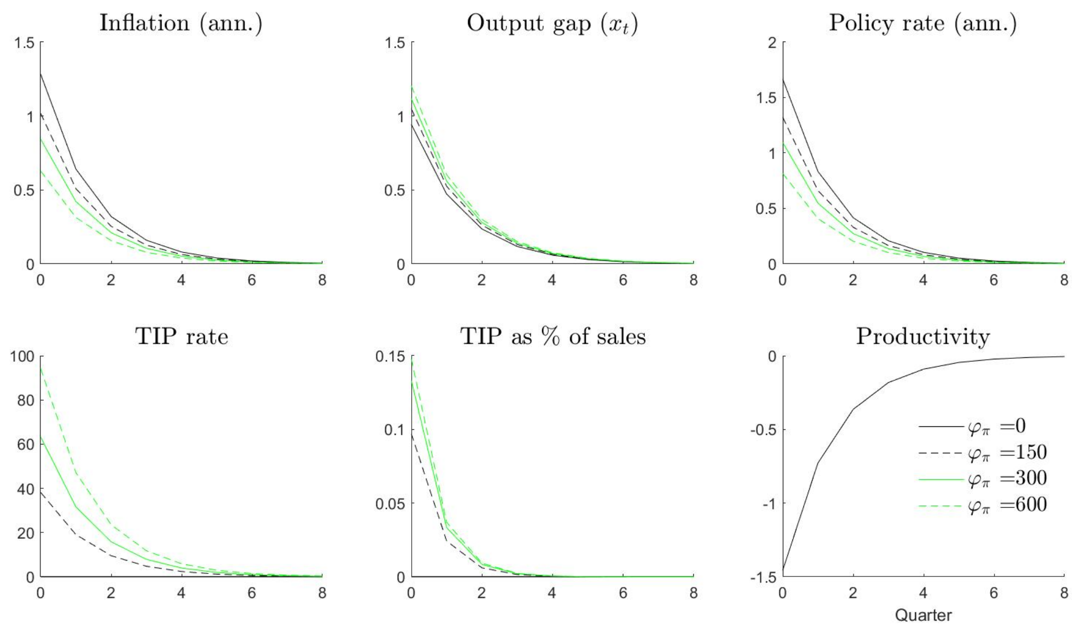
$$-\frac{\varphi_y}{\theta}(1+\beta\phi_\pi)+\kappa-\left(1+\frac{\varphi_\pi}{\theta}\right)(1-\beta)\,\phi_y < \phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)-\beta\frac{\varphi_y}{\theta}+\frac{\tau^{ss}}{\theta}((\sigma+\phi_y)\beta+\epsilon\sigma-(\sigma+\phi_y)\epsilon-\beta\sigma)$$

$$(\phi_\pi-1)\kappa+\frac{\varphi_y}{\theta}\left[(1+\beta\phi_\pi)-\beta-\phi_\pi\right]+\left(1+\frac{\varphi_\pi}{\theta}\right)(1-\beta)\,\phi_y+\frac{\tau^{ss}}{\theta}\phi_y(\beta-\epsilon)>0$$

$$(\phi_\pi-1)\left[\kappa-\frac{\varphi_y}{\theta}(1-\beta)\right]+\left(1+\frac{\varphi_\pi}{\theta}\right)(1-\beta)\,\phi_y>\frac{\tau^{ss}}{\theta}\phi_y(\epsilon-\beta)$$

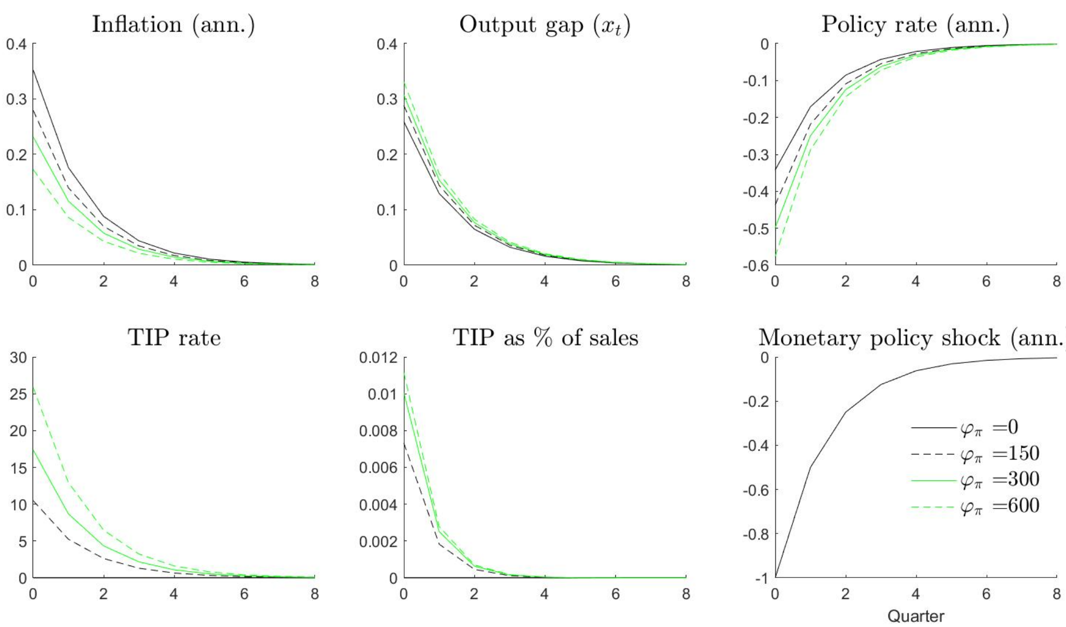
This ends the proof.

C Additional Graphs - Simulations



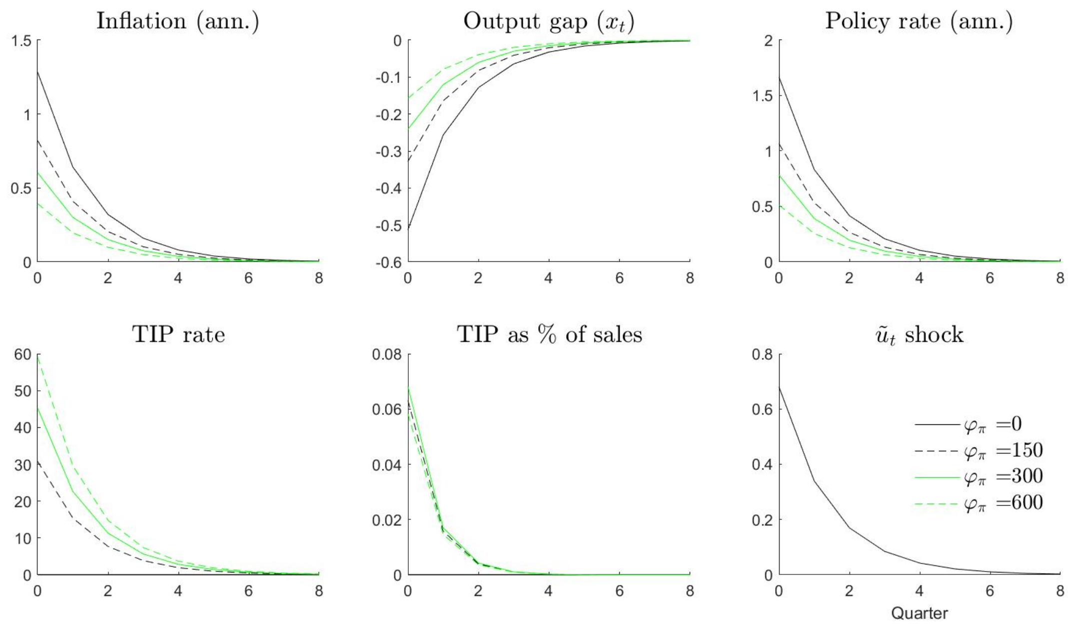
Notes: The initial productivity shock is such that the response of inflation in the baseline without TIP is the same as in the case of a markup shock. Inflation and the policy rate are annualized. Output gap refers to the deviation of output from its efficient level, x_t.

Figure C1: Effects of TIP following a negative productivity shock



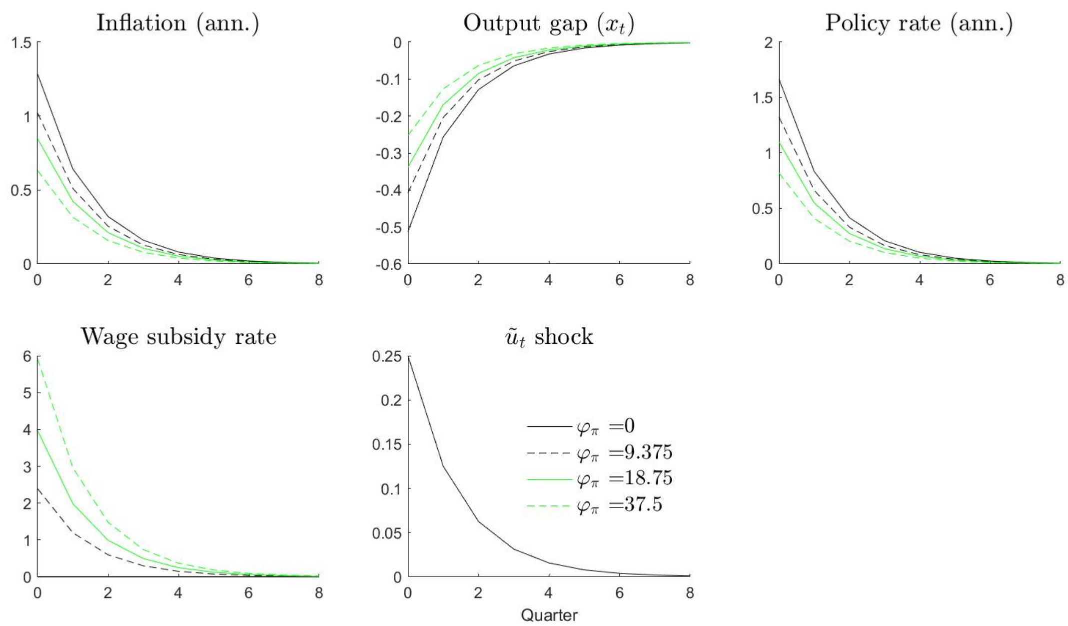
Notes: The initial monetary policy shocks is 0.25p.p. (or 1p.p. annualized). Inflation and the policy rate are annualized. Output gap refers to the deviation of output from its efficient level, x_t.

Figure C2: Effects of TIP following a monetary policy shock



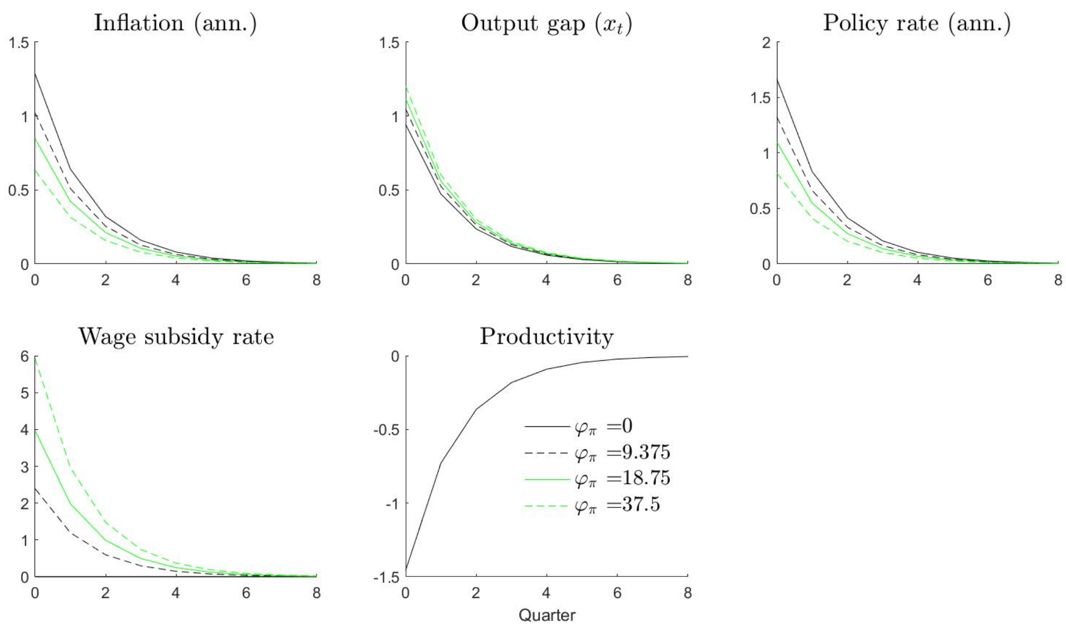
Notes: This figure corresponds to an economy with smaller adjustment costs. We set the Rotemberg parameter such that the equivalent Calvo parameter is equal to 0.5 instead of 0.75. Inflation and the policy rate are annualized. Output gap refers to the deviation of output from its efficient level, x_t.

Figure C3: Effects of TIP following a markup shock with lower price stickiness



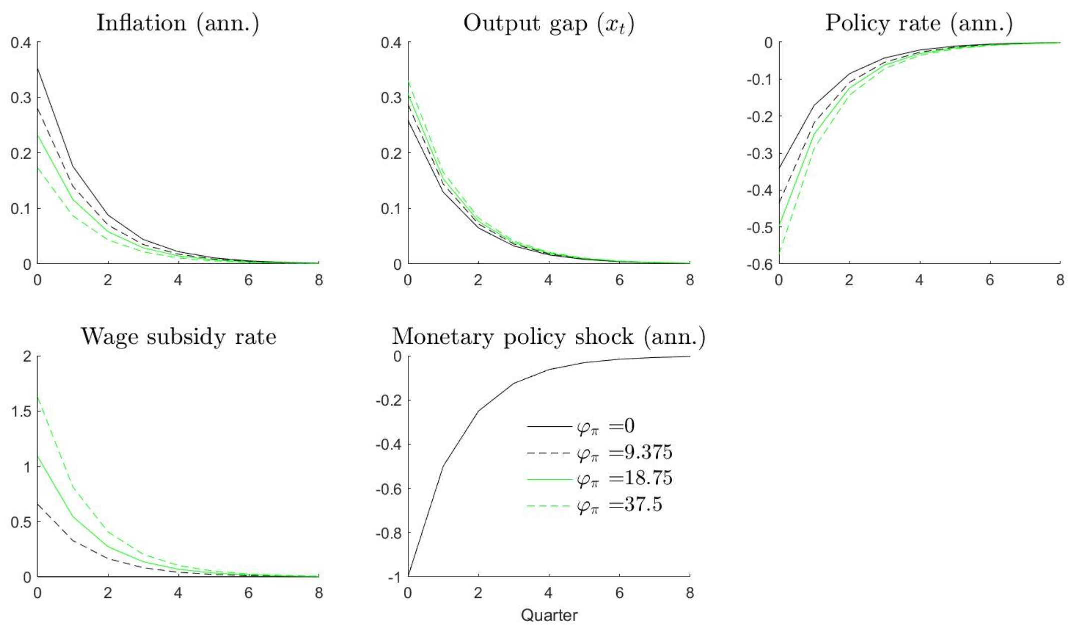
Notes: This figure corresponds to an economy with time-varying payroll subsidies instead of TIP. The shock is the same as in Figure 1. Inflation and the policy rate are annualized. Output gap refers to the deviation of output from its efficient level, x_t.

Figure C4: Effects of payroll subsidies following a markup shock



Notes: This figure corresponds to an economy with time-varying payroll subsidies instead of TIP. The shock is the same as in Figure C1. Inflation and the policy rate are annualized. Output gap refers to the deviation of output from its efficient level, x_t.

Figure C5: Effects of payroll subsidies following a productivity shock



Notes: This figure corresponds to an economy with time-varying payroll subsidies instead of TIP. The shock is the same as in Figure C2. Inflation and the policy rate are annualized. Output gap refers to the deviation of output from its efficient level, x_t.

Figure C6: Effects of payroll subsidies following a monetary policy shock

	No TIP	Moderate TIP	Strong TIP	Extreme TIP
φ_π	0	150	300	600
ϕ_π	1.5	1.5	1.5	1.5
ϕ_y	0.125	0.125	0.125	0.125
$\sigma(\pi_t^{ann})$	0.74	0.59	0.49	0.36
$\sigma(x_t)$	1.09	1.21	1.29	1.39
$\sigma(i_t^{ann})$	0.96	0.76	0.63	0.47
$\sigma(\tau_t)$	0.00	44.06	73.02	108.77
$E(\pi_t\tau_t)$	0.00	0.13	0.18	0.20
$\mathcal{L}^* \times 10^4$	1.32	0.91	0.70	0.50

Notes: Productivity shocks only. The setting is the same as in Figure C1.

Table C2: Evaluation of policy rules under productivity shocks

	No TIP	Moderate TIP	Strong TIP	Extreme TIP
φ_π	0	150	300	600
ϕ_π	1.5	1.5	1.5	1.5
ϕ_y	0.125	0.125	0.125	0.125
$\sigma(\pi_t^{ann})$	0.20	0.16	0.13	0.10
$\sigma(x_t)$	0.30	0.33	0.35	0.38
$\sigma(i_t^{ann})$	0.20	0.25	0.29	0.33
$\sigma(\tau_t)$	0.00	12.10	20.06	29.88
$E(\pi_t\tau_t)$	0.00	0.01	0.01	0.01
$\mathcal{L}^* \times 10^4$	0.08	0.08	0.09	0.10

Notes: Monetary policy shocks only. The setting is the same as in Figure C2.

Table C3: Evaluation of policy rules under monetary policy shocks

D Distortion of Relative Prices

D.1 Model

Setting. We extend the model to include a continuum of sectors indexed by $s \in [0, 1]$. There is also a continuum of firms within each sector which are in monopolistic competition. These firms produce differentiated goods which are used in the production of a new type of goods: the sector goods, which we denote C_s. Sector goods are used for the production of final goods. They are also used by intermediate firms to pay for price increases in their relevant sector. The final good firms have the following technology:

$$\ln Y_t = \int \gamma_s \ln C_{ts} ds$$

where $\int \gamma_s = 1$. There are also aggregator firms in each sector with the following technology:

$$Y_{ts} = \left(\int y_{ts}^{1-1/\epsilon} ds \right)^{\frac{\epsilon}{\epsilon - 1}}$$

Denoting $\mathcal{C}(P_{tis}, P_{t-1is})$ the nominal adjustment costs of firm i in sector s at time t, the market clearing condition in each sector is given by:

$$Y_{ts} = C_{ts}$$

The adjustment cost needs to be paid in terms of final goods.

$$Y_t = C_t + \int_s \frac{\theta_s}{2} \pi_s^2 Y_{ts}$$

Firms' First-Order Condition Consider firms in sector s. The first-order conditions associated with these firm's problem are:

$$(1-\epsilon_{st})\left(\frac{P_{ti}}{P_{st}}\right)^{-\epsilon_{st}} Y_{st} - (1-\epsilon_{st})\left(\frac{P_{ti}}{P_{st}}\right)^{-\epsilon_{st}} Y_{st}\tau_t - \tau_t\epsilon_{st}\frac{P_{t-1i}}{P_{ti}}Y_{st}\left(\frac{P_{ti}}{P_{st}}\right)^{-\epsilon_{st}}$$
$$+\frac{\epsilon_{st}}{1-\alpha}\frac{W_t}{P_{ti}}\left[\left(\frac{P_{ti}}{P_{st}}\right)^{-\epsilon_{st}}\frac{Y_{st}}{A_{st}}\right]^{\frac{1}{1-\alpha}} - \frac{\theta_s}{P_{t-1i}}\left(\frac{P_{ti}}{P_{t-1i}}-1\right)P_tY_{st} + EQ_tV'(P_{ti}) = 0$$

and

$$V'(P_{t-1i}) = \tau_t Y_{st}\left(\frac{P_{ti}}{P_{st}}\right)^{-\epsilon_{st}} + \frac{P_{ti}}{P_{t-1i}^2}\theta_s\left(\frac{P_{ti}}{P_{t-1i}}-1\right)P_tY_{st}.$$

Assuming symmetry gives

$$\left((1-\epsilon_{st})Y_{st} - (1-\epsilon_{st})Y_{st}\tau_t - \tau_t\epsilon_{st}\frac{P_{t-1}}{P_{st}}Y_{st} + \frac{\epsilon_{st}}{1-\alpha}\frac{W_t}{P_{st}}\left[\frac{Y_{st}}{A_{st}}\right]^{\frac{1}{1-\alpha}} - \frac{\theta_s}{P_{t-1,s}}\pi_{st}P_tY_{st}\right)$$
$$+E_tQ_t\left[\tau_{t+1}Y_{t+1s} + (\pi_{t+1s}+1)\frac{P_{t+1}}{P_{ts}}\theta_s\pi_{t+1s}Y_{t+1s}\right] = 0$$
$$\iff \left(1-\epsilon_{st}-\tau_t\left(1-\epsilon_{st}\frac{\pi_{st}}{1+\pi_{st}}\right)+\epsilon_{st}MC_{st}-\theta_s\pi_{st}\frac{(\pi_{st}+1)}{\tilde{P}_{st}}\right)$$
$$+E_tQ_t\left[\tau_{t+1}\frac{Y_{t+1s}}{Y_{st}} + \frac{(\pi_{t+1s}+1)^2}{\tilde{P}_{st+1}}\theta_s\pi_{t+1s}\frac{Y_{t+1s}}{Y_{st}}\right] = 0$$
$$\iff \left((1-\epsilon_{st})\left(1-\frac{\epsilon_{st}}{\epsilon_{st}-1}MC_{st}\right)-\tau_t\left(1-\epsilon_{st}\frac{\pi_{st}}{1+\pi_{st}}\right)-\theta_s\pi_{st}\frac{(\pi_{st}+1)}{\tilde{P}_{st}}\right)$$
$$+E_tQ_t\left[\tau_{t+1}\frac{Y_{t+1s}}{Y_{st}} + \frac{(\pi_{t+1s}+1)^2}{\tilde{P}_{st+1}}\theta_s\pi_{t+1s}\frac{Y_{t+1s}}{Y_{st}}\right] = 0$$
$$\iff \frac{1}{\theta_s}\left((1-\epsilon_{st})(1-\mathcal{M}_{st}MC_{st})-\tau_t\left(1-\epsilon_{st}\frac{\pi_{st}}{1+\pi_{st}}\right)\right)-\pi_{st}\frac{(\pi_{st}+1)}{\tilde{P}_{st}}$$
$$+E_tQ_t\left[\frac{\tau_{t+1}}{\theta_s}\frac{Y_{t+1s}}{Y_{st}} + \frac{(\pi_{t+1s}+1)^2}{\tilde{P}_{st+1}}\pi_{t+1s}\frac{Y_{t+1s}}{Y_{st}}\right] = 0$$

with $\quad MC_{st} = \frac{W_t}{P_{st}(1-\alpha)} \frac{Y_{st}^{\frac{\alpha}{1-\alpha}}}{A_{st}^{\frac{1}{1-\alpha}}} \quad$ and $\quad \mathcal{M}_{st} = \frac{\epsilon_{st}}{\epsilon_{st}-1} \quad$ and $\quad \tilde{P}_{ts} = \frac{P_{st}}{P_t}$.

We denote the steady-state markup $\bar{\mathcal{M}}$. The next step is to linearize this optimality condition around a steady-state with no inflation, constant output, a *non-zero tax* on price changes, a price markup, $\pi = 0, \tau = \tau^{ss}, Y_s = Y_s', MC_{st}\mathcal{M} = 1$. Denoting *mc* the log of the real marginal cost *MC* and μ the log of $\mathcal{M}$, we obtain:

$$
\begin{aligned}
&\frac{1}{\theta_s}\left[(1-\epsilon)\left(1 - *(1 + mc_{st} - mc + \mu_t - \mu)\right) - \hat{\tau}_t - \tau^{ss} + \tau^{ss}\epsilon\pi_{st}\right] \\
&+ \beta\frac{\tau^{ss}}{\theta_s}\left[1 + \hat{q}_t + \frac{\hat{\tau}_{t+1}}{\tau^{ss}} + \hat{y}_{t+1} - \hat{y}_t\right] + \beta\pi_{st+1} = \pi_{st} \\
&\frac{1}{\theta_s}\left[(1-\epsilon)\left(1 - 1*(1 + mc_{st} - mc + \mu_t - \mu)\right) - \hat{\tau}_t - \tau^{ss} + \tau^{ss}\epsilon\pi_{st}\right] \\
&+ \beta\frac{\tau^{ss}}{\theta_s}\left[1 - \pi_{st+1} + \frac{\hat{\tau}_{t+1}}{\tau^{ss}} + (1-\sigma)(\hat{c}_{t+1} - \hat{c}_t)\right] + \beta\pi_{st+1} = \pi_{st} \\
&\frac{1}{\theta_s}\left[(1-\epsilon)\left(-1*(\hat{mc}_{st} + \hat{\mu}_t)\right) - \hat{\tau}_t + \tau^{ss}\epsilon\pi_{st}\right] \\
&+ \beta\frac{\tau^{ss}}{\theta_s}\left[-\pi_{st+1} + \frac{\hat{\tau}_{t+1}}{\tau^{ss}} + (1-\sigma)(\hat{c}_{t+1} - \hat{c}_t)\right] + \beta\pi_{st+1} = \pi_{st}
\end{aligned}
$$

$$
\begin{aligned}
\pi_{st}\left(1 - \epsilon\frac{\tau^{ss}}{\theta_s}\right) = \beta E_t\pi_{st+1}\left(1 - \frac{\tau^{ss}}{\theta_s}\right) + \frac{(\epsilon-1)}{\theta_s}(\hat{mc}_{st} + \hat{\mu}_t) + \frac{1}{\theta_s}\left[\beta E_t\hat{\tau}_{t+1} - \hat{\tau}_t\right] \\
+ \beta\frac{\tau^{ss}}{\theta_s}(1-\sigma)(\hat{c}_{t+1} - \hat{c}_t)
\end{aligned}
$$

Final goods optimal demand for sector goods From the FOC of final goods firms,

$$Y_{ts} = \gamma_s\frac{P_tY_t}{P_{ts}}.$$

we take the logs and obtain

$$
\begin{aligned}
y_{ts} &= \log \gamma_s + p_t + y_t - p_{ts} \\
\Rightarrow y_{ts} - y_t &= \log \gamma_s - (p_{ts} - p_t) \\
\Rightarrow \tilde{y}_{ts} &= \log \gamma_s - \tilde{p}_{ts}
\end{aligned}
$$

where $\tilde{x}_s = x_s - x$ and

$$
\begin{aligned}
y_t &= \int \gamma_s y_{ts} ds \\
p_t &= \int \gamma_s \left(p_{ts} - \log \gamma_s\right) ds
\end{aligned}
$$

We can obtain the log change in the aggregate consumer price index by taking the time difference of the last equation

$$
\pi_t = p_t - p_{t-1} = \int \gamma_s \pi_{ts} ds
$$

Market clearing Since all firms are identical within each sector in equilibrium, the market clearing condition for the sectorial goods market is given by

$$
Y_{st} = C_{st}.
$$

where C_{st} denotes the consumption of sectorial good s by final goods firms. Taking logs

$$
c_{st} = y_{st}
$$

The market clearing for the final goods market is simply given by

$$
Y_t = C_t + \int_s \frac{\theta_s}{2} \pi_{st}^2 Y_s
$$

At first order the approximation is given by

$$c_t = y_t.$$

and at second order the approximation is given by

$$c_t = y_t - \frac{1}{2}E[\theta_s \pi_{st}^2].$$

From the labor market clearing condition $N_t = \int_s \int_{i \in s} N_{ti} dids$, we get

$$\begin{aligned}
N_t &= \int_s \int_{i \in s} \left(\frac{Y_{ti}}{A_{st}} \right)^{\frac{1}{1-\alpha}} dids \\
&= \int_s \left(\frac{Y_{ts}}{A_{st}} \right)^{\frac{1}{1-\alpha}} ds \\
&= \int_s \left(\frac{\gamma_s P_t Y_t}{P_{st} A_{st}} \right)^{\frac{1}{1-\alpha}} ds \\
&= \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_s \left(\frac{\gamma_s P_t A_t}{P_{st} A_{st}} \right)^{\frac{1}{1-\alpha}} ds \\
&= \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \Delta_t
\end{aligned}$$

where $\Delta = \int_s \left(\frac{\gamma_s P_t A_t}{P_{st} A_{st}} \right)^{\frac{1}{1-\alpha}} ds$ captures the costs entailed by price distortions and misallocation of sector goods, and where we used the fact that all firms are ex post identical $Y_{ti} = Y_{st}$ in each sector and the FOC of the final goods firms $Y_{st} = \gamma_s \frac{P_t Y_t}{P_{ts}}$, and where we define A_t the geometric mean of sectorial TFPs, A_{st}: $a_t = \int_s \gamma_s a_{st} ds$.

We then approximate Δ_t. We need to first obtain an expression of relative prices under flexible prices. We start from the optimal markup of a firm in sector s given by

$$\mathcal{M}_{st}^{-1} = \frac{W_t^f}{P_{st}^f} \frac{(Y_{st}^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_{st}^{\frac{1}{1-\alpha}}} = \frac{W_t^f}{P_t^f} \frac{P_t^f}{P_{st}^f} \frac{\left(\gamma_s P_t^f Y_t^f / P_{st}^f\right)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_{st}^{\frac{1}{1-\alpha}}}$$

which implies

$$\frac{P_t^f A_t}{P_{st}^f A_{st}} = \left[\left(\frac{\mathcal{M}_{st} W_t^f}{P_t^f}\right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_t^{\frac{1}{1-\alpha}}}\right]^{\alpha-1} \gamma_s^{-\alpha}$$

We next take this ratio at the power γ_s, take the product over all sectors:

$$\Pi_s^S \left(\frac{P_t^f A_t}{P_{st}^f A_{st}}\right)^{\gamma_s} = \Pi_s^S \left(\left[\left(\frac{\mathcal{M}_{st} W_t^f}{P_t^f}\right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_t^{\frac{1}{1-\alpha}}}\right]^{\alpha-1} \gamma_s^{-\alpha}\right)^{\gamma_s}$$

$$\frac{P_t^f A_t}{\Pi_s^S \gamma_s^{\gamma_s} \Pi_s^S \left(P_{st}^f / \gamma_s\right)^{\gamma_s} \Pi_s^S A_{st}^{\gamma_s}} = \left[\left(\frac{W_t^f}{P_t^f}\right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_t^{\frac{1}{1-\alpha}}}\right]^{\alpha-1} \Pi_s^S \mathcal{M}_{st}^{\gamma_s(\alpha-1)} \gamma_s^{-\alpha\gamma_s}$$

$$\frac{P_t^f A_t}{\Pi_s^S \gamma_s^{\gamma_s} P_t^f A_t} = \left[\left(\frac{W_t^f}{P_t^f}\right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_t^{\frac{1}{1-\alpha}}}\right]^{\alpha-1} \Pi_s^S \mathcal{M}_{st}^{\gamma_s(\alpha-1)} \gamma_s^{-\alpha\gamma_s}$$

$$1 = \left[\left(\frac{\mathcal{M}_t W_t^f}{P_t^f}\right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)A_t^{\frac{1}{1-\alpha}}}\right]^{\alpha-1}$$

with $\mathcal{M}_t = \Pi_s^S \left(\frac{\mathcal{M}_{st}}{\gamma_s}\right)^{\gamma_s}$. Hence

$$\frac{P_t^f A_t}{P_{st}^f A_{st}} = \left[\left(\frac{\mathcal{M}_{st}\mathcal{M}_t W_t^f}{\mathcal{M}_t P_t^f} \right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha) A_t^{\frac{1}{1-\alpha}}} \right]^{\alpha-1} \gamma_s^{-\alpha} = \left[\frac{\mathcal{M}_{st}}{\mathcal{M}_t} \right]^{\alpha-1} \gamma_s^{-\alpha}$$
$$\Rightarrow \tilde{p}_{st}^f = -\tilde{a}_{st} + (1-\alpha)\tilde{\mu}_{st} + \alpha \log \gamma_s$$

We can now go back to Δ and linearize it around the steady-state with $\frac{PA}{P_s A_s} = \left[\frac{\mathcal{M}_s}{\mathcal{M}}\right]^{\alpha-1} \gamma_s^{-\alpha} = \left[\Pi_s^S \gamma_s^{\gamma_s}\right]^{\alpha-1} \gamma_s^{-\alpha}$ where $\mathcal{M}_s$ are equal for all s.

$$\Delta_t = \int_s \left(\frac{\gamma_s P_t A_t}{P_{st} A_{st}} \right)^{\frac{1}{1-\alpha}} ds$$
$$\Delta \exp(\hat{\delta}_t) = \int_s \left(\frac{\gamma_s PA}{P_s A_s} \right)^{\frac{1}{1-\alpha}} \exp\left(-\frac{1}{1-\alpha} \left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st} \right) \right) ds$$
$$= \left[\Pi_s^S \gamma_s^{-\gamma_s} \right] \int_s \gamma_s \exp\left(-\frac{1}{1-\alpha} \left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st} \right) \right) ds$$
$$\hat{\delta}_t = -\frac{1}{1-\alpha} \int_s \gamma_s \left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st} \right) ds$$

where $\hat{\tilde{x}}_{ts}$ denotes the log deviation of "relative x" from its steady state. Importantly, $\hat{\delta}_t$ is zero at first order

$$\hat{\delta}_t = -\frac{1}{1-\alpha} \int_s \gamma_s \left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st} \right) ds = -\frac{1}{1-\alpha} \int_s \gamma_s \left(\tilde{p}_{st} - \tilde{p}_s + \tilde{a}_{st} - \tilde{a}_s \right) ds$$
$$= -\frac{1}{1-\alpha} \int_s \gamma_s \left(\tilde{p}_{st} - \tilde{p}_s - \tilde{p}_{st}^e + \tilde{p}_s^e \right) ds = 0$$

where the last line uses the definition of the price index $p = \int \gamma_s (p_s - \log \gamma_s) ds$ which implies $\int \gamma_s (\tilde{p} - p) = \int \gamma_s \log \gamma_s ds$ which is the same for the four components, hence they cancel out. For future reference we define the log deviation of relative prices from its efficient level

$$\hat{\tilde{p}}_{st} = \tilde{p}_{st} - \tilde{p}_{st}^e.$$

Importantly, relative prices in the steady-state are the same as in the efficient steady-state. Hence we have

$$\hat{\tilde{p}}_{st} = \tilde{p}_{st} - \tilde{p}_s - \tilde{p}^e_{st} + \tilde{p}^e_s \quad \text{and} \quad \int \gamma_s \hat{\tilde{p}}_{st} = 0$$

which will be useful later.

Going back to $\hat{\delta}$, a second order approximation around the steady state gives

$$\begin{aligned}
\hat{\delta}_t &= \int_s \gamma_s \left(-\frac{1}{1-\alpha} \left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st} \right) + \frac{1}{2(1-\alpha)^2} \left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st} \right)^2 \right) ds \\
&= \frac{1}{(1-\alpha)} \left[-E_\gamma \left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st} \right) + \frac{E\left((\tilde{p}_{st} - \tilde{p}^e_{st})^2 \right)}{2(1-\alpha)} \right] \\
&= \frac{E\left((\tilde{p}_{st} - \tilde{p}^e_{st})^2 \right)}{2(1-\alpha)^2}
\end{aligned}$$

where we used $\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st} = \hat{\tilde{p}}_{st}$, the fact that $\hat{\delta}$ is zero at first order and the expectation is taken across sectors under the weights γ_s.

Phillips Curve and Euler Equation We now turn to the first order conditions of the households given by

$$\begin{aligned}
w_t - p_t &= \sigma c_t + \psi n_t \\
c_t &= E_t c_{t+1} - \frac{1}{\sigma} \left(i_t - E_t \pi_{t+1} - \rho \right)
\end{aligned}$$

with $i_t = -\log Q_t$. Combining them with the market clearing condition for the final goods gives

$$\begin{aligned}
w_t - p_t &= \left(\sigma + \frac{\psi}{1-\alpha} \right) y_t - \frac{\psi}{1-\alpha} \log a_t + \psi \delta_t \\
y_t &= E_t y_{t+1} - \frac{1}{\sigma} \left(i_t - E_t \pi_{t+1} - \rho \right)
\end{aligned}$$

with $\delta_t = \log \Delta_t$. From the definition of markup, we obtain

$$
\begin{aligned}
mc_{st} &= w_t - p_t + p_t - p_{st} + \frac{\alpha}{1-\alpha} y_{st} - \frac{1}{1-\alpha} a_{st} - \log(1-\alpha) \\
&= \left(\sigma + \frac{\psi}{1-\alpha}\right) y_t + \psi\delta_t + p_t - p_{st} - \frac{\psi}{1-\alpha} a_t + \frac{\alpha}{1-\alpha} y_{st} - \frac{1}{1-\alpha} a_{st} - \log(1-\alpha) \\
&= \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) y_t + \psi\delta_t - (p_{st} - p_t) - \frac{1+\psi}{1-\alpha} a_t - \frac{1}{1-\alpha}(a_{st} - a_t) \\
&\qquad + \frac{\alpha}{1-\alpha}(y_{st} - y_t) - \log(1-\alpha) \\
&= \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) y_t + \psi\delta_t - \tilde{p}_{st} - \frac{1+\psi}{1-\alpha} a_t - \frac{1}{1-\alpha}\tilde{a}_{st} + \frac{\alpha}{1-\alpha}\tilde{y}_{st} - \log(1-\alpha) \\
&= mc_t + \psi\delta_t - \tilde{p}_{st} - \frac{1}{1-\alpha}\tilde{a}_{st} + \frac{\alpha}{1-\alpha}\tilde{y}_{st}
\end{aligned}
$$

where the second line uses the first order condition of households and the market clearing condition for labor, where the third line uses $\tilde{x}_s = x_s - x$ and the last line uses $mc_t = \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) y_t - \frac{1+\psi}{1-\alpha} a_t - \log(1-\alpha)$. Finally we can use the first order condition of the final goods firms to substitute for $\tilde{y}_{st}$:

$$
mc_{st} = mc_t + \psi\delta_t - \frac{1}{1-\alpha}\tilde{a}_{st} - \left(\frac{1}{1-\alpha}\right)\tilde{p}_{st}
$$

We next take the difference with the steady-state:

$$
\begin{aligned}
\hat{mc}_{st} &= \hat{mc}_t + \psi\hat{\delta}_t - \left(\frac{1}{1-\alpha}\right)(\tilde{p}_{st} - \tilde{p}_s + \tilde{a}_{st} - \tilde{a}_s) \\
\hat{mc}_{st} &= \hat{mc}_t + \psi\hat{\delta}_t - \left(\frac{1}{1-\alpha}\right)\hat{\tilde{p}}^e_{st}.
\end{aligned}
$$

Using this to substitute for the marginal cost in the sector-level Phillips curve:

$$
\begin{aligned}
\pi_{st}\left(1 - \epsilon\frac{\tau^{ss}}{\theta_s}\right) &= \beta E_t \pi_{t+1s}\left(1 - \frac{\tau^{ss}}{\theta_s}\right) + \frac{\epsilon - 1}{\theta_s}\left[\hat{mc}_t - \left(\frac{1}{1-\alpha}\right)\hat{\tilde{p}}^e_{st}\right] \\
&\quad + \frac{1}{\theta_s}\left[\beta E_t \hat{\tau}_{t+1} - \hat{\tau}_t + u_t\right] + \beta\frac{\tau^{ss}}{\theta_s}(1-\sigma)(\hat{c}_{t+1} - \hat{c}_t) \qquad (41)
\end{aligned}
$$

where we used the fact that at first order $\hat{\delta}_t = 0$. Taking the (γ_s-weighted) integral of the Phillips curves over all sectors gives

$$\pi_t \left(1 - \epsilon \frac{\tau^{ss}}{\theta_s}\right) = \beta E_t \pi_{t+1} \left(1 - \frac{\tau^{ss}}{\theta_s}\right) + \hat{mc}_t \int_s \frac{\epsilon - 1}{\theta_s} \gamma_s - \frac{1}{1-\alpha} \int_s \frac{(\epsilon - 1)\gamma_s}{\theta_s} \hat{\tilde{p}}^e_{st}$$
$$+ [u_t + \beta E_t \hat{\tau}_{t+1} - \hat{\tau}_t] \int_s \frac{\gamma_s}{\theta_s} ds + \beta \frac{\tau^{ss}}{\theta_s} (1 - \sigma)(\hat{c}_{t+1} - \hat{c}_t) \quad (42)$$

D.2 Second-order approximation of welfare

We next show that the welfare loss function of the household depends on the dispersion of relative prices around their efficient levels and on the average inflation rates across sectors.

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\sigma + \frac{\alpha + \psi}{1-\alpha} \right) (\hat{x}_{st})^2 + \frac{1}{2} E[\theta_s \pi_{st}^2] + \frac{1}{2} \frac{E\left((\tilde{p}_{st} - \tilde{p}^e_{st})^2 \right)}{1-\alpha} - \Phi \hat{x}_t \right]$$

The log deviation of the representative household is given by

$$\frac{U_t - U}{U_c C} = \left(\hat{c}_t + \frac{1-\sigma}{2} \hat{c}_t^2 \right) + \frac{U_N N}{U_c C} \left(\hat{n}_t + \frac{1+\psi}{2} \hat{n}_t^2 \right)$$

.

We next use a second order approximation of the goods market clearing condition to get an approximation for the first bracket

$$\hat{c}_t = \hat{y}_t - \frac{1}{2} E[\theta_s \pi_{st}^2].$$

We next use the labor market clearing condition to get an approximation for the second bracket:

$$\hat{n}_t = \frac{1}{1-\alpha} (\hat{y}_t - \hat{a}_t) + \hat{\delta}_t$$

Combining both, and keeping only the terms of first and second order, we obtain

$$\frac{U_t - U}{U_c C} = \left(\hat{y}_t - \frac{1}{2} E[\theta_s \pi_{st}^2] + \frac{1-\sigma}{2} (\hat{y}_t)^2 \right) + \frac{U_N N}{U_c C} \left(\frac{1}{1-\alpha} (\hat{y}_t - \hat{a}_t) + \hat{\delta}_t + \frac{1+\psi}{2} \left(\frac{1}{1-\alpha} \right)^2 (\hat{y}_t - \hat{a}_t)^2 \right)$$

Denote Φ the steady-state distortion implicitly defined by $-\frac{U_N N}{U_C C(1-\alpha)} = 1 - \Phi$. We obtain:

$$\frac{U_t - U}{U_c C} = \left(\hat{y}_t - \frac{1}{2} E[\theta_s \pi_{st}^2] + \frac{1-\sigma}{2} (\hat{y}_t)^2 \right) - (1-\Phi) \left((\hat{y}_t - \hat{a}_t) + (1-\alpha)\hat{\delta}_t + \frac{1+\psi}{2} \left(\frac{1}{1-\alpha} \right) (\hat{y}_t - \hat{a}_t)^2 \right)$$

Under the "small distortion" assumption (so that the product of Φ with a second-order term can be ignored as negligible)

$$\begin{aligned}
\frac{U_t - U}{U_c C} &= \Phi \hat{y}_t - \frac{1}{2} E[\theta_s \pi_{st}^2] + \left(\frac{1-\sigma}{2} (\hat{y}_t)^2 \right) - \left(-\hat{a}_t + \frac{1+\psi}{2(1-\alpha)} (\hat{y}_t - \hat{a}_t)^2 \right) - (1-\alpha)\hat{\delta} + t.i.p \\
&= \Phi \hat{y}_t - \frac{1}{2} E[\theta_s \pi_{st}^2] - \frac{1}{2} \left(\sigma + \frac{\alpha+\psi}{1-\alpha} \right) (\hat{y}_t)^2 + \frac{1+\psi}{(1-\alpha)} \hat{y}_t \hat{a}_t - (1-\alpha)\hat{\delta} + t.i.p \\
&= \Phi \hat{x}_t + \Phi \hat{y}_t^e - \frac{1}{2} E[\theta_s \pi_{st}^2] - \frac{1}{2} \left(\sigma + \frac{\alpha+\psi}{1-\alpha} \right) \left[(\hat{y}_t)^2 - 2\hat{y}_t \hat{y}_t^e \right] - (1-\alpha)\hat{\delta} + t.i.p \\
&= \Phi \hat{x}_t - \frac{1}{2} E[\theta_s \pi_{st}^2] - \frac{1}{2} \left(\sigma + \frac{\alpha+\psi}{1-\alpha} \right) (\hat{x}_t)^2 - (1-\alpha)\hat{\delta} + + t.i.p
\end{aligned}$$

note that $\hat{a}_t$, $\hat{a}_t^2$ and $\hat{y}_t^e$ are in t.i.p. because they depend only on exogenous TFP shocks. We used the definition of y_t^e, x_t, the fact that $\hat{y}_t^e = y_t^e - y^e, x_t = y_t - y^e, \hat{x}_t = x_t - x = \hat{y}_t - \hat{y}_t^e$ where x is the steady-state welfare-relevant output gap.

Substituting the second order approximation of $\hat{\delta}_t$

$$\hat{\delta}_t = \frac{E\left((\tilde{p}_{st} - \tilde{p}^e_{st})^2\right)}{2(1-\alpha)^2}$$

into the previous expression gives the result

$$\frac{U_t - U}{U_c C} = \Phi \hat{x}_t - \frac{1}{2}E[\theta_s \pi^2_{st}] - \frac{1}{2}\left(\sigma + \frac{\alpha + \psi}{1-\alpha}\right)(\hat{x}_t)^2 - \frac{E\left((\tilde{p}_{st} - \tilde{p}^e_{st})^2\right)}{2(1-\alpha)} + t.i.p.$$

Finally the term $E[\theta_s \pi^2_{st}]$ can be decomposed into an aggregate and sector-specific components:

$$\begin{aligned}
E[\theta_s \pi^2_{st}] &= E[\theta_s(\pi_t + (\pi_{st} - \pi_t))^2] \\
&= E\left[\theta_s \pi^2_t + \theta_s(\pi_{st} - \pi_t)^2 + 2\theta_s \pi_t(\pi_{st} - \pi_t)\right] \\
&= \left(E\left[\theta_s \pi^2_t\right] + E\left[\theta_s \tilde{\pi}^2_{st}\right] + 2cov\left[\pi_t, \theta_s \tilde{\pi}_{st}\right]\right) \\
&= \pi^2_t E\left[\theta_s\right] + E\left[\theta_s \tilde{\pi}^2_{st}\right]
\end{aligned}$$

where the last line uses the fact that in the cross-section π_t is constant.

D.3 Proof of Proposition (5).

Substituting $\theta_s = \theta$ in the sector-s Phillips curve just derived we obtain

$$\begin{aligned}
\pi_{st}\left(1 - \epsilon\frac{\tau^{ss}}{\theta}\right) &= \beta E_t \pi_{t+1s}\left(1 - \frac{\tau^{ss}}{\theta}\right) + \frac{\epsilon - 1}{\theta}\left[\hat{mc}_t - \left(\frac{1}{1-\alpha}\right)\hat{\tilde{p}}^e_{st}\right] \\
&+ \frac{1}{\theta}\left[\beta E_t \tau_{t+1} - \tau_t + u_t\right] + \beta\frac{\tau^{ss}}{\theta}(1-\sigma)(\hat{c}_{t+1} - \hat{c}_t) \qquad (43)
\end{aligned}$$

Taking the difference between equations (43) and (42) to get an expression for relative price inflation $\tilde{\pi}_{st} = \pi_{st} - \pi_t$ we obtain:

$$\tilde{\pi}_{st}\left(1 - \epsilon\frac{\tau^{ss}}{\theta}\right) = \beta E_t \tilde{\pi}_{t+1s}\left(1 - \frac{\tau^{ss}}{\theta}\right) - \frac{\epsilon - 1}{\theta}\frac{1}{1-\alpha}(\tilde{p}_{st} + \tilde{a}_{st})$$

which gives

$$\left(1 - \epsilon\frac{\tau^{ss}}{\theta}\right)(\tilde{p}_{st} - \tilde{p}_{st-1}) = \left(1 - \frac{\tau^{ss}}{\theta}\right)\beta E_t(\tilde{p}_{st+1} - \tilde{p}_{st}) - \frac{\epsilon - 1}{\theta}\frac{1}{1-\alpha}(\tilde{p}_{st} + \tilde{a}_{st}).$$

This is a linear difference equation in $\tilde{p}_{st}$ which depends only on θ, $\tau^{ss}, \epsilon, \beta, \alpha$ and the stochastic process for $\tilde{a}_{st}$. Importantly, it is independent of the deviation of TIP from the steady-state $\hat{\tau}_t$. We see that the condition $\theta_s = \theta$ is important to get this result, otherwise we wouldn't be able to subtract the aggregate and sector-specific Phillips curve.

If in addition, $\tau^{ss} = 0$ then the difference equation—hence relative prices—is completely independent of TIP:

$$(\tilde{p}_{st} - \tilde{p}_{st-1}) = \beta E_t(\tilde{p}_{st+1} - \tilde{p}_{st}) - \frac{\epsilon - 1}{\theta}\frac{1}{1-\alpha}(\tilde{p}_{st} + \tilde{a}_{st})$$

Looking now at average (squarred) price changes $E[\theta_s \pi_{st}^2] = \pi_t^2 E[\theta_s] + E\left[\theta_s \tilde{\pi}_{st}^2\right]$, under the same conditions, the second term $E\left[\theta_s \tilde{\pi}_{st}^2\right]$ is independent of aggregates and of TIP. Therefore $E[\theta_s \pi_{st}^2]$ varies with TIP only through its effect on aggregate inflation π_t^2.

Note that if we had initially allowed for heterogeneous elasticities of substitution, ϵ, they would have had to be the same across sectors as well for the proof to go through.

D.4 Stationary distribution of price distortions in the linearized model

Using the following difference equation

$$\left(1 - \epsilon\frac{\tau^{ss}}{\theta}\right)(\tilde{p}_{st} - \tilde{p}_{st-1}) = \left(1 - \frac{\tau^{ss}}{\theta}\right)\beta E_t(\tilde{p}_{st+1} - \tilde{p}_{st}) - \frac{\epsilon - 1}{\theta}\frac{1}{1-\alpha}(\tilde{p}_{st} + \tilde{a}_{st})$$

we can compute the stationary distribution of relative prices $\tilde{p}_{st}$ and distortions, $\tilde{p}_{st} + \tilde{a}_{st}$ for a stochastic process for $\tilde{a}_{st}$. We assume it is drawn from a normal distribution with mean 0 and standard deviation $\sigma_a = 1$ and i.i.d. over time and over sectors:

$$\tilde{a}_{st} \sim i.i.d.\mathcal{N}(0,1)$$

To solve the previous difference equation, we guess the following AR(1) process for relative prices:

$$\tilde{p}_{st} = \chi_p \tilde{a}_{st-1} + \chi_a \tilde{a}_{st}$$

Replacing in the previous equation, we get:

$$\left(1 - \epsilon\frac{\tau^{ss}}{\theta} + (1-\chi_p)\beta\left(1 - \frac{\tau^{ss}}{\theta}\right) + \frac{\epsilon - 1}{\theta}\frac{1}{1-\alpha}\right)\tilde{p}_{st} = \left(1 - \epsilon\frac{\tau^{ss}}{\theta}\right)\tilde{p}_{st-1} - \frac{\epsilon-1}{\theta}\frac{1}{1-\alpha}\tilde{a}_{st}$$

We then solve for χ_p and χ_a using:

$$\chi_p = \frac{\left(1 - \epsilon\frac{\tau^{ss}}{\theta}\right)}{\left(1 - \epsilon\frac{\tau^{ss}}{\theta} + (1-\chi_p)\beta\left(1 - \frac{\tau^{ss}}{\theta}\right) + \frac{\epsilon-1}{\theta}\frac{1}{1-\alpha}\right)}$$
$$\chi_a = -\chi_p \frac{\frac{\epsilon-1}{\theta}\frac{1}{1-\alpha}}{\left(1 - \epsilon\frac{\tau^{ss}}{\theta}\right)}$$

This gives

$$\chi_p = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$
$$A = -\beta\left(1 - \frac{\tau^{ss}}{\theta}\right)$$
$$B = \left(1 - \epsilon\frac{\tau^{ss}}{\theta} + \beta\left(1 - \frac{\tau^{ss}}{\theta}\right) + \frac{\epsilon-1}{\theta}\frac{1}{1-\alpha}\right)$$
$$C = -\left(1 - \epsilon\frac{\tau^{ss}}{\theta}\right)$$

Note that we keep only the positive solution to the quadratic equation because the other one gives a negative AR(1) coefficient, which isn't economically sensible.

With this we can solve for the stationary distribution of price distortions: $\tilde{a}_{st} + \tilde{p}_{st}$

$$\begin{aligned} \tilde{a}_{st} + \tilde{p}_{st} &= \chi_p(\tilde{a}_{st-1} + \tilde{p}_{st-1}) + (1+\chi_a)\tilde{a}_{st} - \chi_p\tilde{a}_{st} + \tilde{a}_{st-1} \\ \Rightarrow & E(\tilde{a}_{st} + \tilde{p}_{st}) = 0 \quad \text{and} \quad V(\tilde{a}_{st} + \tilde{p}_{st}) = \frac{(1+\chi_a)^2 + \chi_p^2}{1-\chi_p^2} \end{aligned}$$

where we used the i.i.d. assumption across sectors and time periods.

Using the same calibration described in Appendix B.3, we can compute the sum of squared price deviations $V(\tilde{a}_{st} + \tilde{p}_{st})$ for different values of τ^{ss} ranging from 0 to 100%. Results are reported below. We see that a positive steady-state TIP τ^{ss} has no meaningful quantitative effect on price distortions. If anything it slightly decreases price distortions by lowering the persistence of relative prices χ_p and by increasing the response of relative prices to sector-specific TFP shocks $|\chi_a|$. We see that this result remains true when decreasing β from .99 to .8 and when decreasing the adjustment cost parameter θ from 372 to 200.

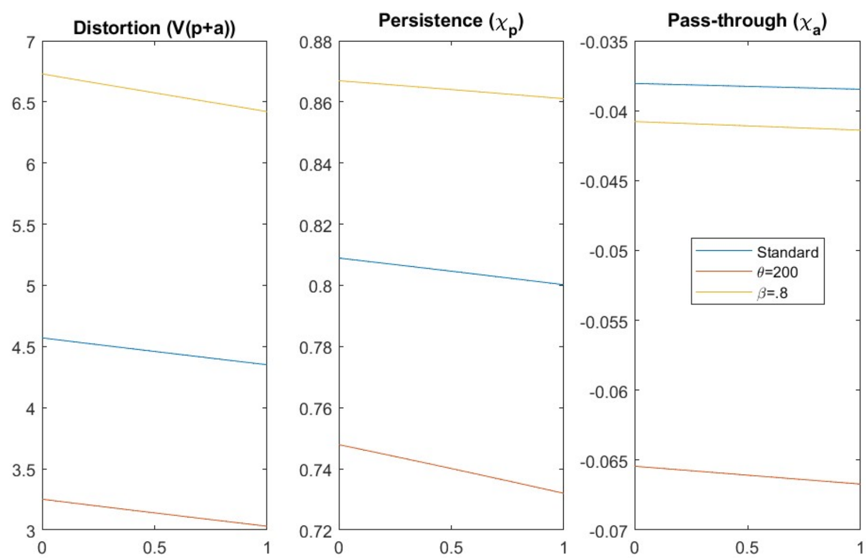


Figure D7: Steady-state distortions for different values of τ^{ss}

D.5 Calibration

Our data used to calibrate the model is a panel of Personal Consumption Expenditures (PCE) price indices for different types of product from the Bureau of Economic Analysis (NIPA Table 2.3.5). Following the BEA classification, we group products into 13 sectors, which are listed in the first column of Table D4. We focus on core inflation and exclude food and fuel. We also exclude services provided by non-profit institutions servicing households. We do so to be consistent with the model's assumptions that firms have monopolistic power to set prices and seek to maximize profits. Our sample covers the period 1960Q1 to 2019Q4. We exclude the COVID-19 years because the large drop and rebound of economic activity could bias our estimates.

For each sector, we compute the standard deviation and autocorrelation coefficients of their relative price changes. We construct relative price changes in the following way. Let's denote P_{st} the price index of sector s at the time t. We compute the log difference of the price index of each product

Sector	PCE Weight	$\hat{\sigma}_s^{PCE} \times 100$	$\hat{\rho}_s^{PCE}$	Simulated $\sigma_s^{PCE} \times 100$	Simulated ρ_s^{PCE}	θ_s	$\sigma_s \times 100$
Durable goods							
Motor vehicles and parts	5.2%	0.72	0.32	0.64	0.33	549.6	55.71
Furnishings and durable household equipment	3.2%	0.53	0.26	0.51	0.27	166.8	8.78
Recreational goods and vehicles	3.6%	0.48	0.23	0.47	0.25	122.9	6.40
Other durable goods	1.8%	0.72	0.33	0.64	0.33	549.7	55.69
Nondurable goods							
Clothing and footwear	4.4%	0.55	0.02	0.56	0.03	15.1	1.62
Other nondurable goods	9.1%	0.48	0.36	0.43	0.35	858.1	56.84
Services							
Housing and utilities	21.1%	0.37	0.33	0.40	0.34	675.9	21.23
Health care	17.7%	0.39	0.34	0.35	0.36	908.8	30.49
Transportation services	3.9%	0.58	0.12	0.57	0.13	35.2	2.96
Recreation services	4.4%	0.33	0.13	0.34	0.15	39.5	1.89
Food services and accommodations	7.4%	0.38	0.10	0.39	0.11	28.3	1.71
Financial services and insurance	8.6%	1.21	-0.05	1.25	-0.04	8.3	2.54
Other services	9.7%	0.42	0.14	0.42	0.15	42.5	2.48

Table D4: Empirical Targets, Model-implied Moments and Parameter Values

category, $\log(P_{st}/P_{st-1})$, and then take the difference with the overall price index: $\log(P_{st}/P_{st-1}) - \log(P_t/P_{t-1})$. Next we remove category-specific trends by applying an HP filter. We use the resulting panel of de-trended relative price log changes to compute both moments: the standard deviation σ_s^{PCE} and the autocorrelation coefficient $\hat{\rho}_s^{PCE}$.

Using a simulated method of moments, for each sector we look for the pair of parameters (σ_s, θ_s) that generates the standard deviation and autocorrelation coefficient $(\sigma_s^{PCE}, \rho_s^{PCE})$ that best match their empirical counterparts. Before doing so, we first need to set the persistence of productivity ρ. We then draw random samples of θ_s and σ_s. We simulate the stationary distribution of price changes and production in each sector. We compute the model-implied moments $(\sigma_s^{PCE}, \rho_s^{PCE})$. We then use this initial set of moments to update our guess for the parameters. We iterate until the simulated moments match the target. Once convergence is achieved for all sectors, we update our guess for the persistence parameter ρ until the weighted average of θ_s across all sectors is of the same order of magnitude as in the representative model in Section 3.3. In practice we obtain $\rho = .43$ and a weighted average θ_s of 440.

Table D4 displays the empirical targets, the model-implied moments and the resulting parameter values for each sector.

D.6 Numerical simulations of non-linear model

We now describe how we solve numerically for the steady state and the transitional dynamics. We start with describing the equilibrium conditions used in the algorithm.

Policy function of firms. In each sector s, we solve for the firms' pricing rule $\tilde{P}_{s,t}(A_{s,t}, \tilde{P}_{s,t-1})$ given the paths of real wages $\{RW^t\}$, aggregate outputs $\{Y^t\}$, aggregate inflation rates $\{\pi^t\}$ (normalize $P_{-1} = 1$ such that $P_t = \exp \sum_{h=1}^{t} \pi_h$), aggregate markup shocks $\{\mu^t\}$, and TIP rates $\{\tau^t\}$ using the following first order condition:

$$\left((1-\epsilon_s)\left(1-\mu_t \mathcal{M}_s MC_{s,t}\right) - \tau_t \left(1-\epsilon_s \frac{\pi_{s,t}}{1+\pi_{s,t}}\right)\right) - \theta_s \pi_{s,t} \frac{(\pi_{s,t}+1)}{\tilde{P}_{s,t}}$$
$$+E_t Q \left[\tau_{t+1} \frac{Y_{s,t+1}}{Y_{s,t}} + \theta_s \frac{(\pi_{s,t+1}+1)^2}{\tilde{P}_{s,t+1}} \pi_{s,t+1} \frac{Y_{s,t+1}}{Y_{s,t}} \right] = 0, \tag{44}$$

with $\tilde{P}_{s,t} = \frac{P_{s,t}}{P_t}$, $\pi_{s,t} = \log(\tilde{P}_{s,t}) - \log(\tilde{P}_{s,t-1}) + \pi_t$ and

$$MC_{s,t} = \frac{RW_t}{\tilde{P}_{s,t}(1-\alpha)} \frac{Y_{s,t}^{\frac{\alpha}{1-\alpha}}}{A_{s,t}^{\frac{1}{1-\alpha}}}, \quad Y_{s,t} = \frac{\gamma_s Y_t}{\tilde{P}_{s,t}}, \quad \mathcal{M}_s = \frac{\epsilon_s}{\epsilon_s - 1} \tag{45}$$

and subject to the following exogenous stochastic process for the sector-specific TFP, $A_{s,t}$,

$$\log(A_{s,t}) = (1-\rho)\log A_s + \rho \log(A_{s,t-1}) + \nu_{s,t}$$

where $v_{s,t}$ is drawn from a normal distribution with standard deviation σ_s and where A_s is chosen such that in the stationary equilibrium each sector's average relative prices is equal to one.

Aggregate output and inflation. We next aggregate all sectors and define aggregate output. It is given by

$$Y_t = (A_t)^{(1+\psi)k}(1-\alpha)^{(1-\alpha)k}(\Gamma_t)^{-\sigma(1-\alpha)k}(\Delta_t)^{-\psi(1-\alpha)k}(\mathcal{M}_t)^{-(1-\alpha)k}, \quad (46)$$

with $k = \frac{1}{\psi+(1-\alpha)\sigma+\alpha}$ and

$$\Gamma_t = 1 - \int \gamma_s \frac{1}{\tilde{P}_s}\frac{\theta_s}{2}\pi_{s,t}^2 ds \quad (47)$$

$$\Delta_t = \int_s \left(\frac{\gamma_s A}{\tilde{P}_{s,t}A_{s,t}}\right)^{\frac{1}{1-\alpha}} ds \quad (48)$$

$$\mathcal{M}_t = \exp\left(\int \gamma_s \log\left(\mathcal{M}_{s,t}\right) ds\right) \quad (49)$$

$$\mathcal{M}_{s,t} = \frac{1}{MC_{s,t}} \quad (50)$$

$$A_t = \exp\left(\int_s \gamma_s \log(A_{s,t}) ds\right) \quad (51)$$

$$RW_t = \frac{W_t}{P_t} = (Y_t)^{\sigma+\frac{\psi}{1-\alpha}} A_t^{-\frac{\psi}{1-\alpha}} (\Delta_t)^{\psi}(\Gamma_t)^{\sigma} \quad (52)$$

The aggregate price level and inflation are given by

$$P_t = \exp p_t \quad (53)$$

$$p_t = \int \gamma_s \left(p_{s,t} - \log \gamma_s\right) ds \quad (54)$$

$$\pi_t = \int \gamma_s \pi_{s,t} ds \quad (55)$$

$$\tilde{p}_t = \int \gamma_s \tilde{p}_{s,t} ds \quad (56)$$

Aggregate demand, monetary policy and TIP. We close the model with the Euler Equation, the targeting rules for monetary policy and TIP:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - \xi_t\right) \tag{57}$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t \tag{58}$$

$$\tau_t = \varphi_\pi \pi_t \tag{59}$$

where ξ_t is a generic demand shock centered around ρ, and the Taylor rule targets deviations of inflation and output from the steady state, $\hat{y}_t$ and TIP targets inflation.

Steady state. In steady-state, we normalize the aggregate price level to 1. The algorithm is then as follows: (i) start from a guess for steady-state output Y, (ii) using equation (44) solve for the firms' pricing rule $\tilde{P}_s(A_s, \tilde{P}_{s,-1})$ given aggregate output and the price index; (iii) using equation (46) update the aggregate output Y'; (iv) iterate until output $Y = Y'$ converges.

Impulse responses. We next solve for the transition path of the economy following a persistent shock to markup demand or productivity, starting from and going back to the steady-state equilibrium. A transition is a set of vectors $x^t = (x_1, x_2 ... x_T)$ where T is the horizon of the transition and where $x \in \{Y, RW, \Gamma, \Delta, \mathcal{M}, i, P, \pi, \tilde{p}, \cdots\}$. The algorithm is as follows:

1. *Guess a sequence of inflation rates.* Let $\pi^{t(i)}$ be the guess for π^t in the beginning of the i-th iteration. We make use of the sequence-space Jacobian approach developed by Auclert et al. (2021) to make an accurate first guess $\pi^{t(1)}$. We give more details below.

2. *Compute the path of output, interest rate and TIP.* Using the Euler equation and the targeting rules for monetary policy and TIP we compute the implied paths for $y^{t(i)}$, $i^{t(i)}$ and $\tau^{t(i)}$.

3. *Update the path of real wages, $rw^{(i)}$.* We next use the equilibrium expression of real wages

$$rw^{t(i)} = \left(\sigma + \frac{\psi}{1-\alpha}\right) y^{t(i)} - \left(\frac{\psi}{1-\alpha}\right) a^t + \psi \ln \Delta^{t(i-1)} + \sigma \ln \Gamma^{t(i-1)}. \tag{60}$$

   where we use $\Gamma^{t(i-1)}$ and $\Delta^{t(i-1)}$, which are already known from the last iteration, instead of guessing a new sequence for $\Gamma^{t(i)}$ and $\Delta^{t(i)}$.

4. *Compute the implied policy rules.* Use the paths of $\{a^t, y^{t(i)}, \pi^{t(i)}, rw^{t(i)}, u^t, \tau^{t(i)}\}$ to solve for the firms' policy rules recursively $\tilde{P}_{s,t}(A_{s,t}, \tilde{P}_{s,t-1})$ for all periods $t = 1...T$ using equation (44).

5. *Compute the paths of aggregate variables.* Use the paths of $\{a^t, y^{t(i)}, \pi^{t(i)}, rw^{t(i)}, u^t, \tau^{t(i)}\}$ and the updated policy rules to compute the paths of aggregate price index. This gives a new path of inflation rates $\pi^{t(i)*}$.

6. *Update guess of inflation path, $\pi^{t(i+1)}$ and iterate until convergence.* We update the path of inflation $\pi^{t(i+1)}$ based on the previous guess $\pi^{t(i)}$ and the new path of inflation rates. Our updating rule is explained below and uses the sequence-space Jacobian. We iterate until the path of inflation and output are consistent with their guesses, $\pi^{t*} = \pi^t$. This implies that the path of output $y^{t(i)}$ converges.

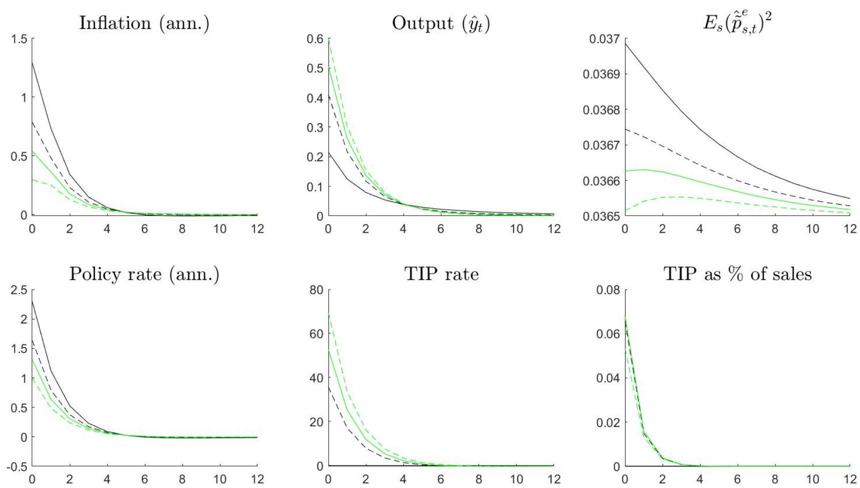
Sequence-space Jacobian. We make use of the sequence-space Jacobian approach developed by Auclert et al. (2021) for two purposes. First we use the sequence space Jacobian to compute the impulse responses of the aggregate variables of the linearized economy. We use this response as our initial guess of the path of inflation $\pi^{t(1)}$ in the loop to compute the solution to the non-linear model. It turns out to be a very accurate first guess, suggesting that the model is close to linear at the aggregate level.

Second we use the sequence space Jacobian to update our guess of inflation $\pi^{t(i)}$ at every iteration of the loop. Recall that the implied path of inflation at the end of iteration (i) is denoted $\pi^{t(i)*}$. We denote $\mathbf{A}$ the matrix encoding the total derivative of $\pi^{t(i)*}$ with respect to $\pi^{t(i)}$: $d\pi^{t(i)*} \approx \mathbf{A}d\pi^{t(i)}$. Our updating rule is given by

$$\pi^{t(i+1)} = \pi^{t(i)} + \iota(\mathbf{I} - \mathbf{A})^{-1}(\pi^{t(i)*} - \pi^{t(i)}), \tag{61}$$

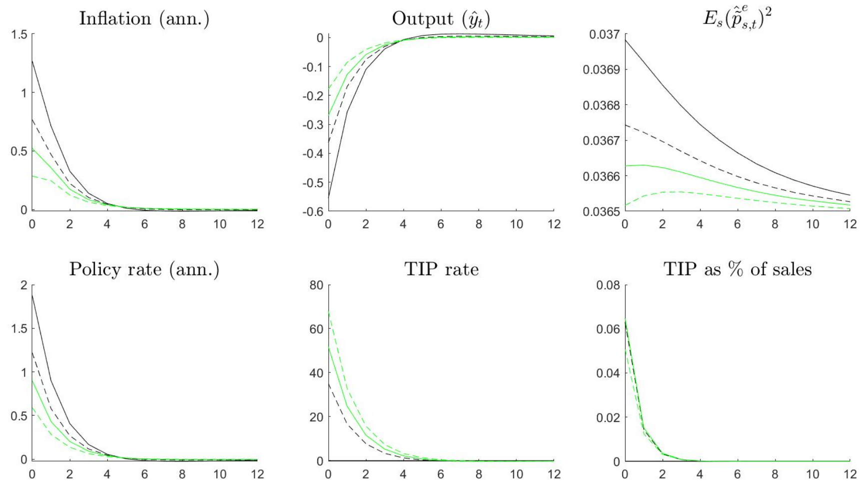
where ι is a parameter controlling the speed of adjustment and $\mathbf{I}$ is the identify matrix.

D.7 Additional Graphs



Notes: The initial markup shock is calibrated to match the initial inflation response in Figure 1. Inflation and the policy rate are annualized. Output refers to the deviation of output from its steady-state level, $\hat{y}_t$.

Figure D8: Effects of TIP following a demand shock in the multi-sector economy



Notes: The initial markup shock is calibrated to match the initial inflation response in Figure 1. Inflation and the policy rate are annualized. Output refers to the deviation of output from its steady-state level, $\hat{y}_t$.

Figure D9: Effects of TIP following a TFP shock in the multi-sector economy

E Time-dependent Calvo pricing

In this appendix we derive the TIP that can implement the first best in a setting with time-dependent Calvo frictions and we show that the Phillips curve implied by Calvo pricing is the same at first order as in the economy with Rotemberg adjustment costs. Following the notations in Galí (2015), the maximization problem of a firm having the opportunity to reset its price is given by

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t^f \left\{ Q_{t,t+k}(P_{ti}^* Y_{t+k|ti} - \Psi_{t+k}(Y_{t+k|ti})) \right\} - \tau_t (P_{ti}^* - P_{t-1i}) Y_{t|ti}$$
$$- \sum_{k=1}^{\infty} \theta^{k-1}(1-\theta) E_t^f \left\{ Q_{t,t+k} \tau_{t+k} (P_{t+ki}^{**} - P_{ti}^*) Y_{t+k|t+ki} \right\}$$

where we denote $\Psi()$ the cost function, P_{t+ki}^{**} the optimal price of firm i when it gets the chance to reset the price in the future, and $Y_{t+k|ti}$ is the output at period $t+k$ of firm i which resets its price in period t. Note that contrary to the setting without TIP, the optimal price for a firm that gets to reset at t may differ across firms, depending on when they last reset their price. We thus need to keep the i-subscript. The F.O.C. is given by

$$\sum_{k=0}^{\infty}\theta^k E_t^f\left\{Q_{t,t+k}\left((1-\epsilon_t)Y_{t+ki|t}+\epsilon_t\Psi'_{t+k}(Y_{t+k|ti})\right)\right\}-\tau_t\left(1-\epsilon_t\frac{P_{ti}^*-P_{t-1i}}{P_{ti}^*}\right)Y_{t|ti}$$
$$+\sum_{k=1}^{\infty}\theta^{k-1}(1-\theta)E_t^f\left\{Q_{t,t+k}\tau_{t+k}Y_{t+k|t+ki}\right\}=0$$

Dividing by $(1-\epsilon_t)$, and multiplying by P_{ti}^*/P_{t-1} we obtain

$$\sum_{k=0}^{\infty}\theta^k E_t^f\left\{Q_{t,t+k}Y_{t+k|ti}\left(\frac{P_{ti}^*}{P_{t-1}}-\mathcal{M}_t MC_{t+k|ti}\Pi_{t-1,t+k}\right)\right\}+\frac{\tau_t}{\epsilon_t-1}\left(1-\epsilon_t\frac{P_{ti}^*-P_{t-1i}}{P_{ti}^*}\right)\frac{P_{ti}^*}{P_{t-1}}Y_{t|ti}$$
$$-\sum_{k=1}^{\infty}\theta^{k-1}(1-\theta)E_t^f\left\{Q_{t,t+k}\frac{\tau_{t+k}}{\epsilon_t-1}\frac{P_{ti}^*}{P_{t-1}}Y_{t+k|t+ki}\right\}$$

with $MC_{t+k|ti}=\psi_{t+k|ti}/P_{t+k}$ the real marginal cost in period $t+k$ for firm i which last reset its price in period t.

First-best implementation. Together with monetary policy TIP can implement the first best. TIP should follow

$$\tau_t=\frac{\epsilon_t-1}{Y_t}\left[\sum_{k=0}^{\infty}\theta^k E_t^f\left\{Q_{t,t+k}^e Y_{t+k}\left(\mathcal{M}_t\mathcal{M}_t^w-1\right)\right\}+\sum_{k=1}^{\infty}\theta^{k-1}(1-\theta)E_t^f\left\{Q_{t,t+k}^e\frac{\tau_{t+k}}{\epsilon_t-1}Y_{t+k}\right\}\right]$$

and monetary policy should target the neutral rate of interest

$$i_t=(Q_t^e)^{-1}-1\quad with\quad Q_t^e=E_t\left[\beta_t\left(\frac{A_{t+1}}{A_t}\right)^{-\frac{\sigma(1+\psi)}{(1-\alpha)\sigma+\psi+\alpha}}\right].$$

First-order approximation. Going back to the first-order condition of firms that get to reset their price, we then linearize around $\frac{P_t^*}{P_{t-1}}=1$, steady-state output Y and $\tau = 0$.

$$\sum_{k=0}^{\infty} \theta^k \beta^k Y E_t \left\{ p_{ti}^* - p_{t-1} - \hat{mc}_{t+k|ti} + p_{t+k} - p_{t-1} \right\} + \frac{\tau_t}{\epsilon - 1} Y$$
$$- \sum_{k=1}^{\infty} \theta^{k-1}(1-\theta)\beta^k E_t \left\{ \frac{\tau_{t+k}}{\epsilon - 1} Y \right\} = 0$$

We then show that the marginal cost is given by

$$\begin{aligned} mc_{t+k|ti} &= mc_{t+k} - \frac{\alpha}{1-\alpha}(y_{t+k|ti} - y_{t+k}) \\ &= mc_{t+k} - \frac{\epsilon\alpha}{1-\alpha}(p_{ti}^* - p_{t+k}) \end{aligned}$$

and substituting this expression into the optimal pricing decision yields

$$\sum_{k=0}^{\infty} \theta^k \beta^k E_t \left\{ p_{ti}^* - p_{t-1} - \Theta \hat{mc}_{t+k} - (p_{t+k} - p_{t-1}) \right\} + \Theta \frac{\tau_t}{\epsilon - 1}$$
$$- \sum_{k=1}^{\infty} \theta^{k-1}(1-\theta)\Theta\beta^k E_t \left\{ \frac{\tau_{t+k}}{\epsilon - 1} \right\} = 0.$$

Rearranging the previous equation gives

$$p_{ti}^* - p_{t-1} = (1-\beta\theta) \sum_{k=0}^{\infty} (\theta\beta)^k E_t \left\{ \Theta \hat{mc}_{t+k} + (p_{t+k} - p_{t-1}) \right\} - (1-\beta\theta)\Theta \frac{\tau_t}{\epsilon - 1}$$
$$+ (1-\beta\theta) \sum_{k=1}^{\infty} \theta^{k-1}(1-\theta)\Theta\beta^k E_t \left\{ \frac{\tau_{t+k}}{\epsilon - 1} \right\}$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$. We then rewrite the sum as a difference equation

$$
\begin{aligned}
p_{ti}^* - p_{t-1} = {} & (1-\beta\theta)\Theta\hat{mc}_t + (1-\beta\theta)\sum_{k=0}^{\infty}(\theta\beta)^k(p_t - p_{t-1}) - (1-\beta\theta)\Theta\frac{\tau_t}{\epsilon-1} \\
& + \beta\theta(1-\beta\theta)\Theta\frac{\tau_{t+1}}{\epsilon-1} \\
& + (1-\beta\theta)\theta\beta\sum_{k=1}^{\infty}(\theta\beta)^{k-1}E_t\left\{\Theta\hat{mc}_{t+k} + (p_{t+k}-p_t)\right\} - \beta\theta(1-\beta\theta)\Theta\frac{\tau_{t+1}}{\epsilon-1} \\
& + (1-\beta\theta)(1-\theta)\Theta\beta E_t\left\{\frac{\tau_{t+1}}{\epsilon-1}\right\} \\
& + (1-\beta\theta)\beta\theta\sum_{k=2}^{\infty}\theta^{k-2}(1-\theta)\Theta\beta^{k-1}E_t\left\{\frac{\tau_{t+k}}{\epsilon-1}\right\} \\
p_{ti}^* - p_{t-1} = {} & (1-\beta\theta)\Theta\hat{mc}_t + \pi_t - (1-\beta\theta)\Theta\frac{\tau_t}{\epsilon-1} + \beta\theta(1-\beta\theta)\Theta\frac{\tau_{t+1}}{\epsilon-1} \\
& + \beta\theta E_t(p_{t+1}^* - p_t) \\
& + (1-\beta\theta)(1-\theta)\Theta\beta E_t\left\{\frac{\tau_{t+1}}{\epsilon-1}\right\} \\
p_{ti}^* - p_{t-1} = {} & \beta\theta E_t\{p_{t+1i}^* - p_t\} + (1-\beta\theta)\Theta\hat{mc}_t + \pi_t - (1-\beta\theta)\Theta\frac{\tau_t}{\epsilon-1} \\
& + (1-\beta\theta)\beta\Theta E_t\left\{\frac{\tau_{t+1}}{\epsilon-1}\right\}
\end{aligned}
$$

We then use $\pi_t = (1-\theta)(p_t^* - p_{t-1})$ to get

$$
\begin{aligned}
\frac{\pi_t}{1-\theta} &= \beta\theta E_t\frac{\pi_{t+1}}{1-\theta} + (1-\beta\theta)\Theta\hat{mc}_t + \pi_t - (1-\beta\theta)\Theta\frac{\tau_t}{\epsilon-1} + (1-\beta\theta)\beta\Theta E_t\left\{\frac{\tau_{t+1}}{\epsilon-1}\right\} \\
\pi_t &= \beta E_t\pi_{t+1} + \frac{(1-\beta\theta)(1-\theta)}{\theta}\Theta\hat{mc}_t - \frac{(1-\beta\theta)(1-\theta)}{\theta(\epsilon-1)}\Theta\tau_t \\
&\quad + \frac{(1-\beta\theta)(1-\theta)}{\theta(\epsilon-1)}\beta\Theta E_t\left\{\tau_{t+1}\right\} \\
\pi_t &= \beta E_t\pi_{t+1} + \lambda\hat{mc}_t - \zeta\left[\tau_t - \beta E_t\left\{\tau_{t+1}\right\}\right]
\end{aligned}
$$

with $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}\Theta$ and $\zeta = \frac{\lambda}{(\epsilon-1)}$.

Finally, using $\hat{mc}_t = \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)\hat{y}_t$ we get

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{y}_t - \zeta \left[\tau_t - \beta E_t \left\{\tau_{t+1}\right\}\right]$$

with $\kappa = \lambda \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)$.

This shows that using time-dependent frictions instead of state-dependent frictions leads to the same representation and macroeconomic dynamics at first order as with Rotemberg-type frictions. Therefore the results on optimal policies derived in the linearized model—corollary 1, section 3.3, section **??** and proposition 5—are robust to using Calvo-type frictions. The following paragraph gives more details on proposition 5.

Relative price distortions and independence from TIP. In a setting with heterogeneous sectors, we can follow the same reasoning as for Rotemberg adjustment cost and derive the following Phillips curve for each sector:

$$\pi_{st} = \beta E_t \pi_{t+1s} + \lambda_s \left[\hat{mc}_t - \left(\frac{1}{1-\alpha}\right)\hat{p}^e_{st}\right] + \zeta_s \left[\beta E_t \tau_{t+1} - \tau_t\right] + u_t$$

with $\lambda_s = \frac{(1-\beta\theta_s)(1-\theta_s)}{\theta_s}\Theta$ and $\zeta_s = \frac{\lambda_s}{(\epsilon-1)}$. Taking the difference between any two sectors, we see that the sufficient conditions for proposition 5 to hold, i.e. for relative prices to be independent of aggregate and on TIP, are the same, namely $\lambda_s = \lambda$ and $\zeta_s = \zeta$ so ultimately $\theta_s = \theta$. The second bullet point of the proposition is a direct corollary of the first.

F TIP in a Medium-Scale DSGE Model

In this Appendix we explain how we embed TIP into a Smets and Wouters (2007)-type medium scale model (SW hereafter), estimate it and conduct counterfactual analysis.

F.1 Incorporating TIP into SW

NKPC with TIP. To make comparison with SW easier, in this section, we follow their notation unless otherwise noted. Adding TIP to the SW model changes only the price Phillips Curve. The original profit maximization problem for intermediate goods producers (page 2 of SW's online Appendix)[20] is given by:

$$\max_{\tilde{P}_t(i)} E_t \sum_{s=0}^{\infty} \xi_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \left[\tilde{P}_t(i) X_{t,s} - MC_{t+s} \right] Y_{t+s}(i), \tag{62}$$

s.t.

$$Y_{t+s}(i) = Y_{t+s} G'^{-1} \left(\frac{P_t(i) X_{t,s}}{P_{t+s}} \tau_{t+s} \right), \tag{63}$$

$$X_{t,s} = \begin{cases} 1 & \text{if } s = 0 \\ \Pi_{l=1}^{s} (\pi_{t+l-1}^{\iota_p} \bar{\pi}^{1-\iota_p}) & \text{if } s > 0 \end{cases}. \tag{64}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate; $\tilde{P}_t(i)$ denotes the newly set price; $\Pi_{l=1}^{s} \pi_{t+l-1}^{\iota_p} \bar{\pi}^{1-\iota_p}$ captures partial price indexation, where the weights on past inflation (π_{t+l-1}) and trend inflation ($\bar{\pi}$) are ι_p and $1 - \iota_p$, respectively; ξ_p is the Calvo probability of resetting prices; $\frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}}$ is the nominal discount factor; $\tau_t = \int_0^1 G' \left(\frac{Y_t(i)}{Y_t} \right) \frac{Y_t(i)}{Y_t} di$, where $G(\cdot)$ corresponds to the Kimball aggregator; and MC_{t+s} is the marginal cost, which is common to all firms.

Since the model has a constant trend inflation, we adjust the TIP rule such that it is centered around trend inflation. Firms pay TIP taxes in two cases.

[20] https://assets.aeaweb.org/asset-server/articles-attachments/aer/data/june07/20041254_app.pdf

First, when they reset their price $\tilde{P}_t(i)$; second, when aggregate inflation fluctuates around the trend due to partial indexation. However, as we shall see below, the second one has no effect at the first order. Denoting the TIP rate τ_t^{TIP} (since τ_t is already taken in the original SW notation), the maximization problem with TIP is given by

$$\max_{\tilde{P}_t(i)} E_t \sum_{s=0}^{\infty} \xi_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \left[\tilde{P}_t(i) X_{t,s} - MC_{t+s}\right] Y_{t+s}(i) - \tau_t^{TIP}(\tilde{P}_t(i) - \bar{\pi} P_{t-1}(i)) Y_t(i)$$
$$- E_t \sum_{s=1}^{\infty} \xi_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \tau_{t+s}^{TIP} \left[\tilde{P}_t(i) X_{t,s} - \tilde{P}_t(i) \bar{\pi} X_{t,s-1}\right] Y_{t+s}(i)$$
$$- E_t \sum_{s=1}^{\infty} \xi_P^{s-1}(1-\xi_P) \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \tau_{t+s}^{TIP} \left[\tilde{P}_{t+s}(i) - \tilde{P}_t(i) \bar{\pi} X_{t,s-1}\right] Y_{t+s|\tilde{P}_{t+s}}(i). \tag{65}$$

As in Section E, the last term for output $Y_{t+s|\tilde{P}_{t+s}}(i)$ is a function of the new reset price and is thus unaffected by the current reset price $\tilde{P}_t(i)$. The F.O.C. is given by

$$0 = E_t \sum_{s=0}^{\infty} \xi_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \left[X_{t,s} Y_{t+s}(i) + \left(\tilde{P}_t(i) X_{t,s} - MC_{t+s}\right) \frac{\partial Y_{t+s}(i)}{\partial \tilde{P}_t(i)}\right]$$
$$- \tau_t^{TIP} \left[Y_t(i) + (\tilde{P}_t(i) - \bar{\pi} P_{t-1}(i)) \frac{\partial Y_t(i)}{\partial \tilde{P}_t(i)}\right]$$
$$- E_t \sum_{s=1}^{\infty} \xi_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \tau_{t+s}^{TIP} \left(X_{t,s} - \bar{\pi} X_{t,s-1}\right) \left(Y_{t+s}(i) + \tilde{P}_t(i) \frac{\partial Y_{t+s}(i)}{\partial \tilde{P}_t(i)}\right)$$
$$+ E_t \sum_{s=1}^{\infty} \xi_P^{s-1}(1-\xi_P) \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \tau_{t+s}^{TIP} \bar{\pi} X_{t,s-1} Y_{t+s|\tilde{P}_{t+s}}(i). \tag{66}$$

Define $\mathcal{G}_{t+s} = \frac{1}{G'^{-1}(z_{t+s})} \frac{G'(x_{t+s})}{G''(x_{t+s})} = \frac{\tilde{P}_t(i)}{Y_{t+s}(i)} \frac{\partial Y_{t+s}(i)}{\partial \tilde{P}_t(i)}$, $x_t = G'^{-1}(z_t)$, and $z_t =$

$\frac{P_t(i)}{P_t}\tau_t$. Multiply both sides by $\tilde{P}_t(i)$:

$$
\begin{aligned}
0 = {} & E_t \sum_{s=0}^{\infty} \xi_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} Y_{t+s}(i) \left[(1+\mathcal{G}_{t+s}) \tilde{P}_t(i) X_{t,s} - \mathcal{G}_{t+s} MC_{t+s} \right] \\
& - \tau_t^{TIP} \tilde{P}_t(i) Y_t(i) \left[1 + \frac{\tilde{P}_t(i) - \bar{\pi} P_{t-1}(i)}{\tilde{P}_t(i)} \mathcal{G}_t \right] \\
& - E_t \sum_{s=1}^{\infty} \xi_P^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \tau_{t+s}^{TIP} \tilde{P}_t(i) Y_{t+s}(i) \left(X_{t,s} - \bar{\pi} X_{t,s-1} \right) (1+\mathcal{G}_{t+s}) \\
& + E_t \sum_{s=1}^{\infty} \xi_P^{s-1} (1-\xi_P) \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \tau_{t+s}^{TIP} \tilde{P}_t(i) \bar{\pi} X_{t,s-1} Y_{t+s|\tilde{P}_{t+s}}(i). \qquad (67)
\end{aligned}
$$

Following SW's online appendix (equation 32), we detrend this FOC. Let lower case variables denote de-trended real variables, e.g., $mc_t = \frac{MC_t}{P_t}$, $\xi_t = \Xi_t \gamma^{\sigma_c t}$, $y_{t+s}(i) = \frac{Y_{t+s}(i)}{\gamma^s}$, $\bar{\beta}^t = \beta^t \gamma^{-\sigma_c t}$, $\tilde{p}_t(i) = \frac{\tilde{P}_t(i)}{P_t}$, $x_{t,s}^p = X_{t,s} \frac{P_t}{P_{t+s}}$. The de-trended FOC is given by:

$$
\begin{aligned}
0 = {} & E_t \sum_{s=0}^{\infty} \xi_P^s \bar{\beta}^s \gamma^s \frac{\xi_{t+s}}{\xi_t} y_{t+s}(i) \left[(1+\mathcal{G}_{t+s}) \tilde{p}_t(i) x_{t,s}^p - \mathcal{G}_{t+s} mc_{t+s} \right] \\
& - \tau_t^{TIP} \tilde{p}_t(i) y_t(i) \left[1 + \frac{\tilde{P}_t(i) - \bar{\pi} P_{t-1}(i)}{\tilde{P}_t(i)} \mathcal{G}_t \right] \\
& - E_t \sum_{s=1}^{\infty} \xi_P^s \bar{\beta}^s \gamma^s \frac{\xi_{t+s}}{\xi_t} \tau_{t+s}^{TIP} \tilde{p}_t(i) y_{t+s}(i) \left(x_{t,s}^p - x_{t,s}^p \left(\frac{\bar{\pi}}{\pi_{t+s-1}} \right)^{\iota_p} \right) (1+\mathcal{G}_{t+s}) \\
& + E_t \sum_{s=1}^{\infty} \xi_P^{s-1} (1-\xi_P) \bar{\beta}^s \gamma^s \frac{\xi_{t+s}}{\xi_t} \tau_{t+s}^{TIP} \tilde{p}_t(i) x_{t,s}^p \left(\frac{\bar{\pi}}{\pi_{t+s-1}} \right)^{\iota_p} y_{t+s|\tilde{P}_{t+s}}(i). \qquad (68)
\end{aligned}
$$

We next linearize this FOC around $\tau_t^{TIP} = 0$:

$$
\begin{aligned}
0 = {} & E_t \sum_{s=0}^{\infty} \xi_P^s (\bar{\beta}\gamma)^s \left[(1+\mathcal{G}_{t+s}) \tilde{p}_t(i) x_{t,s}^p - \mathcal{G}_{t+s} mc_{t+s} \right] \\
& - \left[\tau_t^{TIP} - E_t \sum_{s=1}^{\infty} \xi_P^{s-1} (1-\xi_P) (\bar{\beta}\gamma)^s \tau_{t+s}^{TIP} \right] \qquad (69) \\
= {} & E_t \sum_{s=0}^{\infty} \xi_P^s (\bar{\beta}\gamma)^s \mathcal{G} \left[\frac{1+\mathcal{G}_{t+s}}{\mathcal{G}_{t+s}} \tilde{p}_t(i) x_{t,s}^p - mc_{t+s} - \frac{\tau_{t+s}^{TIP} - (\bar{\beta}\gamma) \tau_{t+s+1}^{TIP}}{\mathcal{G}} \right] \qquad (70)
\end{aligned}
$$

At first order, the denominator in the last term $\frac{\tau_{t+s}^{TIP}-(\bar{\beta}\gamma)\tau_{t+s+1}^{TIP}}{\mathcal{G}}$ is simply $\mathcal{G}$ because we linearize around $\tau^{TIP}=0$. Define $mc_t^* = mc_t + \frac{\tau_t^{TIP}-(\bar{\beta}\gamma)E_t\tau_{t+1}^{TIP}}{\mathcal{G}}$ the marginal cost including current and future TIP. Using this definition into equation (70), we obtain an equation identical to equation 49 in SW's appendix with mc_{t+s}^* instead of mc_{t+s}. Therefore, following the same approach as SW, we obtain the following fully linearized equation:

$$(1+\bar{\beta}\gamma\iota_p)\hat{\pi}_t = \iota_p\hat{\pi}_{t-1} + \bar{\beta}\gamma E_t\hat{\pi}_{t+1} + A\frac{(1-\xi_p\bar{\beta}\gamma)(1-\xi_p)}{\xi_p}(\hat{mc}_t^* + \hat{\lambda}_{p,t}), \tag{71}$$

where $\hat{\pi}_t$ is the deviation of inflation from trend inflation $\bar{\pi}$, and $A = \frac{1}{1+\lambda_p\epsilon}$ captures the effect of strategic complementarities stemming from the Kimball aggregator. The parameter λ_p is the steady state markup, $\hat{\lambda}_{p,t}$ is the markup shock and $\epsilon = \frac{d\left(\frac{G''}{xG'}\right)}{d(x)}$.

$$\hat{mc}_t^* = \frac{dmc_t^*}{mc} = \frac{dmc_t}{mc} + \frac{\tau_t^{TIP}-(\bar{\beta}\gamma)E_t\tau_{t+1}^{TIP}}{\mathcal{G}mc}. \tag{72}$$

In steady state, $\mathcal{G} = -\epsilon(1)$ and $mc = \frac{\epsilon(1)-1}{\epsilon(1)}$, therefore,

$$\hat{mc}_t^* = \hat{mc}_t - \frac{1}{\epsilon(1)-1}(\tau_t^{TIP} - \bar{\beta}\gamma E_t\tau_{t+1}^{TIP}). \tag{73}$$

Therefore, the NKPC with TIP in the SW model is given by

$$\begin{aligned}(1+\bar{\beta}\gamma\iota_p)\hat{\pi}_t &= \iota_p\hat{\pi}_{t-1} + \bar{\beta}\gamma E_t\hat{\pi}_{t+1} \\ &\quad + A\frac{(1-\xi_p\bar{\beta}\gamma)(1-\xi_p)}{\xi_p}\left[\hat{mc}_t - \frac{\tau_t^{TIP}-\bar{\beta}\gamma E_t\tau_{t+1}^{TIP}}{\epsilon(1)-1} + \hat{\lambda}_{p,t}\right].\end{aligned} \tag{74}$$

Targeting rule for TIP. Since trend inflation $\bar{\pi}$ is strictly positive, we modify the inflation-targeting rule 24 in Section 3.3 to center it around $\bar{\pi}$:

$$\tau_t^{TIP} = \varphi_\pi \hat{\pi}_t = \varphi_\pi(\pi_t - \bar{\pi}). \tag{75}$$

The trend-neutral targeting rule ensures the consistency between the non-zero trend inflation and zero TIP when the economy is on its trend.

F.2 Data and Estimation

We extend the original time series from FRED used in SW to the most recent available data point. Our final quarterly sample covers the period from 1960Q1 to 2024Q3. We closely follow their procedures to clean the data. Relative to SW, we make two modifications before estimating the model.

To address both zero-lower bound periods in our sample during which the Federal Funds rate was at zero and the Fed deployed unconventional tools, we replace the Federal Funds rate with the shadow rate by Wu and Xia (2016), as in Anderson et al. (2017), Wu and Zhang (2019), Avdjiev et al. (2020). We also tried alternative measures for robustness, including the new shadow rate by Jones et al. (2021), the 3-year Treasury yields, and keeping the Federal Funds rate unchanged. Our counterfactual results are virtually unchanged with these alternative measures of the monetary policy stance. Results are available upon request.

We restrict the sample to 1960Q1-2019Q4 to exclude the COVID pandemic whose extreme recession and rebound could bias our parameters' estimates. The model may indeed not be well suite to rationalize the dramatic shocks that occurred during the COVID-19 pandemic, especially in 2020Q2, when quarterly GDP collapsed by 31.6% (annualized) relative to 2020Q1 and then rebounded by 31.3% in 2020Q3. We then back out the seven structural shocks from the entire sample period, from 1960Q1 to 2024Q3.

We estimate the parameters and structural shocks using the Dynare replication code in Johannes Pfeifer's Github repository.[21] Our priors follow SW's original priors. The only exception is the trend inflation $\bar{\pi}$: we replace the original gamma distribution with a normal distribution $N(0.5, 0.05^2)$ to bring the posterior trend closer to 2% per annum. This is an important estimate in our counterfactual analysis because we assume that TIP follows an inflation-targeting rule centered around detrended inflation, and a trend of 2% per annum is a more realistic policy objective. Table F5 reports the priors and posteriors from the baseline estimation.

F.3 Counterfactual analysis.

We now used the estimated parameters and shocks from Section F.2, as well as the extended model with TIP to run counterfactuals when $\varphi_\pi = 300$. Results are reported below.

Historical Counterfactual with TIP. We find that TIP has significant stabilizing effects on inflation, particularly during high-inflation periods such as the 1970s and the early 2020s. The top panel of Figure 5 shows the actual path of inflation in blue, measured by the GDP deflator, and its counterfactual path under a TIP rule with $\varphi_\pi = 300$ in red. Both are the 4-quarter moving averages of annualized log changes. When inflation peaked in 1974Q4 at 10.4% per annum, TIP would have cut it by 2.9%, equivalent to 36% of the deviation from the trend inflation of 2.2%. When inflation peaked again in 1980Q4, TIP would have lowered it by 2.3%, equivalent to 31% of the deviation from trend. During the recent COVID-19 pandemic, TIP would have lowered the peak inflation in 2022Q1 by 2.6%, which is almost 50% of the deviation from the trend. During lower-inflation periods, TIP turns out to be highly effective at reducing the volatility of inflation too. Finally, despite its significant effect on inflation, TIP barely affects output growth in the simulations. The actual output growth and the counterfactual path show little difference in the middle panel of Figure 5.

[21] https://github.com/johannespfeifer/dsge_mod

Table F5: Priors and posteriors

	Prior			Posterior	
	Dist.	Mean	Stdev	Mode	Stdev
ρ_a	beta	0.500	0.2000	0.9909	0.0041
ρ_b	beta	0.500	0.2000	0.8546	0.0339
ρ_g	beta	0.500	0.2000	0.9760	0.0075
ρ_i	beta	0.500	0.2000	0.8217	0.0681
ρ_r	beta	0.500	0.2000	0.1677	0.0614
ρ_p	beta	0.500	0.2000	0.9655	0.0316
ρ_w	beta	0.500	0.2000	0.9833	0.0090
μ_p	beta	0.500	0.2000	0.8992	0.0462
μ_w	beta	0.500	0.2000	0.9664	0.0119
φ	norm	4.000	1.5000	4.2620	1.0924
σ_c	norm	1.500	0.3750	1.4598	0.2039
λ	beta	0.700	0.1000	0.5072	0.0639
ξ_w	beta	0.500	0.1000	0.8006	0.0435
σ_l	norm	2.000	0.7500	1.5198	0.6019
ξ_p	beta	0.500	0.1000	0.7747	0.0453
ι_w	beta	0.500	0.1500	0.6198	0.1353
ι_p	beta	0.500	0.1500	0.3006	0.0926
ψ	beta	0.500	0.1500	0.6627	0.1133
ϕ_p	norm	1.250	0.1250	1.5042	0.0746
r_π	norm	1.500	0.2500	1.9852	0.1473
ρ	beta	0.750	0.1000	0.8395	0.0211
r_y	norm	0.125	0.0500	0.0977	0.0202
$r_{\Delta y}$	norm	0.125	0.0500	0.2340	0.0252
$100(\beta^{-1}-1)$	gamm	0.250	0.1000	0.1026	0.0434
$\bar{\pi}$	norm	0.500	0.0500	0.5591	0.0503
$\bar{l}$	norm	0.000	2.0000	3.5819	1.2550
$\bar{\gamma}$	norm	0.400	0.1000	0.3770	0.0366
ρ_{ga}	norm	0.500	0.2500	0.5234	0.0637
α	norm	0.300	0.0500	0.1769	0.0165
η^a	invg	0.100	2.0000	0.4795	0.0261
η^b	invg	0.100	2.0000	0.0946	0.0110
η^g	invg	0.100	2.0000	0.4522	0.0215
η^i	invg	0.100	2.0000	0.3171	0.0301
η^m	invg	0.100	2.0000	0.2137	0.0119
η^p	invg	0.100	2.0000	0.1277	0.0110
η^w	invg	0.100	2.0000	0.3571	0.0197

This large reduction in the volatility of inflation is achieved by setting high TIP rates in times of high inflation. The peak TIP rate would be as high as 300% in the 1970s and 200% in 2022. By contrast, TIP would be largely negative during the Great Moderation because inflation was constantly below the estimated trend of 2.2%. However, since TIP is applied only to price changes, the implied tax burden remains low. As shown in the last panel, TIP costs would be negligible relative to sales for most of the time. TIP costs would have briefly exceeded 3% of sales when inflation reached its highest level in 1974 and 1980. During the COVID period, the average (peak) cost of TIP would have been 0.85% and 1.9% of sales in 2021 and 2022 respectively, while the average reduction in inflation would have been 1.9% per annum.[22]

The stabilization effects of TIP are robust to alternative ways to dealing with the zero-lower bound. We have run the same analysis using the Jones et al. (2021) shadow rate, the 3-year Treasury yields, and the Federal Funds rate. We find virtually the same results, which are available upon request.

Impulse response functions. We now examine the impulse response functions of the economy to the seven structural shocks using the estimated DSGE model. Figure F10 shows the responses of inflation, output, and the TIP rate. Figure F12 shows the responses of more variables to all seven shocks.

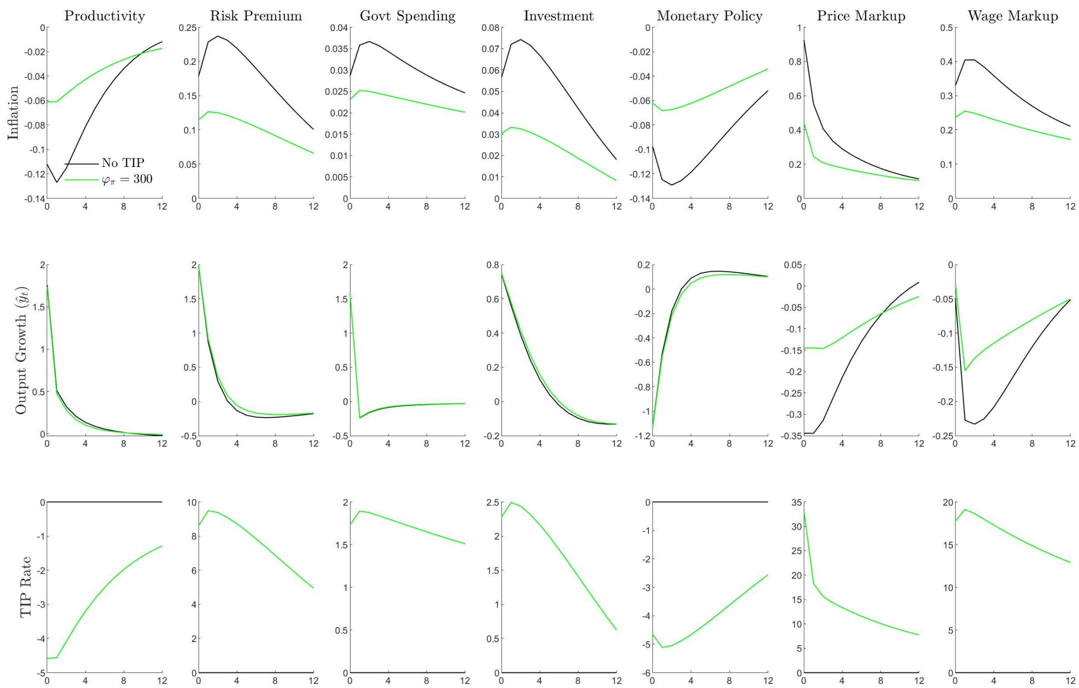
We find that an inflation-targeting TIP rule with φ_π dampens the response of inflation after all seven shocks by 30 to 50% depending on the shock, consistent with the IRFs implied by the small-scale model in Section 3.3. More importantly, TIP remarkably attenuates output losses after price and wage markup shocks, reaffirming the divine coincidence with markup shocks in Section 3.3.1. After all other five non-markup shocks, TIP has only very limited effects on output, suggesting that trade-offs are quantitatively negligible in the medium-scale DSGE model. The main reason is that inflation responses are an order of magnitude smaller than output responses after the five non-markup shocks. While lowering inflation qualitatively decreases the need for monetary policy tightening, quantitatively the magnitude is

[22] TIP would have been quite effective to dampen inflation during this period because price markup shocks were, according to the model, important in driving inflation.

small.

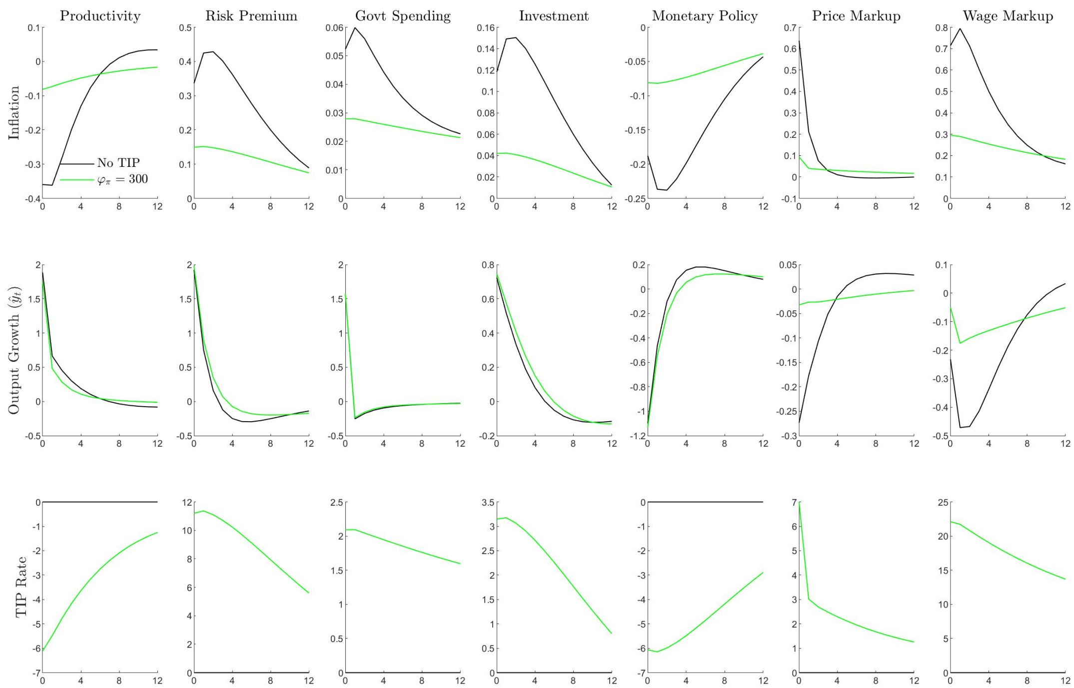
Next, we look at the robustness of our conclusions when prices become more flexible and the Phillips Curve becomes "steeper" as suggested in recent research. To do so, we lower the Calvo pricing parameter ξ_p by 0.25 percentage points, from 0.7747 to 0.5247 as in the exercise in Section 3.3.1. Other parameters remain unchanged. The IRFs are shown in Figure F11.

After a price markup shock, without TIP, the response of inflation on impact is stronger when price are more flexible, from 0.23% in Figure F12 to 0.65% in Figure F11. With TIP, however, the response inflation on impact is slighlty lower, below 0.1% instead of around 0.4%. Even though inflation subsides slightly more slowly with TIP after 1 year than without, the cumulative impact on the price level in the medium run remains significant. When prices are more flexible, TIP is able to lower the cumulative inflation response by more than 40% compared to 30% when price are more sticky. For all other shocks, TIP also becomes more effective at attenuating inflation when prices become more flexible (see Figure F11).



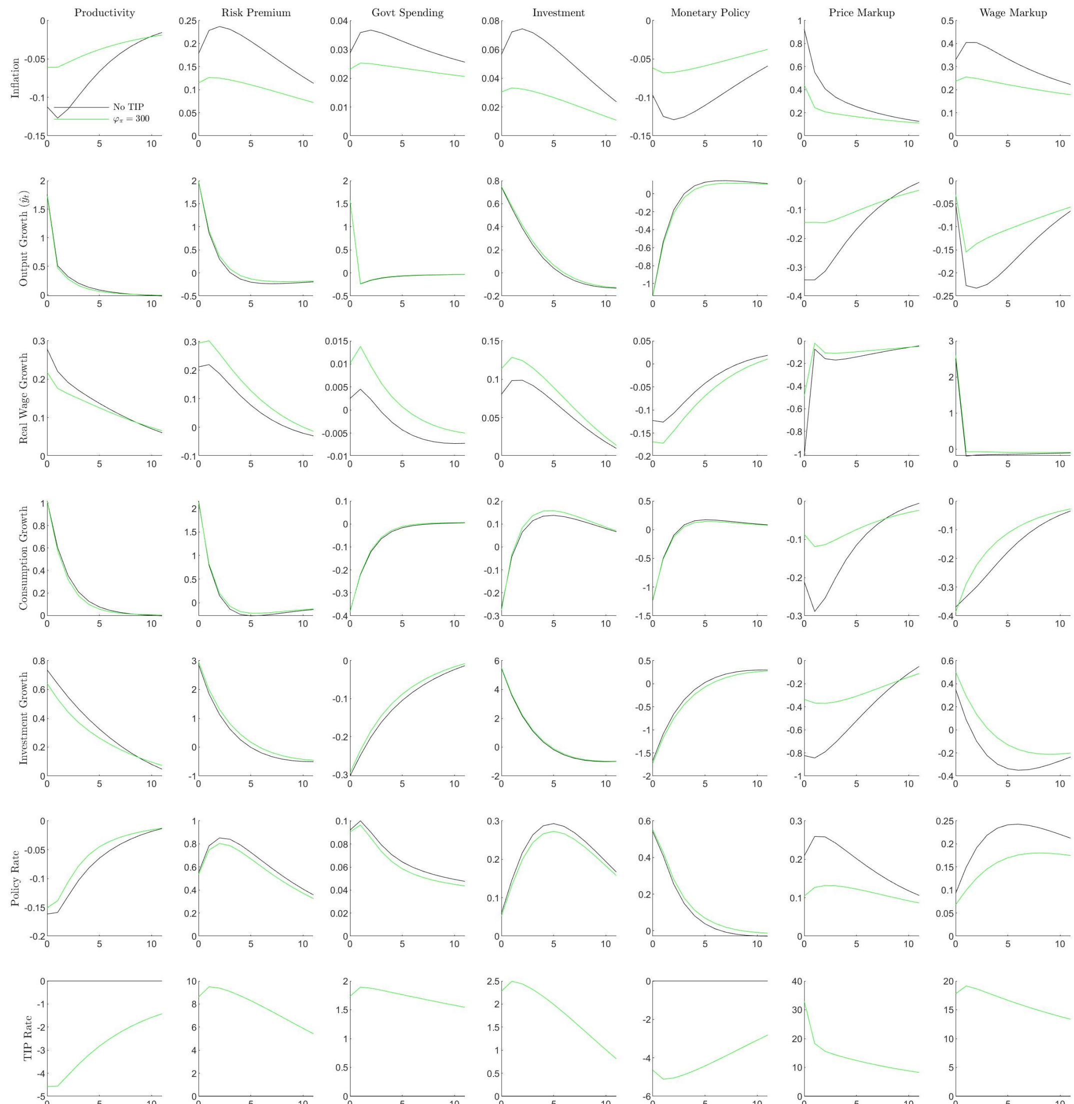
Notes: The initial shock size is one standard deviation. Inflation and the output growth rate are annualized. Output refers to the deviation of output from its steady-state level, $\hat{y}_t$.

Figure F10: Impulse response functions



Notes: The initial shock size is one standard deviation. Inflation and the output growth rate are annualized. Output refers to the deviation of output from its steady-state level, $\hat{y}_t$.

Figure F11: Impulse response functions with lower price stickiness



Notes: The initial shock size is one standard deviation. Inflation, all growth rate, and the policy rate are annualized. Output refers to the deviation of output from its steady-state level, $\hat{y}_t$.

Figure F12: Impulse response functions (continued)

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