The Great Gatsby Goes to College: Tuition, Inequality and Intergenerational Mobility*

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Abstract

This paper analyzes the role of higher education in shaping intergenerational mobility, income inequality and aggregate income. We introduce a model where overlapping generations of heterogeneous households make college choices subject to a borrowing constraint and with heterogeneous colleges that maximize quality. The model is consistent with the observed patterns of sorting of students across colleges and can generate several trends observed in the U.S. since 1980 : a rise in the returns to human capital is predicted to increase the dispersion of spending per-student across colleges, the exclusion of low-income students from top colleges, tuition fees, the intergenerational elasticity of earnings (IGE) and income inequality. Counterfactual simulations show that if all students received the same higher education, the IGE, the income Gini and aggregate income would decrease by up to 21%, 4.2% and 9.7% respectively. Increasing the progressivity of institutional need-based student aid would enhance mobility, decrease inequality and boost GDP by improving the sorting of students and financial resources across colleges.

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1. Introduction

Debates over the impact of higher education on mobility, income inequality and economic activity remain central to discussions on education policy in the U.S.. College is traditionally viewed a key pathway to upward mobility. However, access remains extremely selective and unequal, especially at top-tier colleges. For example, Chetty, Friedman, Saez, Turner, and Yagan (2020) report that children whose parents are in the top 1% of the income distribution are 77 times more likely to attend an Ivy League college than those whose parents are in the bottom income quintile. What are the forces determining the sorting of students and financial resources across colleges? To what extent does parental income matter relative to student ability? How does this sorting in turn shapes intergenerational mobility, inequality and aggregate income at the next generation?

Understanding the role of colleges in intergenerational mobility and income inequality is particularly pressing in light of long-term trends. Over the past forty years, (a) disparities in expenditures per students across colleges have widened (Capelle, 2019); (b) the share of students from the lowest income quintile in top colleges has stagnated (Bailey and Dynarski, 2011; Chetty, Friedman, Saez, Turner, and Yagan, 2020); (c) tuition fees before financial aid have increased by a factor of four in real terms since 1980; (d) intergenerational persistence of income (hereafter IGE) has slightly increased, signaling a decline in intergenerational mobility (Davis and Mazumder, 2017) and (e) market returns to education and income inequality have increased (Autor, Katz, and Kearney, 2008; Piketty and Saez, 2003).

To shed light on these questions, this paper makes four contributions. First we build a tractable model of the equilibrium sorting of students and financial resources across heterogeneous colleges, and of the transmission of human capital over generations. Second we use the model to show analytically how higher education shapes economic mobility and inequality and to offer a unified explanation for the stylized facts (a) to (e): an increase in the returns to human capital. We then estimate a quantitative version of the model to match empirical patterns of the micro-data in the U.S, which we use to run policy counterfactuals. Third we quantify how the endogenous sorting of students and resources across heterogeneous colleges shapes income inequality and intergenerational mobility. Fourth we investigate the optimal degree of progressivity of federal and institutional need-based student aid.

The household side of the model builds on a large theoretical literature that formalizes how human capital transmission across generations perpetuates inequality (e.g., Benabou (2002)). A continuum of heterogeneous households characterized by their human capital transmit—with some randomness—ability to their children and make an educational investment choice subject to a borrowing constraint. The supply side of the market for higher education is a continuum of colleges that differ in quality. Households face an equilibrium tuition schedule that depends on college quality, student ability and parental income.[1] After college, each child becomes an adult, supplies their human capital—a combination of their ability, college quality and a labor market shock—in a competitive labor market and has a child.

A key novelty of our framework is to embed into this general equilibrium dynastic model a distribution of heterogeneous colleges that is endogenous. Colleges seek to maximize the quality they provide to their students. Their quality depends not only on the amount of educational resources spent per student but also on the average ability of the student body, what will be referred to as the "peer effet." Colleges have an incentive to attract high-ability students because of the peer effet, as well as students from rich families who bring in additional resources to finance educational spending. This microfoundation of the college sector builds on a literature that estimates equilibrium models of higher education (e.g., Rothschild and White (1995); Epple, Romano, and Sieg (2006); Cai and Heathcote (2022)). As in Cai and Heathcote (2022), colleges are price-takers and the tuition schedule clears each segment of the higher education market. Finally, we close the model with a government that implements non-linear merit and need-based financial aid to students and non-linear transfers to colleges.

An important contribution of the paper is to provide an analytical characterization of the equilibrium sorting of students and financial resources across heterogeneous colleges, and of the transmission of human capital over generations. Equilibrium in the market for higher education features a hierarchy of colleges differing in education quality, with two-dimensional sorting of students by ability and family income. While sorting on ability is desirable, sorting on parental income is not, reflecting the existence of the borrowing constraint. These two dimensions of sorting—ability and income—also shape inequality and intergenerational persistence. We derive a closed-form expression for the intergenerational elasticity of income (IGE) which shows that

[1] In Benabou (2002), households buy an educational "good" traded at a constant unit price, independent on the households/students' characteristics and there is no notion of quality ladder.

higher education increases intergenerational persistence through an ability-sorting channel, which depends on the importance of peer-effects in the college technology and government merit-based aid, and an income-sorting channel, which depends on the importance of teaching expenditures in the college technology, and need-based aid by colleges and the government.

We then show analytically that the model is consistent with the stylized facts (a) to (e): under weak conditions, an increase in the returns to education—a primitive of the model—is shown to lead to an increase in the inequality of resources across colleges, a decrease in the share of low income students at top colleges, an increase in tuition, a decline in intergenerational mobility and an increase in income inequality. Intuitively, the rising returns to education increase the dispersion of labor earnings for a given distribution of human capital, thereby increasing income inequality. This leads richer households to demand higher quality of education, incentivizing top colleges to raise tuition, increasing the dispersion of revenues and educational spending across colleges. This in turn feeds back into more inequality in human capital at the following generation. Individuals from low-income background are priced out of top colleges, hence the stagnation of their shares and the decline in mobility.[2]

We then estimate a quantitative version of the model that we use to run policy counterfactuals. We allow parents to choose how much financial bequests to give to their child and for individuals to not go to college. The model is estimated using several microdata sources: (i) the restricted-use version of the National Longitudinal Survey of Youth of 1997 (NLSY), a representative panel of high-schoolers, with detailed information on parental background, the children's abilities, their experience through the higher education system and their income in their early thirties; (ii) the National Postsecondary Student Aid Study (NPSAS) provided by the National Center for Education Statistics (NCES), a detailed student-level dataset on net tuition and financial aid; and (iii) the Integrated Postsecondary Education Data System provided by the NCES, a panel of the universe of colleges. We validate the estimated model in two ways. First we show that it quantitatively matches the empirical distribution of parental income by college quintiles in the rich micro-data provided by Chetty, Friedman, Saez, Turner, and Yagan (2020). Second we show that it is quantitatively

[2]Higher education thus contributes to the gradual shift of the U.S. society to the right side of the Great Gatsby curve. The Great Gatsby curve is the negative empirical correlation between cross-sectional income inequality and intergenerational mobility. It has been documented in the cross-section of countries and over time in the U.S.

consistent with the trends (a) to (e).

We then use the estimated model to quantify the importance of the sorting of students and resources across heterogeneous colleges for mobility, inequality and aggregate income. The first counterfactual consists in randomly allocating students to colleges and equalizing spending across institutions. We find that the sorting of students and spending across heterogeneous colleges has a sizable effect on mobility, inequality and aggregate income. In this counterfactual with a uniform higher education, the IGE decreases by 21%, the income Gini by 4.2%, and GDP by 9.7%. With the second counterfactual, we isolate the contribution of the peer effect by equalizing spending across all colleges and perfectly sorting students across colleges by ability. This policy experiment leads to a decrease in the IGE by 15% and in the income Gini by 2.5% and a drop in GDP by 5.7%. Overall the sorting of ability across colleges appears to contribute for more than half of the total effect of higher education on mobility, inequality and aggregate income.

We next analyze the role of existing federal need-based student aid in higher education. This policy can in theory not only decrease intergenerational persistence and inequality but also boost GDP by addressing the misallocation of students and spending implied by the borrowing constraint. This is because income-based policies help neutralize the income-sorting channel and allow high ability students from low-income family to access better colleges. Consistent with this intuition, we find that removing the progressivity of need-based student aid would lead to an increase in intergenerational persistence by 0.7% and inequality by 0.1% and to an decrease in GDP by 0.4%. The decline in GDP shows that these policies (partially) address the misallocation of students across colleges.[3]

Finally we analyze the optimal degree of progressivity of federal and institutional need-based student aid and compare them to their existing levels.[4] Institutional need-based aid, like federal need-based aid, can in principle also boost GDP by addressing the inefficiency introduced by the borrowing constraint. We thus solve for the welfare-maximizing degree of progressivity of federal need-based aid, and then institutional need-based aid. We find that the current degree of progressivity of federal need-based

[3]Merit-based financial aid by the government and institutional need-based aid by colleges have virtually no effect, because they are very small in the *status quo*.

[4]Expansions of federal need-based programs have been debated in recent national elections in the U.S. (*e.g.* the "College for All" proposal). Similarly, increases in institutional need-based aid has been a focus of recent tuition policies by elite colleges.

aid is close to its optimal level. More interestingly, we find that the optimal degree of progressivity of institutional need-based aid is significantly higher than its current level. Both optimal federal and institutional need-based aid significantly improves welfare by about 1%. They do so by decreasing intergenerational persistence and inequality, while increasing GDP. This increase in GDP stems from the improvement in the quality of sorting of students across colleges, as the policy neutralizes the effect of parental income on the sorting of students.

Literature. The present paper contributes to the literature that investigates the determinants of educational choice, mobility and inequality in intergenerational frameworks (Benabou, 1996; Fernandez and Rogerson, 1996; Abbott, Gallipoli, Meghir, and Violante, 2013; Kotera and Seshadri, 2017; Caucutt and Lochner, 2017; Hendricks and Leukhina, 2017; Guerrieri and Fogli, 2017; Durlauf and Seshadri, 2018; Blandin and Herrington, 2022; Lee and Seshadri, 2019; Eckert and Kleineberg, 2019). The closest paper is Restuccia and Urrutia (2004) which focuses on the role of higher education. While in their paper there is a single college with an exogenous tuition fee and quality, we allow for rich heterogeneity of colleges, endogenize the ladder of quality and tuition fees and account for the complex set of government interventions in higher education. As a result, the model is able to replicate the (untargeted) distribution of parental income across heterogeneous colleges (Chetty, Friedman, Saez, Turner, and Yagan, 2020) and trends (a)-(e) affecting higher education, mobility and economic inequality.[5] Our results show that college heterogeneity and the endogeneity of the quality ladder are quantitatively important features of the market for higher education, shaping economic inequality, intergenerational persistence and aggregate income.

Our paper builds on the literature that models the admission and tuition decisions of colleges and the equilibrium of the higher education market. This literature has investigated the impact of financial aid policies (Epple, Romano, and Sieg, 2006; Fillmore, 2016), of a change in the supply of seats in public colleges (Fu, 2014) and affirmative action policies (Kapor, 2015) on the sorting of students, and the role of rising inequality on tuition Cai and Heathcote (2022). The main contribution relative

[5] Jovanovic (2014) studies an economy where long-term growth depends on the quality of assignment between workers and managers. In our model, (i) aggregate income depends on the quality of sorting of students to colleges qualities, which are a (endogenously-determined) bundle of teaching expenditures and students mean ability; (ii) students are heterogeneous in two dimensions (abilities and parental income) and not just in ability, and (iii) the source of the misallocation is a financial friction, not an exogenous noise in the assignment process.

to this literature is to adopt an intergenerational approach and to analyze the role of higher education in shaping inequality and mobility in the long-run. To the best of our knowledge, it is the first paper to embed the sorting of heterogeneous students across heterogeneous colleges into an intergenerational setting. An appealing property of our approach is that we analytically characterize the equilibrium allocation and how higher education shapes inequality and mobility.[6]

This paper also contributes to the literature that studies the determinants of tuition fees. Lucca, Nadauld, and Shen (2015) stresses the role of the expansion of credit supply, Gordon and Hedlund (2017) the importance of financial aid and Jones and Yang (2016) the rising cost of universities input and professors—the Baumol's disease, a mechanism our model accommodates. We stress the role of the increase in the returns to education to explain the rise in the average and the dispersion of tuition fees, due to the increase in demand by households for higher quality of higher education, especially at the top of the distribution. This mechanism is akin to the revenue theory of cost by Bowen (1980), whereby universities raise as much tuition fees as they can and then spend it on projects that enhance quality. Martin, Hill, and Waters (2017) estimate that this mechanism accounts for two third of the increase in tuition fees.[7]

The rest of the paper is organized as follows. Section 2 presents the model and section 3 analyzes the properties of its equilibrium. Section 4 generalizes the model to include public transfers to colleges and public and institutional financial aid to students. Section 5 derives an important analytical comparative statics: an increase in the return to human capital generates facts (a) to (e). Section 6 extends the model,

[6]This has three advantages: i) one can analyze in a transparent manner how technology and policy parameters shape the sorting of students, inequality and mobility, ii) the identification of structural parameters in the estimation is very transparent and iii) existence and uniqueness properties of the equilibrium—two issues that have plagued the theoretical and quantitative literature on clubs—can be characterized.

[7]A large reduced-form literature provides evidence on the returns to college quality and selectivity. Most papers find significant returns on the labor and marriage markets as well as for children' achievements (Black and Smith, 2006; Long, 2010; Hoekstra, 2009; Zimmerman, 2014; Bleemer, 2019). Another group of papers have cast doubt on these findings and the debate is still on-going (Dale and Krueger, 2011; Hickman and Mountjoy, 2019). Our results suggest moderate amplification effects of higher education. Another literature has shown that parental background matters a lot for achievements and access in top colleges (Bailey and Dynarski, 2011; Chetty, Friedman, Saez, Turner, and Yagan, 2020; Hoxby and Turner, 2019) and that financial aid policy has a significant impact on college decisions (Dynarski, 2003; Angrist, Autor, Hudson, and Pallais, 2016).

explains the estimation procedure and derives the quantitative results. Section 7 concludes.

2. Human Capital Transmission with a Hierarchy of Colleges

The economy is populated by two types of agents: dynastic households and colleges. At each generation, households imperfectly transmit human capital to their child and decide which college to send them to after high school. Colleges choose their pool of students as well as educational spending to maximize the quality they deliver. For clarity, this section highlights the properties of the *laissez-faire* economy. We introduce taxes, transfers and financial aid in section 4.

2.1. Households

There is a continuum of dynasties, indexed by $i \in \mathscr{I}$. Individuals live for two periods: one as a child and one as an adult. Each adult has one child. A generation $t \in \mathbb{N}$ household of dynasty i is characterized by its level of human capital h_{it} and the child's human capital at the end of high school z_{it}. They choose consumption c_{it}, labor supply ℓ_{it} and college quality q_{it} for their child. When no confusion results, we drop the generation and dynasty subscripts and denote the state variables of the next generation with a prime. The current generation value $\mathscr{U}(h, z)$ is solution to

$$\mathscr{U}(h, z) = \max_{c,\ell,q} \left\{ [\ln c - \ell^{\eta}] + \beta E\left[\mathscr{U}(h', z')\right] \right\} \tag{1}$$

where β denotes the intergenerational discount factor. A child's human capital at the end of high school is modeled as a log-linear combination of parents' human capital h and the birth shock ξ_z, capturing the randomness of the transmission process:

$$z = (\xi_z h)^{\alpha_z}. \tag{2}$$

Households are subject to a lifetime budget constraint. Their income y is a function of their level of human capital h and their supply of raw labor ℓ:

$$y = h^{\lambda} \ell \tag{3}$$

where λ parametrizes the elasticity of output to human capital. We will refer to it as the "returns to human capital." It plays an important role in the rest of the paper. We argue in section 5 that an increase in λ is able to rationalize the trends observed in higher education and explained in introduction.[8]

Their income can be spent on consumption and on tuition fees. The tuition schedule $e(q, y, z)$ is an equilibrium object which depends on college quality q, household income y and the child ability z. Normalizing the price of the final good to one, it is given by

$$y = c + e(q, z, y) \tag{4}$$

This budget constraint implies that households face an intergenerational borrowing constraint, *i.e.* the current adults cannot leave bequest or pass-on debt along to their offspring. This assumption draws on a large set of evidence that borrowing constraints do matter for college choices. Lochner and Monge-Naranjo (2012) review the evidence on borrowing constraint in education. Although this specification rules out net financial transfers across generation, the quantitative version we introduce later partially relaxes this assumption.[9]

The adulthood human capital of the child after college is a log-linear combination of its pre-college ability, the quality of the college they went to and a labor market shock ξ_y.[10] It is given by

$$h' = zq^{\alpha_q}\xi_y. \tag{5}$$

There are two sources of randomness in the accumulation process of human capital. The birth shock ξ_z is known before the college quality decision has to be made and the labor market shock ξ_y is realized once the child enters the labor market. Both

[8] Although simple, this functional form is also the reduced-form expression of a more sophisticated production function with physical capital and/or the payoff to a household involved in an aggregate production process with some degree of complementarity across heterogeneous tasks.

[9] It rules out net financial transfers across generations but not gross transfers. For example, children are allowed to borrow from their parents early in life and repay them later. Similarly, it doesn't rule out student loans as long as they are exactly offset by a parental transfer of the same amount.

[10] There is empirical evidence that the law of accumulation of human capital is characterized by complementarities between pre-college ability and college quality. Dillon and Smith (2018) finds evidence of such complementarities for long-term earnings. Lee and Seshadri (2019) estimate that the elasticity of substitution across periods of the human capital accumulation process is one, which amounts to a Cobb-Douglas functional form.

shocks are i.i.d across generations and households and log-normally distributed:[11]

$$\ln \xi_z \sim \text{i.i.d.}\mathcal{N}\left(-\sigma_z^2/2, \sigma_z^2\right) \tag{6}$$

$$\ln \xi_y \sim \text{i.i.d.}\mathcal{N}\left(-\sigma_y^2/2, \sigma_y^2\right). \tag{7}$$

2.2. Colleges

Technology. There is a mass one of *ex-ante* identical colleges indexed by $j \in [0,1]$. They are all of a fixed size.[12] A college is a technology that delivers to its students a quality q_j that depends on educational services per student I_j and the average of student ability $\bar{z}_j$, which will be referred to as the "peer effect." Furthermore, we assume that quality depends negatively on the degree of dispersion of abilities and parental income within the college, σ_u^2, which we define later. The production function of quality is given by

$$\ln q_j = \ln I_j^{\omega_I} \bar{z}_j^{\omega_z} - \sigma_{u,j}^2 \tag{8}$$

where $\omega_I, \omega_z > 0$.

Colleges are clubs because who belongs to the college matters for the quality delivered to all members, through $\bar{z}_j$. There is empirical evidence that peers enter the production function of college quality. For example, Sacerdote (2011), Smith and Stange (2016) and Mehta, Stinebrickner, and Stinebrickner (2018) find evidence of peer effets, especially from roommates, for achievements while in college. Zimmerman (2019) finds evidence that the network and social capital built while in college matters for labor market outcomes. Peer effects are also supported by the fact that colleges compete for the best students (Hoxby, 2009, 2013).

We make two assumptions about the negative impact of student heterogeneity on quality. First we assume that the peer effects are a geometric average of student

[11] This formulation of the household problem draws from and extends Benabou (2002) in several dimensions: we allow for heterogeneous colleges, a wide range of policies in higher education and allow for a birth shock.

[12] As we explained in Appendix A.7, it is natural to set the size of a college to the cardinality of the continuum $\mathfrak{c} = \aleph_1$ as the paper analyzes equilibria in which there is a continuum of heterogeneous students in each college.

abilities which therefore punishes heterogeneity relative to an arithmetic average:

$$\ln \bar{z}_j = E_{\phi_j(.)}[\ln(z)]$$

where $\phi_j(.)$ denotes the distribution of student abilities within college j. Secondly, through $\sigma_{u,j}$, we explicitly assume that the more heterogeneous the class in terms of student ability and economic background the more difficult it is for a college to deliver a given quality to its students. This is supported by empirical evidence showing that classroom heterogeneity makes peer interactions and teaching harder (Figlio and Page, 2002; Duflo et al., 2011). We define $\sigma^2_{u,j}$ as the within-college variance of a weighted average of (log) ability and parental background, $\sigma^2_{u,j} = \frac{\omega_I}{2} V_{\phi(.)}\left(\log z^{\frac{\omega_z}{\omega_I}} y^{-\frac{\omega_y}{\omega_I}}\right)$. Defining $\sigma^2_{u,j}$ in this manner ensures tractability by making $I_j \times e^{-\sigma^2_{u,j}}$ a geometric average of tuition fees. The solution to this problem would therefore be the same if colleges maximized a weighted geometric average of tuition and student ability.[13]

Educational services I_j are financed through the collection of tuition fees from all students. The static budget constraint of a college is

$$I_j = E_{\phi_j(.)}[e(q, z, y)].$$

Objective and Problem. Colleges seek to maximize the quality q_j they deliver to their students. This objective follows the literature that studies the behavior of universities (Epple, Romano, and Sieg, 2006; Fu, 2014). We assume that colleges are competitive and take the equilibrium tuition schedule as given as in Cai and Heathcote (2022). Like this paper, we assume a constant returns to scale technology which implies that the size of a college is irrelevant.

The college's problem can be decomposed into two stages. In the second stage, given a quality q, a college chooses the amount of educational services per student I_j and the composition of the student body $\phi_j(z, y)$—a density over (z, y), which determines the average student ability $\bar{z}_j$. In the first stage, a college chooses its

[13]From this perspective, the college's problem has a flavor of Fu (2014), where colleges maximize a weighted average of average student ability and a quadratic function of net tuition.

quality q_j in a positioning game with other colleges. The problem of college j is:

$$\max_{I_j, \bar{z}_j, \phi_j} \mathscr{V}_j = q_j \tag{9}$$

$$\text{subject to:} \quad \ln q_j = \ln I_j^{\omega_I} \bar{z}_j^{\omega_z} - \sigma_u^2 \tag{10}$$

$$I_j = E_{\phi_j}[e(q, z, y)] \tag{11}$$

$$\ln \bar{z}_j = E_{\phi_j}[\ln(z)] \tag{12}$$

and subject to a positioning constraint which we introduce below.[14]

Entry and Positioning Game. At each period, before operating, colleges play a positioning game on the line of qualities. Taking the position of all other colleges as given, each college sequentially chooses which quality to offer, $q \in \mathbb{R}_+$. The order in which they choose is exogenous and denoted $o(j) : [0, 1] \to [0, 1]$. Since colleges are otherwise identical, the order is arbitrary and inconsequential. The payoff for operating a given quality is given by (9) and is assumed to be $\mathscr{V} = 0$ if the college is not operating.

A subgame perfect Nash equilibrium of this positioning game is a mapping from the set of colleges $j \in [0, 1]$ to the set of qualities $\mathbb{R}^+$ such that given the positioning of all other colleges, no college wants to change its position. In the equilibrium of this game, colleges choose their quality in descending order and the positioning constraint is thus given by $q \leq \chi_t^{-1}(o(j))$ where $\chi_t(q)$ is the equilibrium density of students across college qualities. In Appendix A.7 we give a game-theoretic formalization of the positioning game.

This structure for entry ensures that all positive qualities are offered in equilibrium. The assumption that all colleges must be of a fixed size ensures that colleges do not agglomerate at the highest quality level with each one of them operating with an

[14]This formulation for the college problem abstracts from several potentially relevant issues, such as the heterogeneity of tracks, colleges and fields of study within the same institution, the existence of congestion forces, and the choice of a size, and the existence of a fixed factor of production (*e.g.*, endowments) and imperfect information. Allowing for within-college heterogeneity is an interesting avenue for future research and one that would require detailed data about the exact peer-group of a student within a college as well as about expenditures by field of study. Regarding the issues of size, congestion and the existence of inelastic factors of production of higher education, they matter in the short-term but are less of a concern for the long-run which is the focus of this paper. Finally, imperfect information about the type of applicants (z, y) is also an important issue studied in prior work Fu (2014). We focus instead on another important source of inefficiency: the borrowing constraint.

infinitely small mass of students.[15]

2.3. Equilibrium

An equilibrium path is a sequence of tuition schedules $e_t(q, z, y)$, household's policy functions $c_t(h, z)$, $\ell_t(h, z)$, $q_t(h, z)$, colleges' policy functions $\phi_t(q, z, y), I_t(q)$, a distribution of human capital $f_t(h)$ and a distribution of students over college quality $\chi_t(q)$ such that i) given $e_t(q, z, y)$, the household's policy functions $c_t(h, z)$, $\ell_t(h, z)$, $q_t(h, z)$ are solution to (1), ii) given $e_t(q, z, y)$, the college's policy functions $\phi_t(q, z), I_t(q)$ are solution to (9), and the sorting of colleges along the quality line $\chi_t(q)$ is a subgame perfect Nash equilibrium of the positioning game, iii) the education markets clear so that the sorting of colleges across qualities $q_t(h, z)$ is consistent with the equilibrium distribution of students $\chi_t(q)$ and the final good market clears, iv) the evolution of the distribution of human capital, $f_t(h)$, is consistent with the intergenerational law of motion of human capital and the sorting rule, $q_t(h, z)$.

3. Properties of the Decentralized Equilibrium

This section analytically characterizes the equilibrium allocation and the law of motion of the distributions of human capital over generations. Equilibrium in the market for higher education features a hierarchy of colleges differing in education quality, with two-dimensional sorting of students by ability and family income. Sorting on parental income is not efficient and reflect the existence of the borrowing constraint. In turn these two dimensions of sorting——ability and income——shape economic inequality and intergenerational persistence.

3.1. Equilibrium Tuition Schedule

Consider a college that decides to supply quality q. It has to choose the optimal combination of inputs—educational services I and the distribution of students' quality

[15] Intuitively, if there were no lower bound to their size, all colleges would locate at the highest quality level and operate with virtually no students. In other words, all colleges would like to be Harvard but there is only one Harvard. One can see the positioning game with quality-maximizing colleges as the equivalent of the free-entry/non-profit condition with profit-maximizing colleges. A key difference, however, is that while free entry equalizes profits to zero for all colleges, in a subgame perfect Nash equilibrium of the positioning game in our setup, colleges receive heterogeneous payoffs if they offer different qualities.

consistent with q. Given the substitutability between educational resources and student ability, a college will trade off lower tuition for higher student ability. The first-order conditions with respect to the density over student types and to the level of spending in the college's problem reflect this trade-off. The following proposition gives the unique equilibrium tuition schedule that is compatible with all colleges being at an interior solution. It takes a log-linear form and, incidentally, implies that all colleges are indifferent between all student types.[16]

Proposition 3.1. *The equilibrium tuition schedule is given by*

$$e_t(q, z, y) = q^{\frac{1}{\omega_I}} z^{-\frac{\omega_z}{\omega_I}} \tag{13}$$

and all colleges are indifferent between all types.

Tuition fees are increasing in quality q and decreasing in student ability z with respective elasticities of $\frac{1}{\omega_I}, \frac{\omega_z}{\omega_I}$. These elasticities are intuitive. Colleges of higher quality need to finance higher expenses, hence require higher tuition. If educational spending are important for the production of college quality, ω_I is high hence $\frac{1}{\omega_I}$ is low, tuition will not be very elastic to quality, because a small increase in revenues implies a large increase in quality. The elasticity $-\frac{\omega_z}{\omega_I}$ captures the importance of the peer effet relative to educational spending: if peers significantly matter, colleges have strong incentives to subsidize high ability students to attract them. Finally tuition is independent of parental income, y, because colleges have no incentives to price-discriminate based on this characteristic.

3.2. Household Policy Functions

Given the equilibrium tuition schedule (13), households choose where to send their offspring. Since the tuition schedule is monotonic in q, this decision amounts to

[16] We construct an equilibrium in which the distribution of human capital is log-normal. A necessary and sufficient condition for this distribution to remain log-normal over generations is for the tuition schedule to be a log-linear function of college quality q, student ability z and parental income y. Given the assumptions laid out in the previous section, the unique tuition schedule compatible with the equilibrium conditions and colleges being in an interior solution is log-linear. These two restrictions—log-normality of human capital and interior solutions for colleges—ensure the tractability of the equilibrium expressions.

Although it is natural to focus on interior solutions, we cannot rule out the existence of other equilibria outside of this class. Looking at a more general class of equilibria is potentially interesting, but it is beyond the scope of this analysis and would defeat a key purpose of this paper, as all tractability would be lost.

choosing how much of their income to spend on higher education. Define the spending rate of a household of type (z, y) going to college of quality q: $s_t(q, z, y) = \frac{e_t(q,z,y)}{y}$.

An attractive feature of the class of models with unitary elasticity of intergenerational substitution, log-normal innovations and log-linear technologies is the possibility to obtain analytic expressions for the optimal spending rate and labor supply.[17] The following proposition characterizes the solution to the F.O.Cs associated with the households' problem.

Proposition 3.2. *Defining $U = \frac{\partial \ln \mathscr{U}}{\partial \ln h}$, the elasticity of the value function to human capital, one has that, in equilibrium, for all households, the households' spending rate, labor supply and marginal value of human capital U are given by:*

$$s_t = \frac{\beta \alpha_q \omega_I U_{t+1}}{1 + \beta \alpha_q \omega_I U_{t+1}} \tag{14}$$

$$\ell_t = \left[\frac{1}{\eta} \left(1 + \beta \alpha_q \omega_I U_{t+1} \right) \right]^{\frac{1}{\eta}} \tag{15}$$

$$\textit{with} \quad U_t = \sum_{k=0}^{\infty} \beta^k \lambda_{t+k} \prod_{m=0}^{k-1} \alpha_{h,t+m} \tag{16}$$

$$\textit{and} \quad \rho_t = \alpha_z + \alpha_q \left[\omega_z \alpha_z + \omega_I \lambda_t \right]$$

where ρ_t is IGE of human capital at generation t. The spending rate and labor supply are independent of the household type and depends positively on U_{t+1} which is also common to all households. The latter depends positively on all future ρ's, which is the IGE. The higher the future IGEs the more incentive the current generation has to invest in human capital and work. Importantly U_t—thus s_t—is also increasing in both the current and future returns to education—λ_t. It will play a key role in the dynamics of human capital afterwards.

3.3. Equilibrium Sorting Rule

By combining the equilibrium tuition schedule and the equilibrium positioning of colleges on the quality line—the "supply side"—with the household spending rule—the "demand side"—one obtains the equilibrium sorting rule, a mapping from the set of household and student types into the set of qualities of higher education.

[17]This paper draws on a long tradition that uses log preferences and lognormal distributions in dynastic frameworks to derive analytically tractable expressions, *e.g.* Glomm and Ravikumar (1992).

Proposition 3.3. *In equilibrium, the sorting rule is given by*

$$q_t(y, z) = (s_t y)^{\omega_I} \, z^{\omega_z} \tag{17}$$

Equation (17) tells us which quality of higher education a student from family background y and ability z gets. The elasticity of quality to income and ability capture the strength of what we call, respectively, the income-sorting and ability-sorting channel. The income-sorting channel captures the fact that richer households are able to buy a higher quality of college. This income-sorting channel arises because of the borrowing constraint: in the absence of constraint, poor students could freely borrow and parental income would not matter for sorting. For parental income to matter, it also has to be the case that colleges need financial resources and that they are ready to trade-off these resources for ability. The ability-sorting channel captures the desire of colleges to attract high ability students because of the peer effect.

3.4. Law of Motion of Human Capital

Having described the static equilibrium conditions, we now derive the law of motion for the distribution of human capital. Since the first two moments of this distribution are the only aggregate states, it also describes the dynamics of the aggregate economy. We start with the law of motion of human capital at the individual level.

Intergenerational Transmission of Status. Plugging the expression for the equilibrium sorting rule (17) into the law of accumulation of human capital (5) and gathering all terms in $\ln h$ gives the following intergenerational law of motion of human capital: $\ln h_{t+1} = \rho_t \ln h_t + \ln \xi_y + (\alpha_z + \alpha_q \omega_z) \ln \xi_z + \alpha_q \omega_I \ln (s_t \ell)$ with $\alpha_{h,t}$ the IGE.

The IGE is a linear combination of the before and during college transmission of human capital. This paper focuses on and opens the box of the transmission of economic status through college. The transmission during college decomposes itself into the two sub-channels introduced in the previous paragraph: the income-sorting channel that emphasizes the role of parental income and the ability-sorting channel that emphasizes the role of ability in the sorting of students across the ladder of college quality.

$$\rho_t = \underbrace{\alpha_z}_{\text{Before College}} + \underbrace{\alpha_q(\underbrace{\alpha_z\omega_z}_{\text{Ability-Sorting Channel}} + \underbrace{\omega_I\lambda_t}_{\text{Income-Sorting Channel}})}_{\text{College}}$$

Aggregate Law of Motion of Human Capital. Using the assumption of log-normality of both shocks, (6) and (7), if the economy starts from a log-normal distribution then human capital stays log-normally distributed along the equilibrium path:

Proposition 3.4. *If* $\ln h_t \sim \mathscr{N}\left(m_{h,t}, \Sigma_{h,t}^2\right)$ *then*

$$\ln h_{t+1} \sim \mathscr{N}\left(m_{h,t+1}, \Sigma_{h,t+1}^2\right) \tag{18}$$

$$m_{h,t+1} = \rho_t m_{h,t} + X_{1,t} \tag{19}$$

$$\Sigma_{h,t+1}^2 = \alpha_{h,t}^2 \Sigma_{h,t}^2 + X_2 \tag{20}$$

$$\textit{where} \quad \rho_t = \alpha_z + \alpha_z\alpha_q\omega_z + \alpha_q\omega_I\lambda_t$$

$$X_{1,t} = -\frac{\sigma_y^2}{2} - \alpha_z\left(\alpha_q\omega_z + 1\right)\frac{\sigma_z^2}{2} + \alpha_q\omega_I \ln\left(\ell_t s_t\right)$$

$$X_2 = \sigma_y^2 + \left(\alpha_z[1+\alpha_q\omega_z]\right)^2\sigma_z^2.$$

It is intuitive that the shifter in the law of motion of the mean of the distribution (19) is increasing in the spending rate s_t and labor supply ℓ_t. The law of motion of the variance (20) is the mathematical expression of the Great Gatsby curve: the positive relationship between the level of inequality Σ_h and the strength of the intergenerational transmission of status, ρ.

3.5. Distribution of Students along the Quality Ladder and Within-College Distribution of Students

Recall facts (a) and (b) noted in introduction: the dispersion of expenditures per student across colleges has increased and the share of low-income students at top colleges has stagnated. One can actually derive analytical expressions for the distribution of students across college qualities (and the implied distribution of expenditures) and for the within-college distributions of parental income and student ability. These closed-form solutions enable us to shed light on the forces that determine these two objects and will prove useful for the derivation of comparative statics in the next section. These three distributions are log-normal and their first and second moments depend on the aggregate states, directly and indirectly through the income-sorting and ability-sorting elasticities,

$$\theta_{I,t} = \omega_I \lambda_t \quad \text{and} \quad \theta_A = \omega_z \alpha_z.$$

As the following proposition establishes, the dispersion of qualities is an increasing function of both of these variables. But the dispersion of parental income and ability within a college is a function of their ratio. The former is increasing with the ratio θ_A/θ_I while the latter is decreasing: the more students sort into colleges based on parental income, the less economic diversity there is in a college and the more students sort into colleges based on abilities, the lower the dispersion of abilities.

Proposition 3.5. *1. The distribution of college quality is given by*

$$\ln q \sim \mathscr{N}\left(\mu_{1,t}(m_{h,t}, \Sigma_{h,t}), \sigma^2_{1,t}(\Sigma_{h,t}, \theta_{I,t}, \theta_A)\right)$$

with $\sigma_{1,t}$ increasing in θ_A, $\theta_{I,t}$ and $\Sigma_{h,t}$.

2. Within a college of quality q, the distribution of parents' (log) human capital is:

$$\ln h|q \sim \mathscr{N}\left(\mu_{2,t}(m_{h,t}, \Sigma_{h,t}), \sigma^2_{2,t}(\Sigma_{h,t}, \theta_{I,t}, \theta_A)\right)$$

with $\mu_{2,t}$ increasing in q; $\sigma_{2,t}$ increasing in θ_A and $\Sigma_{h,t}$ and decreasing in $\theta_{I,t}$.

3. Within a college of quality q, the distribution of students' (log) abilities is:

$$\ln z|q \sim\sim \mathscr{N}\left(\mu_{3,t}(m_{h,t}, \Sigma_{h,t}), \sigma^2_{3,t}(\Sigma_{h,t}, \theta_{I,t}, \theta_A)\right)$$

with $\mu_{3,t}$ increasing in q; $\sigma_{3,t}$ increasing in $\theta_{I,t}$ and $\Sigma_{h,t}$ but decreasing in θ_A.

4. Taxes, Transfers and Financial Aid in Higher Education

In this section, we introduce a government which implements non-linear transfers of income across households and provides merit and need-based financial aid to students as well as subsidies to colleges. We also allow colleges to provide need-based aid by assuming they have a social objective. We use log-linear tax and transfer schedules as introduced by Persson (1983) and Benabou (2002) and estimated more recently by Heathcote, Storesletten, and Violante (2017). They fit well the empirical schedules and they preserve the tractability of the framework introduced in the previous section.

4.1. Government

The government implements four kind of taxes: two are specific to higher education (non-linear merit-based and need-based financial aid to college students and non-linear transfers to colleges) and two that are more standard (a linear consumption tax and a progressive income tax).

Progressive Income Tax Schedule. The household labor income is subject to a progressive tax schedule with a_y the average tax rate and τ_y its progressivity. The after-tax and transfers lifetime earnings is given by

$$y = (1 - a_y) y_m^{1-\tau_y} T_y \tag{21}$$

where T_y is a normalizing aggregate endogenous factor ensuring that a_y parametrizes the average income tax rate. The non-linear schedules for financial aid and the college subsidy are in the same spirit as this income tax schedule.

Merit and Need-Based Financial Aid. Financial aid is allowed to be progressive with income and merit-based with abilities:

$$e(q, z, y) = T_e z^{-\tau_m} y^{\tau_n} \frac{e_u(q, z, y)}{(1 + a_n)} \tag{22}$$

where $e(q, z, y)$ is the after financial aid net tuition faced by households, as specified in (4) and $e_u(q, z, y)$ is the before financial aid price, commonly referred to as the sticker price. τ_m is the rate of progressivity (or rather regressivity) of the merit-based subsidy, τ_n is the rate of progressivity of the need-based subsidy and T_e ensures that a_n is the average financial aid to students.

Need-based policies are interesting interventions, not only because they are implemented in practice, but also because they can help address the efficiency implied by the borrowing constraint faced by households. Recall that this friction implies that parental income shapes and worsens the sorting of students and resources across colleges. Need-based policies, by correcting this distortion, could thus improve efficiency. We investigate quantitatively whether progressive aid can improve the decentralized allocation in section 6.

4.2. College Need-based Aid and Social Objective

We also allow colleges to give need-based aid. This is an interesting policy to consider because private and public universities increasingly claim that they are making efforts to recruit low-income students. This is also interesting because institutional need-based aid could help correct the distortion implied by the borrowing constraint and thus improve efficiency, which we investigate quantitatively in section 6. To endogenize need-based aid by colleges, we assume that they have a social objective.[18] The social objective is modeled as follows. A college's payoff is increasing in the quality of higher education, as in the previous section, and decreasing in the (geometric) average of parental incomes, $\bar{y}_j$, and this penalty is parametrized by $\omega_y > 0$:[19]

$$\ln \mathscr{V}_j = \ln q_j - \omega_y \ln \bar{y}_j \tag{23}$$

where $\bar{y}_j$ is the geometric average parental income of students:

$$\ln \bar{y}_j = E_{\phi_j(.)}[\ln(y)] \tag{24}$$

A college maximizes (23) subject to the technology for quality (10), the definition of the peer effet (12), the after-subsidy budget constraint (25) and the definition of average parental income (24).

Transfers to Colleges. Financial transfers to colleges by states and the federal government are large and highly progressive, in the sense that colleges that spend less per student receive relatively more subsidies, as is documented in a companion paper Capelle (2019). This progressivity is closely related to the location of public and private colleges in the distribution of quality. Other papers modeling the higher

[18]Colleges could give need-based to students not because of a social objective but because of parents with higher income are less elastic to prices and therefore higher mark-up, as in Epple, Romano, and Sieg (2006). Colleges do not discriminate by parental income in Cai and Heathcote (2022).

[19]There is no *a priori* restrictions on ω_y. But it will become clear in the next paragraphs that to rationalize the strong sorting on parental income, it cannot be too large.

education sector differentiate between public and private colleges. In contrast, we do not specify any *ex ante* differences across colleges.[20] In our model, the bottom and middle of the distribution of qualities, *i.e.* the colleges that receive relatively more transfers from the government, can be interpreted as public colleges. This way of modeling government transfers allows me to keep the model tractable while capturing most of the heterogeneity in government transfers along the quality distribution. Taking into account these transfers, the budget constraint of a college is:

$$I = T_u(1 + a_u) \left(E_{z,y}[e_u(q, z, y)]\right)^{1-\tau_u} \tag{25}$$

where τ_u is the degree of progressivity of subsidies to universities and T_u ensures that a_u is the average amount of transfers per student received by colleges. The budget constraint presented in the college problem, (11), is the special case when $\tau_u = 0$. We show in Capelle (2019) that the functional form assumption in (25) is a good approximation of the data.

Government Budget Constraints. There are two kinds of constraints. The first one is the aggregate budget constraint that states that revenues (income tax and consumption tax) must equal spending (transfers to colleges and students) at any period. The other three constraints pin down T_u, T_y, T_e such that a_y, a_n, a_u are respectively the average rate of income tax, financial aid and transfers to college. We give more details in Appendix A.3.1.

4.3. Properties of the Decentralized Equilibrium

Equilibrium Tuition Schedule. In this generalized framework, the log-linear form of the tuition schedule is preserved.

[20] It is unclear what differentiate public colleges' objectives and constraints from non-profit private ones beyond the fact that the former receive public subsidies but not the latter. One common additional assumption in the literature is that tuition fees at public universities are subject to specific constraints. For example, Epple, Romano, and Sieg (2006) and Cai and Heathcote (2022) assume that tuition fees at public colleges are exogenous, which corresponds to the notion that tuition fees are fixed by States' legislatures. But decentralization policies have given public colleges significant autonomy in their tuition and hiring policies (Mc Guinness, 2011). For-profit colleges do display different behavior, but they make up a very small part of total enrollment.

Proposition 4.1. *The equilibrium before-financial-aid tuition schedule is given by*

$$e_{u,t}(q,z,y) = \left(\frac{1}{(1+a_{u,t})T_{u,t}} q^{\frac{1}{\varepsilon_{I,t}}} z^{-\frac{\varepsilon_{z,t}}{\varepsilon_{I,t}}} \left(\frac{y}{\kappa_t}\right)^{\frac{\varepsilon_{y,t}}{\varepsilon_{I,t}}} \right)^{\frac{1}{1-\tau_{u,t}}} \tag{26}$$

$$\text{where} \quad \varepsilon_{l,t} = \frac{\omega_l}{1-\nu_t \omega_y} \quad \forall\, l = I, z, y$$

with ν_t *the endogenous elasticity of mean parental income within a college to quality*

$$\bar{y}_t(q) = \kappa_t q^{\nu_t}$$

and all colleges are indifferent between all types.

There are three new elements relative to the previous section. First, tuition are increasing in parental income because of the social objective. The elasticity $\frac{\varepsilon_y}{\varepsilon_I(1-\tau_u)} = \frac{\omega_y}{\omega_I(1-\tau_u)}$, depends on the strength of the social objective: the larger ω_y, the more progressive tuition fees are.[21] Secondly, tuition decreases with the average subsidies to colleges a_u, and the elasticity of tuition fees with quality is increasing in the degree of progressivity of the college subsidy schedule, τ_u.

Thirdly, the elasticities of tuition with respect to quality, student ability and parental income—$\varepsilon_{I,t}, \varepsilon_{z,t}, \varepsilon_{y,t}$—are equilibrium objects that depend on current aggregate states, in particular the dispersion of human capital in the economy Σ_h, and the policy parameters. Mathematical expressions for ν_t and κ_t are given in Appendix A.4.1. κ_t depends not only on current states but also on all future states through the labor supply decision ℓ_t. The notation $\nu_t(\Sigma_{h,t})$ makes explicit that the elasticity of mean parental income to quality depends on the dispersion of human capital in the economy. As we show in Appendix A.9, it is increasing in the latter. It also depends on the current policy parameters and λ the returns to human capital. Note that when colleges have no social objective, $\omega_y = 0$, then $\varepsilon_l = \omega_l$ for all $l = I, z, y$ and the ε's are independent of the state of the economy.

The term $\frac{1}{1-\nu_t\omega_3}$ that transforms ω_I into ε_I reflects the cross-subsidization from high-income to low-income families within a college implied by the social objective. Tuition fees for a family with a given income y increase with a lower elasticity with respect to quality when colleges have a social objective. This family becomes poorer

[21]The equilibrium tuition function turns out to be similar to the one in Epple, Romano, and Sieg (2006). While the progressivity of tuition fees with parental income originates from market power in their framework, it comes from the social objective in this paper.

and poorer relative to the within-college mean parental income as one climbs the college quality ladder (since parental income increases in equilibrium with quality). This effect is all the more pronounced as the social objective parameter ω_y and the equilibrium elasticity of parental income to college quality ν are large.[22]

Household Policy Functions and Sorting Rule For conciseness, and because it is very similar to its expression in the previous section, the equilibrium spending rate of households is given by equation (39) in Appendix A.1. Combining the household decision with the equilibrium tuition schedule gives the equilibrium sorting rule.

Proposition 4.2. *In equilibrium, the sorting rule is given by*

$$q_t = \left(\frac{s_t y_t^{1-\tau_{n,t}} z_t^{\tau_{m,t}} (1 + a_{h,t})}{T_{e,t}} \right)^{\varepsilon_{I,t}(1-\tau_{u,t})} ((1 + a_{u,t}) T_{u,t})^{\varepsilon_{I,t}} z_t^{\varepsilon_{z,t}} \left(\frac{y_t}{\kappa_t} \right)^{-\varepsilon_{y,t}} \tag{27}$$

The elasticity of quality to income, $\varepsilon_{I,t}(1-\tau_u)(1-\tau_n) - \varepsilon_{y,t}$, which capture the strength of the income-sorting and the elasticity of quality to ability, $\varepsilon_{z,t} + \tau_m(1-\tau_u)\varepsilon_{I,t}$, which capture the strength of the ability-sorting channel now captures the progressivity of taxes and financial aid.

Relative to the framework without government intervention and a social objective for college, the income-sorting channel is tempered by government subsidies to colleges that are progressive with slope $(1-\tau_u)$, by need-based financial aid that are progressive with slope τ_n, financial aid by colleges that is progressive with slope ω_y. In theory, this elasticity, ε_I, could be negative, if the social objective parameter, ω_y, was large enough, such that $\varepsilon_{I,t}(1-\tau_u)(1-\tau_n) < \varepsilon_{y,t}$. The elasticity with respect to ability—associated with the ability-sorting channel—is amplified by the merit-based subsidy of the government, τ_m.

Intergenerational Transmission of Status. The IGE is now given by

$$\tilde{\rho}_t = \underbrace{\alpha_z}_{\text{Before College}} + \underbrace{\alpha_q(\underbrace{\alpha_z(\varepsilon_{z,t} + \varepsilon_{I,t}(1-\tau_{u,t})\tau_{m,t})}_{\text{Ability-Sorting Channel}} + \underbrace{(\varepsilon_{I,t}(1-\tau_{u,t})(1-\tau_{n,t}) - \varepsilon_{y,t})(1-\tau_{y,t})\lambda_t}_{\text{Income-Sorting Channel}})}_{\text{College}}$$

[22] If inequality increases for exogenous reasons—as will be the case in our comparative statics with respect to the returns to education λ—the endogenous increase in ν provides a partial mitigating force by making colleges willing to endogenously redistribute more across students, provided $\omega_y > 0$.

Law of Motion of Distribution of Human Capital. Like in the simpler version of the model, human capital remains log-normally distributed over the equilibrium path:

Proposition 4.3. *If* $\ln h_t \sim \mathscr{N}\left(m_{h,t}, \Sigma_{h,t}^2\right)$ *then*

$$\ln h_{t+1} \sim \mathscr{N}\left(m_{h,t+1}, \Sigma_{h,t+1}^2\right) \tag{28}$$

$$m_{h,t+1} = \rho_t m_{h,t} + X_1 \tag{29}$$

$$\Sigma_{h,t+1}^2 = \tilde{\rho}_t^2 \Sigma_{h,t}^2 + X_{2,t} \tag{30}$$

where ρ_t *has the same expression as in the previous section. Expressions for* X_1 *and* $X_{2,t}$ *can be found in Appendix A.5.*

In general the expression (29) is not a linear recursive formulation for the law of motion of m_h because s and ℓ are forward looking variables that depend on all the future Σ_h's via the ε's. In contrast, the law of motion of Σ_h, given by (30), is still recursive—although in general not linear since both the autoregressive coefficient and the shifter depend on Σ_h. The full system, (29) and (30), is therefore block-recursive which allows us to characterize the existence and uniqueness of the equilibrium path after the exposition of the government budget constraints.

Government Budget, Educational Sector and Market Clearing The aggregate government budget constraint (44) imposes, in all periods, a restriction on the path of the consumption tax rate $a_{c,t}$ given an exogenous path of income tax $a_{y,t}$, higher education subsidies $a_{h,t}, a_{u,t}$ and endogenous spending rate s_t. Analytical expressions for this constraint as well as for equations (45),(46) and (47) defining respectively $T_{y,t}, T_{e,t}$ and $T_{u,t}$ are derived in Appendix A.3.1.

Existence and Uniqueness of the Equilibrium Path Existence and uniqueness of the steady-state and of the macroeconomic equilibrium path are slightly harder to obtain than in the previous section. Although existence and local stability is obtained under an intuitive sufficient condition, a sufficient condition for global stability is that ω_y be small enough.

Proposition 4.4. *Existence and Uniqueness of Equilibrium Path*

- *If* $\lim_{\Sigma_h \to \infty} \tilde{\rho} < 1$*, there exists at least one locally stable steady-state.*

- *For ω_y small enough, there exists a unique globally stable steady-state and a unique equilibrium path.*

with $\lim_{\Sigma_h^2 \to \infty} \tilde{\rho} = \alpha_z + \alpha_z \alpha_q(\omega_z + \tau_m(1-\tau_u)\omega_I) + \alpha_q[\omega_I(1-\tau_u)(1-\tau_n) - \omega_y](1-\tau_y)\lambda$

A high ω_y might lead to multiple equilibria by making inequality Σ_h potentially grow too fast in some parts of the state-space, *i.e.* by making the derivative of the right-hand-side of (30) higher than 1, thus failing to meet the crucial defining feature of a contraction mapping. This stems from the fact that ν is increasing in Σ_h, hence that ε_l for $l = I, z, y$, ρ and X_2 are increasing in Σ_h.

5. Rationalizing Trends in Higher Education

The previous sections highlighted important cross-sectional features of the market for higher education, including the two dimensional sorting of students across heterogeneous colleges by ability and parental income, and the role of higher education in shaping economic mobility and inequality. This section shows analytically that the model is able to replicate trends affecting higher education, intergenerational mobility and inequality observed in the past decades. More specifically, we show that an increase in the market returns to education λ generates (a) the increase in the dispersion of expenditures per students across colleges (Capelle, 2019); (b) the stagnation of the share of students from the lowest income quintile in top colleges despite the increase in financial aid (Bailey and Dynarski, 2011; Chetty, Friedman, Saez, Turner, and Yagan, 2020); (c) the increase in real terms of tuition fees before and after financial aid; (d) the slight increase in the intergenerational elasticity of income mobility (Davis and Mazumder, 2017); and (e) the increase in income inequality (Piketty and Saez, 2003; Autor, Katz, and Kearney, 2008). It is natural to focus on the increase in the returns to education as it is widely recognized to be one of the main sources of the increase in inequality (Katz and Murphy, 1992; Autor, Katz, and Kearney, 2008).[23] Proposition 5.1 formally states the key comparative static result.

[23]We do not take a stand on the exact source of increase in the returns to human capital. Many factors have contributed to this rise: a skill-biased technological change, capital-skill complementarity, an improvement in the assortative matching of workers and firms, an increase in assortative mating and in the number of single households and an increase in the substitutability across skills due to international trade or due to better communication technology.

Proposition 5.1. *Assume the economy starts from a steady-state at $t = 0$. Consider a weakly increasing sequence $\{\lambda_t\}_0^{+\infty}$. If $\omega_I(1-\tau_n)(1-\tau_u) > \omega_y$, along the equilibrium path,*

a) The Gini coefficient of colleges' (log) expenditures per student (and quality) increase.

b) The ratio of variance of (log) income within a college over variance of (log) income in economy decreases.

c) The average expenditure for college as a share of income increases.

d) The intergenerational elasticity increases.

e) The Gini coefficient of human capital and income increase.

The formal proof of this proposition is contained in Appendix A.9. Here we present intuition for the stated effects. Intuitively, when the returns to human capital, λ, increase, the dispersion of households' income rises for a given distribution of human capital [fact (e)]. Given that households all spend the same share of their income for the higher education of their child, it implies an increase in the dispersion of desires to pay for college. Following this change on the demand side of the higher education market, colleges react: top colleges take advantage of the rising willingness to pay of their pool of students by increasing their fees and their spending relative to colleges at the bottom. Inequality of revenues and spending across colleges rise [fact (a)].

Poor but high ability students get priced out of top colleges for two reasons. First tuition fees at top colleges have increased relative to lower ranked colleges. Second their parents' income have decreased relative to the average parental income. More generally this rise in the dispersion of tuition for colleges implies that parental income matters even more to access a higher quality college than it used to, relative to ability. It corresponds to an increase in the elasticity of college quality to income, ε_I, what we described earlier as a strengthening of the income-sorting channel.

Consequently, top colleges become less diverse in terms of economic background because poor students are priced out and students from rich families are able buy their way to the top. More generally, colleges become more segregated and homogeneous in terms of parental income [fact (b)]. Another implication is that intergenerational mobility decreases, as parental income becomes increasingly determinant for the opportunities of children [fact (d)]. This is a direct manifestation of the Great Gatsby curve (Corak, 2013), whereby an increase in income inequality leads to a strengthening of the transmission of economic status, here through access to better quality higher education, which feeds back into higher inequality.

Over time, the initial increase in inequality gets amplified through the higher education system. Students from richer backgrounds get relatively higher quality of higher education, which increases the dispersion of human capital and therefore of income once their generation become adults. The shock propagates over generations as this increased dispersion of human capital translates into a higher dispersion of children abilities which gets amplified by the increasingly unequal distribution of college quality.

The amplification of the initial increase in the returns to human capital, λ, through colleges happens through two channels: the reallocation of resources and the reallocation of students. As we have argued above, financial resources and expenditures become increasingly concentrated at the top of the college distribution. In contrast, high ability students become slightly less concentrated at the top of the college ladder, partially mitigating the amplification.

Why do colleges accommodate the increased dispersion in desires to pay for colleges? They are led to do so by their desire to maximize the quality they provide, despite their social objective. Even if an individual college at the top of the distribution didn't raise its tuition fees relative to, say, the median college, another college would fill up this gap, offering higher quality for higher tuition fees. This mechanism is akin to the revenue theory of cost by Bowen (1980), but now applied to a hierarchy of colleges.[24]

Finally, average tuition fees and the share of total income devoted to higher education increase because higher returns to human capital gives stronger incentives to households to accumulate human capital which drives their demand for higher education up [fact (d)]. It is therefore the same demand-driven mechanism that drives both the average increase in tuition and the rise in inequality across colleges.

[24]Bowen summarizes his theory page 19:1) The dominant goals of institutions are educational excellence, prestige, and influence. 2) In quest of excellence, prestige, and influence, there is virtually no limit to the amount of money an institution could spend for seemingly fruitful educational ends. 3) Each institution raises all the money it can. 4) Each institution spends all it raises. 5) The cumulative effect of the preceding four laws is toward ever increasing expenditure.

6. The Role of Higher Education: Quantitative Results

In the previous sections we developed a tractable model of human capital accumulation with a ladder of colleges which allows for a sharp analytical characterization of the equilibrium sorting of students across colleges, intergenerational mobility, income inequality and aggregate production. We now relax some of the assumptions and extend the model to a richer quantitative environment. We then explain how we estimate this richer model. We use the calibrated model to assess the quantitative relevance of the sorting of students and resources across heterogeneous colleges in shaping intergenerational mobility, inequality and the efficiency of the accumulation of human capital. We then quantify the implications of existing federal need-based student aid. Finally, we solve for the optimal degree of progressivity of federal and institutional need-based aid.

6.1. Quantitative Extension

We extend the model in three dimensions. First the restrictions on intergenerational financial transfers are partially relaxed: positive transfers as well as negative transfers (student debt minus parental transfers) are allowed up to a limit:

$$\mathscr{U}(h, z, a) = \max_{c,\ell,q,a'} \{\ln c - \ell^\eta + \beta E(\mathscr{U}(h', z', a'))\} \tag{31}$$

$$y + (1+r)a = c(1+a_c) + e(q, z, y) + a' \tag{32}$$

$$a' \geq \underline{a} \tag{33}$$

where r denotes the interest rate and $\underline{a}$ is the exogenous borrowing limit.

Households therefore face a portfolio problem: they have to decide upon the optimal combination of bequest and higher education for their offspring. High ability children from a poor background will take up loans and rich families with low ability children will choose to transmit financial wealth instead of buying a high quality college for their kid. Overall, allowing for financial transfers should weaken the link between parental income and the child's position on the college ladder. In contrast with the previous sections, spending on tuition as a share of parental income will be heterogeneous across households and will be an increasing function of the child ability.

Secondly, there is an outside option delivering $\underline{q}$ for free:

$$e(\underline{q}, y, z) = 0 \quad \forall\, (z, y) \tag{34}$$

Some individuals will find it optimal not to go to college and take up the free outside option. This gives rise to a meaningful enrollment decision that was absent from the previous framework where all individuals got at least some arbitrarily low quality of higher education. A direct implication of equation (34) is that, if $\underline{q} > 0$, in equilibrium no individual ever chooses $q < \underline{q}$ and there is a Dirac peak at $\underline{q}$. It is natural to define the enrollment rate as the share of individuals with $q > \underline{q}$.

Finally, we generalize the law of accumulation of human capital given by (5) to allow for a direct transmission of abilities that is independent of ability and college quality and a non-unitary coefficient on z:

$$h' = z^{\zeta} q^{\alpha_q} h^{\alpha_y} \xi_y. \tag{35}$$

This more general law of motion is supported by and better matches the data.

The set of technological constraints faced by the household is otherwise similar to the original problem described in section 4. Formally, the problem of the household consists in maximizing (31) subject to (2)-(7) and the new constraints (32)-(35). The rest of the model remains the same.[25] The original problem is the special case when $a' = \underline{q} = 0$, and $\alpha_y = 0$.

In this version of the model, the households policy functions and the distributions lose their closed-form expressions because of the outside option which introduces a lower bound on the distribution of college qualities, or because of the financial transfer which makes the share of tuition in household expenditure a complex function of parental income and child ability. We show in appendix that the third extension preserves the closed-form expressions.

6.2. Data and Calibration

The core dataset is the restricted-use version of the NLSY-1997, a representative panel of individuals who were 12 to 17 years-old in 1997, whom we follow every year up to now. It features data on parental income, abilities measured by a common comprehensive

[25]In Appendix A.10, we explain how the problem of the colleges is kept tractable in this more general framework.

test-score, the Armed Services Vocational Aptitude Battery or ASVAB, a detailed description of their journey through the higher education system—each college they attended, the time spent and the degree obtained—and their labor earnings. We normalize the ability measure so that it follows a standard normal distribution.[26]

To estimate the parameters related to financial aid, we use the restricted-use NCES-NPSAS in 2000, which is the closest survey to the average year when individuals in the NLSY go to college. It is a representative survey of students that features detailed information about parental income, out-of-pocket college costs and financial aid disaggregated by source—federal government, state, private and institutional.

The publicly available NCES-IPEDS annual surveys provide college-level information on expenditures, revenues, enrollment and the distribution of test scores within each college. We use the 2000 to 2004 surveys. Finally we complement these data with statistics on enrollments from the NCES and measures of aggregate spending for higher education from the OECD.

External Calibration. Out of the 21 parameters to calibrate, we set 12 without solving the model. The list of externally calibrated parameters is given in the first column of Table 1.

From the OECD, we compute a_u by dividing the total amount of public subsidies by the total revenues before public aid. According to the specification for subsidies to university, τ_u can be estimated in a weighted least-square regression of (log) total revenues per student on (log) revenues before public transfers in the cross-section of colleges, where the weights are given by students enrollment. We run this regression in a companion paper Capelle (2019) and find $\tau_u = 0.35$ at the beginning of the 2000s.

The income tax schedule parameters a_y, τ_y are informed by the average income tax rate and the slope of the income tax schedule estimated by Heathcote, Storesletten, and Violante (2017).[27] We calibrate a_y using average income tax rate provided by the CBO:[28] $a_y = 0.2$. In a companion paper, we estimate the average per-student state transfers to college, a_u, and the degree of progressivity of these transfers τ_u (Capelle, 2019).

[26]We can normalize the ability measure without loss of generality because test score is a choice of the test designer.

[27]In order to calibrate τ_y, we take an average between the value estimated by Heathcote, Storesletten, and Violante (2017) (.2) and the ones needed to match the ratio between the market income and after tax and transfers Gini in the U.S in 2000 (.26), which gives $\tau_y = 0.23$.

[28]See CBO.

In the NCES-NPSAS dataset, one observes parental income $y_{m,i}$, test score, institutional aid, government aid as well as out of pocket payment. Regressing the (log) ratio of after-government aid payment on before-government aid payment over parental income and student ability gives τ_n and τ_m. We use the average financial aid received by students from their state and the federal government to calibrate a_n. To estimate the progressivity of institutional financial aid ω_y—"the social objective parameter"—we run a regression of before-government aid payment on college fixed effects, parental income and student ability. The parameter is identified by the elasticity of before-government aid tuition to parental income.

Table 1: Externally calibrated parameters

Parameter	Description	Value	Source
a_n	Average financial aid	0.20	NPSAS
a_u	Average transfer to colleges	0.40	OECD
τ_u	Elasticity of transfers to colleges	0.35	IPEDS
a_y	Average income tax rate	0.20	CBO
τ_y	Progressivity of income tax	0.23	Heathcote et al. (2017)
τ_n	Progressivity of need-based subsidies	0.11	NPSAS
τ_m	Progressivity of merit-based subsidies	0.00	NPSAS
r	Interest rate	3.5%	Standard
ω_y	Social objective of colleges	0.00	NPSAS
λ	Return to human capital	0.67	Own comput.
$\underline{a}$	Borrowing limit (% of GDP)	3.0%	U.S. Dpt. of Education
η	Inverse elasticity of labor supply	2.0	Chetty et al. (2011)

Notes: the calibrated value for λ follows Benabou (1996) and is based on empirical estimates on output elasticity to human capital.

We use estimates of the Frisch elasticity of labor supply from the literature to calibrate η (Chetty, Guren, Manoli, and Weber, 2011).[29] The returns to education is set to $\lambda = 0.67$ following the value used in Benabou (1996) based on empirical estimates of the elasticity of output to human capital. The generation length is set to 30 years. For $\underline{a}$, we target the official limit on student loans, as a percentage of

[29] η is set to match the Frisch elasticity of labor supply, $\varepsilon_{\ell,w} = \frac{1}{\eta-1}$. Empirical estimates of $\varepsilon_{\ell,w}$ range from 0.2 to 0.7 (Chetty, Guren, Manoli, and Weber, 2011). Our preferred estimate is the conservative value $\varepsilon_{\ell,w} = .2$ which implies $\eta = \left(1 + \frac{1}{0.2}\right) = 2$. Ideally, to be consistent with the model, we would target the elasticity of lifetime household income to wages. However all estimates are at the individual and yearly level. These are likely upper bounds to the lifetime household elasticity for two reasons. First they do not capture intra-household substitution. Second they do take into account the intertemporal substitution stemming from temporary fluctuations in wages.

lifetime GDP per capita, which amounts to 3%.

Internal Calibration The algorithm used to estimate the parameters is akin to a Simulated Method of Moments. The results are reported in Table 2. We assume that the economy is in steady state in 2010.

We now make a heuristic identification argument that justifies the choice of moments used in the estimation. Although no parameter can be identified out of a single moment, we stress in this section which moment is important for each parameter.

To calibrate $\underline{q}$—the outside option to college—it is natural to target the enrollment rate: the lower $\underline{q}$, the stronger the incentives to go to college. The immediate enrollment rate, provided by the NCES, in the U.S. in the 2000s is about 70%.

We calibrate the standard deviation of the birth shock σ_z so that the equilibrium standard deviation of log ability, which also depends on the dispersion of human capital h, is one in the model. The Gini coefficient of income is used to inform the variance of labor market shocks, σ_y^2. The best estimate for the Gini coefficient of lifetime labor earnings is from Kopczuk, Saez, and Song (2010) who have access to administrative data.[30]

The intergenerational rate of preference, β, is strongly related to the share of young adults who borrow (65%), as more altruistic parents are likely to transfer more bequests. The elasticity of quality to teaching ω_I governs the strength of the income-sorting channel and is therefore connected to the elasticity of college quality to parental income as can be seen in equation (17). We thus regress the log of teaching expenditures I on the log of parental income and ability and target the coefficient on income. We use teaching expenditures I instead of q in the regression because it is directly observable.

We finally turn to the coefficients governing the law of motion of human capital (35). To estimate the elasticity of child ability to parental income α_z, we match the regression coefficient of children ability on family income in the NLSY97. Likewise, to estimate the elasticity of child earning to parents' income α_y, we match the regression

[30]There would be two issues with the NLSY97: first children labor earnings are observed only up to 2015, *i.e.* in their first years of labor market experience and a lot of them are not in a households yet. Secondly, top income are censored. Kopczuk, Saez, and Song (2010) finds that the eleven-year Gini coefficient is between .45 and .50. This is slightly lower than the annual Gini coefficient, which is between .49 and .57—depending on the exact measure of gross income used—in 2000 according to the CBO, probably because of transitory income shock. We keep a Gini of lifetime labor earnings of .45 as a target.

coefficients of children's future earnings on parents income. The elasticity to college quality α_q is connected to the share of spending of education on GDP. The OECD reports that share of private spending for higher education in GDP in the U.S. over the period 2000-2004 is 1.3%.[31] Finally, the elasticity of child earnings to their own ability ζ is related to the regression coefficient of children's future earnings on their ability.

Table 2: Internally calibrated parameters

Parameter	Description	Value
$\underline{q}$	Quality of outside option	0.01
σ_z	Standard deviation of birth shock	0.97
σ_y	Standard deviation of labor market shock	1.22
β	Intergenerational rate of preference	0.24
ω_I	Expenditure effect in colleges	0.73
α_z	Elasticity of child ability to parents human capital	0.35
ζ	Elasticity of human capital to ability	0.18
α_q	Elasticity of human capital to college quality	0.30
α_y	Elasticity of human capital to parental human capital	0.33

Table 3: Targets for internal calibration

Description	Source	Data	Model
Private spending on higher education	OECD	1.3%	1.3%
Enrollment rate	NCES	70%	70%
Standard deviation of ability	Standardization	1.0	1.06
Income Gini coefficient	Kopczuk et al. (2010)	0.45	0.42
Share of students who borrow	NCES	65%	70%
Elasticity of I to y in sorting rule	NLSY & IPEDS	0.15	0.16
Elasticity of z to y_m	NLSY	0.43	0.41
Elasticity of y'_m to z	NLSY & IPEDS	0.31	0.33
Elasticity of y'_m to y_m	NLSY	0.36	0.35

Notes: *Private spending on higher education is in percent of GDP. Enrollment rate is the first time enrollment rate. The elasticity of I to y is the regression coefficient on $ln(y)$ in the regression of $ln(I)$ on a constant, $ln(y)$ and $ln(z)$.*

[31] For reference, they also report that the share of public spending is 1%, making spending in higher education 2.3% of GDP.

6.3. Model Fit and Validation

As evident from Table 3, the model matches well the targeted characteristics of the higher education system.

Given our aim of analyzing the interaction between the sorting of students across colleges, income inequality and mobility, an important question is the extent to which our model is able to generate a realistic distribution of parental income within each college, which we don't directly target. We collect the distribution of parental income for the universe of colleges in the U.S. from Chetty, Friedman, Saez, Turner, and Yagan (2020) and sort colleges into quintiles based on the average parental income.[32] Figure 1 shows that the model does overall a very good job at replicating the empirical distribution of parental income for each college quintile.

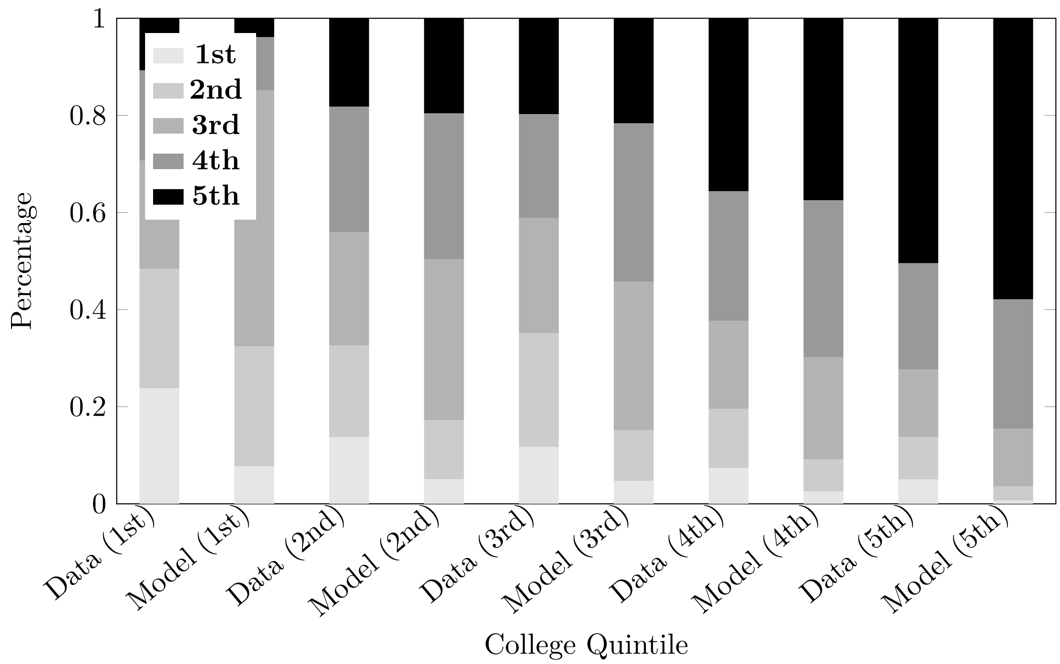

Figure 1: Income Distribution by College Quintile (Data and Model)

Notes: *The figure reports the distribution of parental income within each quantile of college quality. The underlying data source is Chetty, Friedman, Saez, Turner, and Yagan (2020).*

In our last validation exercise, we use the calibrated model to predict the extent to which a reasonably parametrized change in key parameters, including the returns to education, λ, can explain the stylized facts (a)-(e) presented in the introduction. Let's

[32]The ranking of colleges is robust to sorting colleges based on median parental income, average kid income and the college's average spending per student.

denote x_{1980} and x_{2010} the value of parameter x in the original steady-state (1980) and in the final steady-state (2010), respectively. The values in 2010 for all parameters are the one estimated in the previous section. We calibrate the value of the returns to education in the old steady-state, λ_{1980}, to target the change in the Gini coefficient of household net earnings provided by the CBO: $\Delta\text{Gini} = .45 - .38 = .07$. We recalibrate three other parameters. Given the large decline in the degree of progressivity of college transfers documented in a companion paper Capelle (2019), we set $\tau_{u,1980} = .5$. We also recalibrate the quality of the outside option $\underline{q}_{1980}$ to match a 50% enrollment rate in 1980 as reported by the NCES. Finally, we recalibrate the interest rate r_{1980} to match the 22% lower household wealth to output ratio reported by the Bureau of Economic Analysis. We find that it is important to increase the returns to financial investments when changing the returns on human capital. Using the lower interest rate r_{2010} results a sharp decrease in the incentives to save and a low wealth to output ratio in 1980. The resulting parameters are $\lambda_{1980} = 0.56, \underline{q}_{1980} = 0.01, r_{1980} = 3.7\%$.

Comparing the two steady states, we find that the model predicts a 4.3 percentage points increase in the Gini coefficient of expenditure per student, which is close to the 6 percentage points increase observed in the data. The model also implies that the share of students from the lowest income quintile at colleges in the top quintile decreases from 3.2% in 1980 to 0.7% in 2010 and that spending on tuition fees over GDP increases from 0.6% to 1.3%. It also generates a 7.7% rise in the IGE, qualitatively consistent with the empirical increase.[33] Additionally, the model produces an increase in the skill premium of $\Delta \log\left(\frac{w_{college}}{w_{HS}}\right) = 0.5$, which aligns reasonably well with the estimate of 0.2 provided by Autor, Katz, and Kearney (2008) (see Figure 2 in their paper).

6.4. Policy Experiments in Higher Education

In Table 4, we gather the results of the five policy experiments discussed below. We provide the percentage change from the status quo steady-state to the counterfactual steady-state of the Gini coefficient of labor earnings, expenditures per students, the intergenerational elasticity, a measure of the quality of sorting across colleges by abilities, GDP and a measure of welfare. Our measure of sorting quality is $1 - V(\ln z|q)/V(\ln z) \in [0, 1]$ which is equal to 0 when students are randomly allocated

[33] Measures of the IGE in the early 1980s are not as precise, and estimates vary.

across colleges and equal to 1 when there is perfect assortative matching. The social welfare function is the mean of households values.

Contribution of higher education. What is the contribution of the sorting of students and resources across heterogeneous colleges to intergenerational persistence, income inequality and GDP? To address this question, the first policy experiment consists in randomly allocating students across colleges. As a result, spending per student and average student ability are equalized across all colleges and every student receives the same higher education. The common college quality, $\bar{q}$, is given by the production function of quality (10), the average children ability in society and average government transfers per student.[34] Because households optimal spending rate for higher education drops to zero, all the resources spent in the higher education system have to be financed through taxes and transfers to colleges. One therefore needs to take a stand on the level of government subsidies. We assume that there are such that the share of GDP going to higher education remains the same as in the *status quo* allocation.[35]

We find that the sorting of students and resources across heterogeneous colleges plays a quantitatively important role in shaping intergenerational persistence, aggregate income and, to some extent, economic inequality. Quantitatively such policy would reduce the IGE by 21.0% (see Table 4), inequality by 4.2% and aggregate income by 9.7%. It is therefore a sizable effect. The drop in GDP is caused by the misallocation of students and resources across colleges, i.e. the decrease in the positive assortative matching of students on the one hand, as evident from the 100% decline in our measure of sorting quality, and of resources on the other.

With the second policy experiment we are interested in isolating the contribution of the peer effect. To do so, we equalize financial resources across all colleges. Such policy neutralizes the effect that parental income has on the sorting of students across colleges conditional on child ability. As a result, the distribution of student abilities across colleges changes in the counterfactual and the equilibrium allocation features

[34]The random allocation of students not only equalizes college experiences among college-goers but implies that everyone goes to college. It thus neutralizes both the extensive (enrolling or not) and the intensive (quality) margin.

[35]The choice for the level of subsidies does not influence inequality or mobility, but it does have a first order effect on the aggregate level of production. This assumption allows to focus on the effect of misallocation on aggregate production. In practice it means that the level of government subsidies should increase to offset the decline in private spending for higher education.

perfect sorting of colleges by student ability. Like in the previous policy experiment, we assume that government policies exactly offset the decrease in average private spending, so that the aggregate spending rate in higher education remains constant in the two steady-states.

We find that the sorting of ability contribute for three fourth of the total effect on mobility, more than half of the total effect on aggregate income and more than half of the total effect on inequality. In the counterfactual allocation, the IGE is reduced by 15.2% and the Gini coefficient of earnings by 2.5%. On the one hand the mismatch of student abilities is reduced with the elimination of the income-sorting channel, but the equilibrium equalization of spending across colleges leads to a less efficient accumulation of human capital. The net effect on GDP is negative and results in a large decrease of 5.7%.

The role of existing student-based aid. The next policy experiment investigates the implications of existing need-based policies on mobility, inequality and aggregate income. These policies can potentially correct the misallocation of students across colleges due to the borrowing constraint. This constraint implies that parental income plays a role in shaping the sorting of students and resources across colleges and prevents high ability students from low-income families to access better colleges. The next counterfactual eliminates the progressivity of government need-based student aid by setting $\tau_n = 0$.[36]

We find that existing need-based student aid policies not only enhance mobility and decrease inequality, but they also boost output. Quantitatively, removing the progressivity of need-based student aid leads to an increase in the IGE by 0.7%, an increase in the income Gini by 0.1% and decrease in output by 0.4%. As a result welfare decreases by 0.7%. The negative impact on output is an important result because it shows that need-based policies improve the sorting of students and resources across colleges and (partially) addresses the misallocation implied by financial frictions. This can be seen from the 1% decline in the measure of sorting quality.[37] Merit-based

[36]Another source of inefficiency is the missing insurance markets for birth and labor market shocks. It turns out that progressive policies can also partially address this lack of insurance, albeit imperfectly.

[37]We also find that removing the progressivity of need-based student aid decreases the Gini coefficient of college expenditures. This is a surprising result that stems from the fact that the enrollment rate decreases when $\tau_n = 0$ which in turns reduces the dispersion among the smaller pool of students that go to college.

aid plays virtually no role since they are small in the current system, i.e. $\tau_m = 0$. Similarly, institutional need-based aid play no role since they are also negligible, i.e. $\omega_y = 0$.

Optimal progressivity of need-based aid. The last two experiments investigate the optimal degree of progressivity of institutional and federal need-based aid. These are interesting experiments because they are widely discussed in policy circles and, increasingly, colleges advertize their need-based financial aid program. Formally, we solve for the progressivity of federal aid, τ_n, that maximizes welfare, keeping other policy parameters unchanged. We then do the same for the social objective parameter of colleges, ω_y.

We find that the current level of progressivity of federal need-based aid is very close to optimal. While the current level of progressivity of federal student aid is at $\tau_n = 0.11$, the optimal level is around $\tau_n = 0.07$. Consistent with this result, we find that implementing the optimal policy would leave mobility, inequality and aggregate income virtually unchanged at their current values.

Table 4: Policy counterfactuals

	% Change from status quo					
Policy	**Gini Earnings**	**Gini Exp./Stud.**	**Intergen. Elas.**	**Sorting Quality**	**GDP**	**Welfare**
Random admission	-4.2	-100	-21.0	-100	-9.7	-7.8
Equal resources	-2.5	-100	-15.2	32.2	-5.7	-4.7
No need-based aid, $\tau_n = 0$	0.1	-5.3	0.7	-1.0	-0.4	-0.7
Optimal τ_n (Federal)	0,0	-0.3	0.3	-0.2	-0.1	0.1
Optimal ω_y (Institutional)	-0.1	-2.3	-4.0	5.2	0.5	1.2
Status quo levels	*42.4*	*13.3*	*35.5*	*75.6*	-	-

[1] Notes: Sorting Quality is defined as $(1 - V(\ln z|q)/V(\ln z)) \times 100$.

More interestingly, we find that increasing progressive institutional aid from $\omega_y = 0$ to $\omega_y = 0.1$ would significantly improve welfare by decreasing inequality and intergenerational persistence while raising GDP. Quantitatively, we find that implementing the optimal level of progressivity of institutional need-based aid would result in a decrease in the IGE by 4%, a decrease in the income Gini by 0.1%, and increase in GDP by 0.5%. Overall welfare would increase by 1.2%, which is economically sizable. The increase in GDP comes from the decrease in the strength of the income-sorting channel and in the resulting improvement in the assortative matching of students across colleges, as shown by the 5.2% increase in the measure of sorting quality.

7. Conclusion

This paper analyzes the role of higher education in shaping intergenerational mobility, income inequality and aggregate income. It introduces a model where overlapping generations of heterogeneous households make college choices subject to a borrowing constraint and with heterogeneous colleges that maximize quality. The model is consistent with the observed patterns of sorting of students across colleges and can generate several trends observed in the U.S. since 1980. Counterfactual simulations show that if all students received the same higher education, the IGE, the income Gini and aggregate income would decrease by up to 21%, 4.2% and 9.7respectively. Increasing the progressivity of institutional need-based student aid would enhance mobility, decrease inequality and boost GDP by improving the sorting of students and financial resources across colleges.

There are three critical areas of investigation for future research: (i) the allocation of students across colleges in the model works through a system of clearing markets, while the real world displays a mix of price mechanism and quantity restrictions, (ii) beyond the accumulation of human capital and labor market returns, higher education has non-pecuniary returns and there is evidence that households get a direct consumption value from going to college. The implications for welfare and policy analysis are likely to be far-reaching, (iii) the paper has focused on the role played by tuition and public subsidies for the shaping of inequality in higher education: the analysis should be extended to take into account donations and endowments as it is likely to be an additional source of inequality.

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A. Analytical Model - Details

We solve the model using a guess and verify. We guess that the tuition function before government financial aid are given respectively by:

$$e_u(q,z,y) = \left(\frac{1}{(1+a_u)T_u} q^{\frac{1}{\varepsilon_I}} z^{-\frac{\varepsilon_z}{\varepsilon_I}} \left(\frac{y}{\kappa}\right)^{\frac{\varepsilon_y}{\varepsilon_I}} \right)^{\frac{1}{1-\tau_u}} \tag{36}$$

A.1. Solution to the Household Problem

Using the guess (36) and the expression for financial aid (22), the problem of the Households is

$$\mathscr{U} = \max_{s,\ell} \left[\ln \frac{(1-s)(1-a_y)T_y}{1+a_c} + \ln \left(h^\lambda \ell\right)^{1-\tau_y} - \ell^\eta \right] + \beta E \mathscr{U}' \tag{37}$$

$$\begin{aligned} \ln h' &= \ln \xi_y + \alpha_z (1 + \alpha_q(\varepsilon_z + \tau_m(1-\tau_u)\varepsilon_I)) \ln \xi_z + \tilde{\rho} \ln h \\ &+ \alpha_q \varepsilon_I \left(\ln \left(s(1+a_n)/T_e\right)^{1-\tau_u} (1+a_u)T_u \right) + \alpha_q(\varepsilon_I(1-\tau_u)(1-\tau_{n,t}) - \varepsilon_y) \ln \ell^{(1-\tau_y)} \\ &+ \alpha_q \left(\varepsilon_I(1-\tau_u)(1-\tau_n) - \varepsilon_y\right) \left((1-\tau_{y,t}) \ln (1-a_y)T_y)\right) + \alpha_q \varepsilon_y \ln \kappa_t \end{aligned} \tag{38}$$

with $\tilde{\rho} = \alpha_z + \alpha_y + \alpha_z\alpha_q(\varepsilon_z + \tau_m(1-\tau_u)\varepsilon_I) + \alpha_q(\varepsilon_I(1-\tau_u)(1-\tau_n) - \varepsilon_y)(1-\tau_y)\lambda$ and s the spending rate, i.e. the amount of spending for college over income. We then guess that $\mathscr{U}_t = U_t \ln h_t + Z_t \ln \xi_{z,t} + B_t$. Replacing this guess into (37), then using (38) to substitute for $\ln h_{t+1}$ and using (6) and (7)

$$\begin{aligned} U_t \ln h_t + Z_t \ln \xi_{z,t} + B_t &= \max_{s,\ell} \left[\ln \frac{(1-s)(1-a_y)T_y}{1+a_c} + (1-\tau_y) \ln h^\lambda \ell - \ell^\eta \right] \\ &+ \beta \Bigg[U_{t+1} \Bigg(\mu_y + \alpha_z(1+\alpha_q(\varepsilon_z + \tau_{m,t}(1-\tau_u)\varepsilon_I)) \ln \xi_{z,t} + \tilde{\rho}_t \ln h_t \\ &+ \alpha_q \varepsilon_I \left(\ln \left(s(1+a_n)/T_e\right)^{1-\tau_u} (1+a_u)T_u \right) + \alpha_q(\varepsilon_I(1-\tau_u)(1-\tau_{n,t}) - \varepsilon_y) \ln \ell^{(1-\tau_y)} \\ &+ \alpha_q(\varepsilon_I(1-\tau_u)(1-\tau_{n,t}) - \varepsilon_y) \left((1-\tau_{y,t}) \ln (1-a_{y,t})T_{y,t}\right) + \alpha_q \varepsilon_y \ln \kappa_t \Bigg) + Z_{t+1}\mu_b + B_{t+1} \Bigg] \end{aligned}$$

Gathering all the terms in $\ln h_t$ one gets that U_t has to verify
$U_t = \sum_{k=0}^{\infty} \beta^k (1-\tau^y_{t+k})\lambda_{t+k} \prod_{m=0}^{k-1} \alpha^h_{t+m}$

Gathering all the terms in $\ln \xi_{z,t}$, one gets $Z_t = \Big(U_t-(1-\tau_{y,t})\lambda\Big)^{\frac{\alpha_z(1+\alpha_q(\varepsilon_z+\varepsilon_I(1-\tau_u)\tau_{m,t}))}{\alpha_t^h}}$.

Finally gathering the independent terms, the F.O.C for s and ℓ give

$$s_t = \frac{\beta\alpha_q\varepsilon_I(1-\tau_u)U_{t+1}}{1+\beta\alpha_q\varepsilon_I(1-\tau_u)U_{t+1}} \tag{39}$$

$$\ell = \left[(1-\tau_{y,t})\frac{1}{\eta}\left(1+\beta\alpha_q\left(\varepsilon_I(1-\tau_u)(1-\tau_{n,t})-\varepsilon_y\right)U_{t+1}\right)\right]^{\frac{1}{\eta}} \tag{40}$$

A.2. University problem

We first provide a generalized definition of σ_u that takes into account government policies

$$\sigma_u^2 = \frac{\omega_I(1-\tau_u)}{2}E\left(\left(\ln\left(\bar{z}^{\frac{\omega_z}{\omega_I(1-\tau_u)}}\bar{y}^{-\frac{\omega_y}{\omega_I(1-\tau_u)}}\right)-\ln z^{\frac{\omega_z}{\omega_I(1-\tau_u)}}y^{-\frac{\omega_y}{\omega_I(1-\tau_u)}}\right)^2\right) \tag{41}$$

Using this definition and our guess for tuitions (36), one gets the following

$$\sigma_u^2 = \frac{\omega_I(1-\tau_u)}{2}E\left(\left(\ln e_u(q,z,y)-\ln\left(\frac{\tilde{I}}{(1+a_u)T_u}\right)^{\frac{1}{1-\tau_u}}\right)^2\right) = \frac{\omega_I(1-\tau_u)}{2}\tilde{\sigma}_u^2$$

where we define $\ln\tilde{I} = \ln I - \frac{\tilde{\sigma}_u^2}{2}$. We now show that $\tilde{\sigma}_u^2$ is the within-university variance of log tuition. We guess that tuition fees are log-normally distributed within the university. Denoting $\mu_{e,q}, \sigma_{e,q}$ the mean and standard deviation of log tuition within the university of quality q, the budget constraint of the university -given by (25)-becomes

$$I = T_u(1+a_u)\left(E_{z,y}[e_u(q,z,y)]\right)^{1-\tau_u} = T_u(1+a_u)e^{(1-\tau_u)\mu_{eq}+(1-\tau_u)\frac{\sigma_{eq}^2}{2}}$$
$$\iff \frac{1}{(1-\tau_u)}\ln\frac{\tilde{I}}{(1+a_u)T_u}+\frac{\tilde{\sigma}_u^2}{2}-\frac{\sigma_{eq}^2}{2} = \mu_{e,q} = E\ln e_u(z,y)$$

Substituting this last line into the expression of σ_u^2 above gives

$$\tilde{\sigma}_u^2 = \int\phi(z,y)\left(\ln e_u(z,y)-E\ln e_u(z,y)+\frac{\sigma_{eq}^2-\sigma_u^2}{2}\right)^2 dzdy \tag{42}$$

$$\iff \tilde{\sigma}_u^2 = \sigma_{e,q}^2+\left(\frac{\sigma_{eq}^2-\sigma_u^2}{2}\right)^2+0 \quad \Rightarrow \tilde{\sigma}_u^2 = \sigma_{e,q}^2 \quad \text{or} \quad \tilde{\sigma}_u^2 = \sigma_{e,q}^2+4$$

$\tilde{\sigma}_u = \sigma_{e,q}$ is a solution to the quadratic equation. This verifies our guess. $\mu_{e,q} = E \ln e_u(z,y) = \ln \left(\frac{\tilde{I}}{(1+a_u)T_u}\right)^{\frac{1}{1-\tau_u}}$ and $\sigma_u^2 = \sigma_{eq}^2$ are respectively the mean and standard deviation of within-university log tuitions. Therefore we can now rewrite the problem of the university replacing I with $\tilde{I}$

$$\max_{\tilde{I},\bar{z},\bar{y},r(.)} \tilde{I}^{\omega_I} \bar{z}^{\omega_z} \bar{y}^{-\omega_y} \tag{43}$$

$$\ln \tilde{I} \int_0^1 r_{z,y} dz\, dy = \int r_{z,y} \Big((1-\tau_u) \ln(e_u)^i + \ln(1+a_u)T_u\Big)\, dz\, dy$$

$$\ln \bar{z} \int_0^1 r_{z,y} dz\, dy = \int_0^1 r_{z,y} \ln z\, dz\, dy \quad \text{and} \quad \ln \bar{y} \int_0^1 r_{z,y}\, dz\, dy = \int_0^1 r_{z,y} \ln y\, dz\, dy$$

where $r_{z,y}$ denotes the mass of individuals of type (z,y).

The F.O.Cs are $\frac{\omega_I}{\tilde{I}} + \frac{\lambda_1}{\tilde{I}} = 0, \qquad \frac{\omega_z}{\bar{z}} + \frac{\lambda_2}{\bar{z}} = 0$ and $-\frac{\omega_y}{\bar{y}} + \frac{\lambda_3}{\bar{y}} = 0$

$$r_{z,y} = \begin{cases} 0 \quad \text{if} \quad \left(\frac{1}{(1+a_u)T_u} q^{\frac{1}{\omega_I}} z^{-\frac{\omega_z}{\omega_I}} (y/\bar{y})^{\frac{\omega_y}{\omega_I}}\right)^{\frac{1}{1-\tau_u}} < e_u(q,z,y) \\ c \in \mathbb{R} \quad \text{if equality} \\ +\infty \quad \text{if strictly larger} \end{cases}$$

where we have solved for the Lagrange multipliers. We guess that in equilibrium, $\bar{y} = \kappa_t q^{\nu_t}$. Therefore whenever a college admits a certain student type, the tuition formula is:

$$e_u(q,z,y) = \left(\frac{1}{(1+a_u)T_u} q^{\frac{1-\nu\tilde{\omega_y}}{\omega_I}} z^{-\frac{\omega_z}{\omega_I}} y^{\frac{\omega_y}{\omega_I}} \kappa_t^{-\frac{\omega_y}{\omega_I}}\right)^{\frac{1}{1-\tau_u}} = \left(\frac{1}{(1+a_u)T_u} q^{\frac{1}{\varepsilon_I}} z^{-\frac{\varepsilon_z}{\varepsilon_I}} y^{\frac{\varepsilon_y}{\varepsilon_I}} \kappa_t^{-\frac{\varepsilon_y}{\varepsilon_I}}\right)^{\frac{1}{1-\tau_u}}$$

$$\textit{with} \quad \varepsilon_I = \frac{\omega_I}{1-\nu\omega_y} \qquad \varepsilon_z = \frac{\omega_z}{1-\nu\omega_y} \qquad \varepsilon_y = \frac{\omega_y}{1-\nu\omega_y}$$

We can solve for ν_t and κ_t using the equilibrium outcome given by the mean income in proposition A.4. We do this later in appendix A.4.1. This confirms the guess for tuition fees (36). Given this guess for tuition, a university is always at the interior solution, therefore always indifferent between all types.

A.3. Other Equilibrium Conditions

A.3.1. Government Budget Constraints

There are two kinds of constraints. The first one is the aggregate budget constraint that states that revenues (income tax and consumption tax) must equal spending (transfers to colleges and students) at any period.

$$\int_0^1 a_y y(i) + a_c c(i) + e(i) di = \int_0^1 e(i)(1 + a_u)(1 + a_n) di \tag{44}$$

The other three constraints, (46), (45) and (47) pin down T_u, T_y, T_e such that a_y, a_n, a_u parametrize respectively the average rate of income tax, financial aid and transfers to college. Denoting f_q the mass of students in colleges of quality q

$$\int_0^1 y(i)^{1-\tau_y} T_y di = \int_0^1 y(i) di \tag{45}$$

$$(1 + a_n) \int_0^1 e(i) di = \int_0^1 e_u(i) di \tag{46}$$

$$\int E_{z,y}[e_u(q, z, y)] f_q dq = \int T_u \left(E_{z,y}[e_u(q, z, y)]\right)^{1-\tau_u} f_q dq \tag{47}$$

Lemma 1. *Along the equilibrium path, the government budget constraints* (44),(45),(46) *and* (47) *are given by*

$$\frac{a_{c,t}(1 - s_t)}{(1 + a_{c,t})} = s_t(1 + a_{u,t})(1 + a_{h,t}) - \frac{a_{y,t}}{1 - a_{y,t}} - s_t \tag{48}$$

$$\ln T_y = \tau_y \ln \ell + \tau_y \lambda m_h + \frac{\lambda^2}{2}(2 - \tau_y)\tau_y \Sigma_{h,t}^2 \tag{49}$$

$$\ln T_e = (-\tau_n \lambda + \alpha_z \tau_m) m_h + \frac{\alpha_z \tau_m}{2}(\alpha_z \tau_m - 1)\sigma_z^2 - \tau_n(\ln \ell(1 - a_y)) \tag{50}$$

$$+ \left[\lambda^2(1 - \tau_y)^2(\tau_n - 2)\tau_n + 2\lambda(1 - \tau_n)(1 - \tau_y)\tau_m \alpha_z + (\alpha_z \tau_m)^2 - \tau_n \lambda^2(2 - \tau_y)\tau_y\right] \frac{\Sigma_h^2}{2} \tag{51}$$

$$\ln T_u = \tau_u \left(\ln \ell s(1 + a_n)(1 - a_y) + \lambda m_h + \lambda^2 \frac{\Sigma_h^2}{2}\right) + \frac{\tau_u}{1 - \tau_u} \frac{\sigma_I^2}{2} \tag{52}$$

1. Solving for the aggregate state budget constraint is immediate
2. Solving for T_y. Using (45), and the expression for market income y_m, (21), and using the guess that $\ln h$ is normally distributed one gets:

$$\int_0^1 (\ell h^\lambda)^{1-\tau_{y,t}} T_y di = \int_0^1 \ell h^\lambda di \iff T_y e^{\lambda(1-\tau_y)m_h + \frac{(\lambda(1-\tau_y))^2}{2}\Sigma_h^2} = \ell^{\tau_y} e^{\lambda m_h + \frac{\lambda^2}{2}\Sigma_h^2}$$

3. Solving for T_e. Using (46), one gets:

$$(1+a_n)\int_0^1 e^i di = \int_0^1 (e_u)^i di \iff (1+a_n)\int_0^1 sy_I di = \int_0^1 \frac{sy(1+a_n)}{T_e z^{-\tau_m} y^{\tau_n}} di$$

$$T_e(1-a_y)^{\tau_n}(\ell)^{\tau_n(1-\tau_y)}(T_y)^{\tau_n} e^{\lambda(1-\tau_y)m_h+(\lambda(1-\tau_y))^2\frac{\Sigma_h^2}{2}}$$
$$= e^{(\lambda(1-\tau_y)(1-\tau_n)+\tau_m\alpha_z)m_h-\alpha_z\tau_m\frac{\sigma_z^2}{2}+(\lambda(1-\tau_y)(1-\tau_n)+\tau_m\alpha_z)^2\frac{\Sigma_h^2}{2}+\frac{(\alpha_z\tau_m)^2}{2}\sigma_z^2}$$

4. Solving for T_u. Substituting (25) into (47), one gets

$$\int E_{z,y}[e_u(q,z,y)]f_q dq = \int T_u\left(E_{z,y}[e_u(q,z,y)]\right)^{1-\tau_u} f_q dq$$
$$\iff \int\left(\frac{I_q}{(1+a_u)T_u}\right)^{\frac{1}{1-\tau_u}} f_q dq = \int \frac{I_q}{(1+a_u)} f_q dq$$
$$\left(\frac{1}{(1+a_u)T_u}\right)^{\frac{1}{1-\tau_u}} \int I_i^{\frac{1}{1-\tau_u}} di = \frac{1}{(1+a_u)}\int I_i di$$
$$\iff \left(\frac{1}{1+a_u}\right)^{\frac{\tau_u}{1-\tau_u}} \int I_i^{\frac{1}{1-\tau_u}} di = (T_u)^{\frac{1}{1-\tau_u}} \int I_i di$$

where i indexes households. We then guess that I_i is log-normally distributed with mean μ_I and variance σ_I^2 - we give an expression for these variables in appendix A.6):

$$\left(\frac{1}{1+a_u}\right)^{\frac{\tau_u}{1-\tau_u}} e^{\frac{\mu_I}{1-\tau_u}+\frac{\sigma_I^2}{2(1-\tau_u)^2}} = (T_u)^{\frac{1}{1-\tau_u}} e^{\mu_I+\frac{\sigma_I^2}{2}}$$
$$\Rightarrow \ln T_u = \tau_u \ln\left(\frac{1}{1+a_u}\right) + \mu_I\tau_u + \frac{\sigma_I^2}{2}\frac{\tau_u(2-\tau_u)}{(1-\tau_u)}$$

Using the guess and from appendix **??**, one gets

$$\ln E(I) = \mu_I + \frac{\sigma_I^2}{2} = \ln \ell s(1+a_n)(1+a_u)(1-a_y) + \lambda m_h + \lambda^2\frac{\Sigma_h^2}{2}$$
$$\text{Hence}\quad \mu_I = \ln \ell s(1+a_n)(1+a_u)(1-a_y) + \lambda m_h + \lambda^2\frac{\Sigma_h^2}{2} - \frac{\sigma_I^2}{2}$$

Substituting back into the expression for T_u gives

$$\ln T_u = \tau_u\left(\ln \ell s(1+a_n)(1-a_y) + \lambda m_h + \lambda^2\frac{\Sigma_h^2}{2}\right) + \frac{\tau_u}{1-\tau_u}\frac{\sigma_I^2}{2}$$

I derive the expression for σ_I^2 in appendix A.6

A.4. Quality distribution and within-college distributions

Parent's education and income. Taking the logarithm of (17): $\ln q = (\theta_I + \theta_A)\ln h + \theta_A \ln \xi_z + x$ with
$x = \theta_I \left(\ln\left(s\frac{(1+a_n)}{T_e}\right)^{1-\tau_u}((1+a_u)T_u)\right) + (\theta_I(1-\tau_u)(1-\tau_n)-\varepsilon_y)\ln(\ell)^{1-\tau_y}T_y(1-a_y) + \varepsilon_y \ln\kappa_t$, where $\theta_I = (\varepsilon_I(1-\tau_u)(1-\tau_{n,t})-\varepsilon_y)(1-\tau_y)\lambda$ and $\theta_A = \alpha_z(\varepsilon_z + \tau_m(1-\tau_u)\varepsilon_I)$. All pairs (h, ξ_y) that verify this condition will go to a university with quality q. The distribution of parents human capital within a given university of quality q can therefore be computed explicitly. The mass of individuals with $\ln h$ and going to $\ln q$ is given by:

$$
\begin{aligned}
& f\left(\frac{1}{\theta_A}(\ln q - x - (\theta_I+\theta_A)\ln h) \cap \ln h\right) = f_{\xi_z}\left(\frac{1}{\theta_A}(\ln q - x - (\theta_I+\theta_A)\ln h)\right) f_h(\ln h) \\
&= \phi\left(\frac{\ln q - x - (\theta_I+\theta_A)\ln h}{\theta_A}, \mu_b, \sigma_z^2\right)\phi\left(\ln h, m_h, \Sigma_h^2\right) \\
&= \phi\left(\ln h, \underbrace{\frac{\ln q - x - \theta_A\mu_b}{\theta_I+\theta_A}}_{\mu_1^q}, \underbrace{\left(\frac{\theta_A\sigma_z}{\theta_A+\theta_I}\right)^2}_{\sigma_1^2}\right)\phi\left(\ln h, m_h, \Sigma_h^2\right) \\
&= \phi\left(\ln h, \mu_1^q, \sigma_1^2\right)\phi\left(\ln h, m_h, \Sigma_h^2\right) = \phi\left(\mu_1^q, m_h, \sigma_1^2+\Sigma_h^2\right)\phi\left(\ln h, \mu_2^q, \sigma_2^2\right)
\end{aligned}
$$

where the RHS is the mass of individuals going to quality q and the LHS is the density of people whose parents have human capital h conditional on college q.

$$
f(\ln h|q) \sim \mathcal{N}\left(\frac{\Sigma_h^{-2}m_h + \left(\frac{\theta_A}{\theta_I+\theta_A}\right)^{-2}\sigma_z^{-2}\frac{(\ln q - x - \theta_A\mu_b)}{\theta_I+\theta_A}}{\Sigma_h^{-2} + \left(\frac{\theta_A}{\theta_I+\theta_A}\right)^{-2}\sigma_z^{-2}}, \frac{\Sigma_h^2\left(\frac{\theta_A}{\theta_I+\theta_A}\right)^2\sigma_z^2}{\Sigma_h^2 + \left(\frac{\theta_A}{\theta_I+\theta_A}\right)^2\sigma_z^2}\right) \sim \mathcal{N}\left(\mu_2^q, \sigma_2^2\right)
$$

For future reference we introduce $\mu_2^q = \mu_{2,1}m_h + \mu_{2,2}(\ln q - x - \theta_A\mu_b)$

$$
\text{with} \quad \mu_{2,1} = \frac{\Sigma_h^{-2}}{\Sigma_h^{-2} + \left(\frac{\theta_A}{\theta_I+\theta_A}\right)^{-2}\sigma_z^{-2}} \quad \text{and} \quad \mu_{2,2} = \frac{\left(\frac{\theta_A}{\theta_I+\theta_A}\right)^{-2}\sigma_z^{-2}}{\left[\Sigma_h^{-2} + \left(\frac{\theta_A}{\theta_I+\theta_A}\right)^{-2}\sigma_z^{-2}\right](\theta_I+\theta_A)}
$$

where the second line stems from independence of h and ξ_z. The mass of individuals studying in college of quality q is $\phi(\mu_1^q, m_h, \sigma_1^2 + \Sigma_h^2)$ and the density of $\ln h$ conditional on being in this college is $\phi(\ln h, \mu_2^q, \sigma_2^2)$. From the distribution of parents' human capital within a college, the distribution of parents' income is

$$\ln y \sim \mathscr{N}\left(\ln(1-a_y)+(1-\tau_y)\left[\lambda\mu_2^q+\ln\ell\right]+\ln T_y,(1-\tau_y)^2\lambda^2\sigma_2^2\right).$$

Distribution of college quality Since $\phi\left(\mu_1^q, m_h, \sigma_1^2+\Sigma_h^2\right)$—with $\mu_1=\frac{1}{\theta_I+\theta_A}\left(\ln q-x-\theta_A\mu_b\right)$ and $\sigma_1^2=\left(\frac{\theta_A}{\theta_A+\theta_I}\right)^2\sigma_z^2$—is the mass of students in college of quality q, the distribution of quality is given by : $\ln q \sim \mathscr{N}\left((\theta_I+\theta_A)m_h+x+\theta_A\mu_b, \theta_A^2\sigma_z^2+(\theta_I+\theta_A)^2\Sigma_h^2\right)$

Students' abilities From the definition of abilities $\ln z=\alpha_z\ln h+\alpha_z\ln\xi_z$ and the sorting rule used above $\ln q=(\theta_I+\theta_A)\ln h+\theta_A\ln\xi_z+x$, one gets

$$\ln z=\frac{\alpha_z}{\theta_A}\left(\ln q-\theta_I\ln h-x\right)\Rightarrow \ln z|q\sim\mathscr{N}\left(\frac{\alpha_z}{\theta_A}\left(\ln q-\theta_I\mu_2^q-x\right),\left(\frac{\alpha_z\theta_I}{\theta_A}\right)^2\sigma_2^2\right)$$

A.4.1. Solving for κ_t and ν

The initial guess was that $\bar{y}=\kappa_t q^{\nu_t}$. Recall that $\ln\bar{y}$ is the mean log (after tax) income within a college $\ln\bar{y}=\ln(1-a_y)\left(\ell\right)^{1-\tau_y}T_y+(1-\tau_y)\lambda\left(\mu_{2,1}m_h+\mu_{2,2}(\ln q-x-\theta_A\mu_b)\right)$

Identifying coefficients with the guess $\ln\bar{y}=\ln\kappa_t+\nu_t\ln q$, one gets:

$$\nu=(1-\tau_y)\lambda\mu_{2,2}=(1-\tau_y)\lambda\frac{\left(\frac{\theta_A}{\theta_I+\theta_A}\right)^{-2}\sigma_z^{-2}}{\left[\Sigma_h^{-2}+\left(\frac{\theta_A}{\theta_I+\theta_A}\right)^{-2}\sigma_z^{-2}\right](\theta_I+\theta_A)}$$

$$\Longleftrightarrow\ \nu=\frac{1}{\left[\Sigma_h^{-2}\left(\frac{\theta_A}{\theta_I+\theta_A}\right)^2\sigma_z^2+1\right]\left[(\omega_I(1-\tau_u)(1-\tau_n)-\omega_y)+\frac{\omega_z}{(1-\tau_y)\lambda}\right]+\omega_y}$$

ν is therefore only a function of Σ_h^2. Identifying the coefficient independent of $\ln q$, and recalling that x_t is a linear function of $\ln\kappa_t$, and defining $\tilde{x}=x-\varepsilon_y\ln\kappa_t$, one gets:

$$\begin{aligned}\kappa_t&=(1-a_y)\left(\ell\right)^{1-\tau_y}T_y e^{(1-\tau_y)\lambda(\mu_{2,1}m_h-\mu_{2,2}(x+\theta_A\mu_y)}\\&=\left((1-a_y)\left(\ell\right)^{1-\tau_y}T_y e^{(1-\tau_y)\lambda(\mu_{2,1}m_h-\mu_{2,2}(\tilde{x}+\theta_A\mu_b)}\right)^{1-\nu\omega_y}\end{aligned}$$

A.5. Law of motion

Replacing κ_t, T_y, T_e and T_u obtained above in the law of motion for human capital

$$
\begin{aligned}
&\ln h' = \ln \xi_y + \alpha_z(1+\alpha_q(\varepsilon_z + \tau_m(1-\tau_u)\varepsilon_I)) \ln \xi_z + \tilde{\rho} \ln h \\
&+ \alpha_q \omega_I \left(\ln s + \ln(1+a_n) + \ln(1+a_u) + \tau_u \left(\ln \ell(1-a_y) + \lambda m_h + \lambda^2 \frac{\Sigma_h^2}{2} \right) + \frac{\tau_u}{1-\tau_u} \frac{\sigma_I^2}{2} \right) \\
&+ \alpha_q \omega_I (1-\tau_u)(1-\tau_n) \left(\ln \ell + \ln(1-a_y) + \tau_y \lambda m_h + \frac{\lambda^2}{2}(2-\tau_y)\tau_y \Sigma_h^2 \right) \\
&+ \alpha_q \omega_y \nu \theta_A \frac{\sigma_z^2}{2} + \alpha_q \omega_y (1-\tau_y) \lambda \mu_{2,1} m_h \\
&+ \alpha_q \omega_I (1-\tau_u) \Bigg[(\tau_n \lambda - \alpha_z \tau_m) m_h + \frac{\alpha_z \tau_m}{2}(1-\alpha_z \tau_m)\sigma_z^2 + \tau_n(\ln \ell(1-a_y)) \\
&- \left[\lambda^2 (1-\tau_y)^2 (\tau_n - 2)\tau_n + 2\lambda(1-\tau_n)(1-\tau_y)\tau_m \alpha_z + (\alpha_z \tau_m)^2 - \tau_n \lambda^2 (2-\tau_y)\tau_y \right] \frac{\Sigma_h^2}{2} \Bigg]
\end{aligned}
$$

We now take the expectation, we factorize out all the terms in m_h as well as all the terms in σ_z^2. The next steps consist in simplifying the coefficient in front of σ_z^2, of factorizing out all the terms in Σ_h^2 and in using the expression in (A.6) for σ_I^2. We also use the fact that $\mu_{2,1} = 1 - \mu_{2,2}(\theta_I + \theta_A)$. One obtains

$$
\begin{aligned}
&m_h' = \rho m_h - \frac{\sigma_y^2}{2} \\
&+ \left[\frac{\tau_u}{1-\tau_u} \left(\frac{\alpha_z(1-\tau_u)}{(1-\nu\omega_y)} (\tau_m + \omega_z(1-\tau_n)\nu) \right)^2 - \alpha_z \left(\alpha_q \left(\omega_z + \omega_I(1-\tau_u)(\tau_m)^2 \alpha_z \right) + 1 \right) \right] \frac{\sigma_z^2}{2} \\
&+ \alpha_q \omega_I \left(\ln \ell (1-a_y) s (1+a_u)(1+a_n) \right) \\
&+ \alpha_q \omega_I \Bigg[\lambda^2 + \frac{\tau_u}{1-\tau_u} \left(\frac{\alpha_z(1-\tau_u)}{(1-\nu\omega_y)} (\tau_m + \omega_z(1-\tau_n)\nu) \right)^2 \left(\frac{\omega_I}{\omega_z} + 1 \right)^2 \\
&\qquad - (1-\tau_u) \left[\lambda(1-\tau_y)(1-\tau_n) + (\alpha_z \tau_m) \right]^2 \Bigg] \frac{\Sigma_h^2}{2}
\end{aligned}
$$

with $\rho = \alpha_z + \alpha_y + \alpha_z \alpha_q \omega_z + \alpha_q \omega_I \lambda$. Finally taking the variance gives the expression for the law of motion of Σ_h^2: $\Sigma_h^{2'} = \sigma_y^2 + (\alpha_z(1+\alpha_q(\varepsilon_z + \tau_m(1-\tau_u)\varepsilon_I)))^2 \sigma_z^2 + \tilde{\rho}^2 \Sigma_h^2$

Gathering all our results, the law of motion of human capital is given by

$$\begin{aligned}
&\ln h_{t+1} \sim \mathcal{N}\left(m_{h,t+1}, \Sigma^2_{h,t+1}\right)\\
&m_{h,t+1} = \rho_t m_{h,t} + X_1\\
&\Sigma^2_{h,t+1} = \bar{\rho}_t^2 \Sigma^2_{h,t} + X_{2,t}(\Sigma_{h,t})\\
&\rho_t = \alpha_z + \alpha_y + \alpha_z\alpha_q\omega_z + \alpha_q\omega_I\lambda_t\\
&X_{1,t} \;= -\frac{\sigma_y^2}{2} + \left[\frac{\tau_{u,t}}{1-\tau_{u,t}}\left(\frac{\alpha_z(1-\tau_{u,t})}{(1-\nu_t\omega_y)}(\tau_{m,t}+\omega_z(1-\tau_{n,t})\nu_t)\right)^2\right.\\
&\qquad\qquad\left. -\alpha_z\left(\alpha_q\left(\omega_z+\omega_I(1-\tau_{u,t})(\tau_{m,t})^2\alpha_z\right)+1\right)\right]\frac{\sigma_z^2}{2}\\
&\quad + \alpha_q\omega_I \ln \ell_t(1-a_{y,t})s_t(1+a_{u,t})(1+a_{h,t})\\
&\quad + \alpha_q\omega_I\left[\lambda_t^2 + \frac{\tau_{u,t}}{1-\tau_{u,t}}\left(\frac{\alpha_z(1-\tau_{u,t})}{(1-\nu_t\omega_y)}(\tau_{m,t}+\omega_z(1-\tau_{n,t})\nu_t)\right)^2\left(\frac{\omega_{I,t}}{\omega_{A,t}}+1\right)^2\right.\\
&\qquad\qquad\left. -(1-\tau_{u,t})\left[\lambda(1-\tau_{y,t})(1-\tau_{n,t})+(\alpha_z\tau_{m,t})\right]^2\right]\frac{\Sigma^2_{h,t}}{2}\\
&X_{2,t} = \sigma_y^2 + \left(\alpha_z(1+\alpha_q(\varepsilon_z+\tau_m(1-\tau_u)\varepsilon_I))\right)^2\sigma_z^2
\end{aligned}$$

A.6. From the distribution of $\ln q$ to the distribution of $\ln I$.

Using the definition of q and the expression for $\bar{z}$ obtained earlier,

$$\ln q = \ln \tilde{I}^{\omega_I}\bar{z}^{\omega_z} = \omega_I \ln \tilde{I} + \omega_z\left(\frac{\alpha_z}{\theta_A}(\ln q - \theta_I\mu_2^q - x)\right)$$

which implies

$$\ln \tilde{I} = \frac{1}{\omega_I}\left(\ln q\left(1-\alpha_z\frac{\omega_z}{\theta_A}\right)+\alpha_z\frac{\omega_z}{\theta_A}(\theta_I\mu_2+x)\right).$$

Given the expression for μ_2^q $\mu_2^q = \mu_{2,1}m_h + \mu_{2,2}(\ln q - x - \theta_A\mu_b)$, one gets

$$\ln \tilde{I} = \frac{1}{\omega_I}\left(\ln q\left(1-\alpha_z\frac{\omega_z}{\theta_A}+\alpha_z\frac{\omega_z}{\theta_A}\theta_I\mu_{2,2}\right)+\alpha_z\frac{\omega_z}{\theta_A}\left(\theta_I\mu_{2,1}m_h+(1-\theta_I\mu_{2,2})x-\theta_I\mu_{2,2}\theta_A\mu_b\right)\right)$$

Hence from the distribution $\ln q$ we can recover the distribution of $\ln \tilde{I} \sim \mathcal{N}\left(\mu_{\tilde{I}}, \sigma^2_{\tilde{I}}\right)$ with

$$\mu_{\tilde{I}} = \frac{1}{\omega_I}\left(\mu_q\left(1-\alpha_z\frac{\omega_z}{\theta_A}+\alpha_z\frac{\omega_z}{\theta_A}\theta_I\mu_{2,2}\right)+\alpha_z\frac{\omega_z}{\theta_A}\left(\theta_I\mu_{2,1}m_h+(1-\theta_I\mu_{2,2})x-\theta_I\mu_{2,2}\theta_A\mu_b\right)\right)$$

and $\sigma_{\tilde{I}}^2 = \left(\frac{\alpha_z(1-\tau_u)}{(1-\nu\omega_y)}\left(\tau_m + \omega_z(1-\tau_n)\nu\right)\right)^2\left(\sigma_z^2 + \left(\frac{\omega_I}{\omega_z}+1\right)^2\Sigma_h^2\right)$

The last line stems from

$$\begin{aligned}\frac{1}{\omega_I}\left(1-\alpha_z\frac{\omega_z}{\theta_A}+\alpha_z\frac{\omega_z}{\theta_A}\theta_I\mu_{2,2}\right) &= \frac{1}{\omega_I}\left(1-\alpha_z\frac{\omega_z}{\theta_A}+\alpha_z\frac{\omega_z}{\theta_A}\theta_I\mu_{2,2}\right)\\ &= \frac{\alpha_z(1-\tau_u)}{\theta_A(1-\nu\omega_y)}\left(\tau_m+\omega_z(1-\tau_n)\nu\right)\end{aligned}$$

where we used $\nu = (1-\tau_y)\lambda\mu_{2,2}$ and $\varepsilon_l = \frac{\omega_l}{1-\nu\omega_y}$. Finally

$$\begin{aligned}\sigma_{\tilde{I}}^2 &= \left(\frac{\alpha_z(1-\tau_u)}{\theta_A(1-\nu\omega_y)}\left(\tau_m+\omega_z(1-\tau_n)\nu\right)\right)^2\sigma_q^2\\ &= \left(\frac{\alpha_z(1-\tau_u)}{(1-\nu\omega_y)}\left(\tau_m+\omega_z(1-\tau_n)\nu\right)\right)^2\left(\sigma_z^2+\left(\frac{\omega_I}{\omega_z}+1\right)^2\Sigma_h^2\right)\end{aligned}$$

Since $\ln\tilde{I} = \ln I - (1-\tau_u)\frac{\sigma_u^2}{2}$ and σ_u^2 is common across all colleges, we have $\ln I \sim \mathcal{N}\left(\mu_I, \sigma_I^2\right)$with $\mu_I = \mu_{\tilde{I}} + (1-\tau_u)\frac{\sigma_u^2}{2}$ and $\sigma_I^2 = \sigma_{\tilde{I}}^2$

Expression for σ_u^2 Given that all households save a fraction s of their disposable income and the selection equation into college, one gets

$$\begin{aligned}\ln e_u = \ln\frac{(1+a_n)s}{T_e} &+ \tau_m\frac{\alpha_z}{\theta_A}\left(\ln q - x\right) + \ln h^{(1-\tau_n)(1-\tau_y)\lambda-\tau_m\frac{\alpha_z}{\theta_A}\theta_I)}\\ &+ (1-\tau_n)\ln T_y(1-a_y)\ell^{(1-\tau_y)}\end{aligned}$$

Hence the within-university variance of tuitions is given by:

$$\sigma_u^2 = \left((1-\tau_n)(1-\tau_y)\lambda - \tau_m\frac{\alpha_z}{\theta_A}\theta_I\right)^2\sigma_2^2 = \left((1-\tau_y)\lambda\frac{(1-\tau_n)\omega_z+\tau_m\omega_y}{\omega_z+\omega_I(1-\tau_u)\tau_m}\right)^2\sigma_2^2$$

which is indeed constant across universities wince σ_2^2 is an aggregate constant.

A.7. Details on the Positioning Game

In this appendix we give a formal explanation of the positioning game as well as a characterization of the equilibrium. Recall the general environment. There is a continuum of colleges $j \in [0,1]$. At each generation $t \in \mathbb{N}$, they play a positioning game. The games played at any two generations $t > t'$ are independent of each other.

At a given generation $t \in \mathbb{N}$, and before playing the positioning game, each college is given a real number $o \in [0,1]$. The positioning game is sequential and o is the order in which colleges play. Without loss of generality, since all colleges are identical, one can relabel colleges $j = o$ so that their label is also their order.[38] Colleges play sequentially in descending order: j plays before j' if and only if $j > j'$. Each college plays once.

All colleges have the same set of actions: the line of qualities $q \in \mathbb{R}_+$. The history of the sequential game up to college j's turn is a (injective) function $\mathscr{H}_j^+ : (j,1] \to \mathbb{R}_+$ that describes the colleges' actions up to j's turn. A strategy for college j is a choice of quality $q \in \mathbb{R}_+$ whenever it is its turn; abusing notation we denote it $q_j(\mathscr{H}_j^+)$. Denoting $\mathscr{H}_j : [j,1] \to \mathbb{R}_+$ the history including college j's action, one has, for all $k > j$, $\mathscr{H}_j(k) = \mathscr{H}_j^+(k)$ and $\mathscr{H}_j(j) = q_j(\mathscr{H}_j^+)$. $\mathscr{H}_0$ denotes a terminal history.

We now introduce the notion of the *set of available students* at quality q at history $\mathscr{H}_j$. Denote $S(q) \subset \mathscr{I}$ the subset of students who demand quality q and $card(S(q))$ its cardinal, similarly denote $S(q, \mathscr{H}_j)$ the subset of students demanding quality q who are not in a college yet after history $\mathscr{H}_j$ (we call it the set of available students).

The cardinality of the set of available students at each quality to colleges that play later $j' \leq j$ is a function of the positions of colleges that have already played, $j' > j$, because when college j chooses quality q it takes a subset of these students, $S(q, \mathscr{H}_j) \subset S(q, \mathscr{H}_j^+)$. More specifically, we assume that college j picks a subset of students of cardinality $\aleph_1$ (its assumed size). We further assume that at any history $\mathscr{H}_j^+$, if $card(S(q), \mathscr{H}_j^+) \leq \aleph_1$ and j chooses q, then college j takes all the students at quality q and $card(S(q, \mathscr{H}_{j'})) = 0$ for all $j' \leq j$.[39] If $card(S(q, \mathscr{H}_j^+)) > \aleph_1$, we assume that college j picks a subset of students of cardinality $\aleph_1$ which implies $card(S(q), \mathscr{H}_j) > \aleph_1$.[40]

[38] This assumption of an order across colleges captures in a very direct way the notion that colleges do not start on an equal foot in the competition for prestige. In the real world, there are slow-moving state variables that gives an advantage to some colleges in this race, such as their reputation, their faculty member, their stock of publications, their endowment. Our assumption should be seen as a reduced-form expression of this *ex ante* hierarchy of advantages created by these state variables that this paper abstracts from.

[39] This is indeed a restriction, and not a tautology. It would be possible for a countable number of colleges to offer the same quality q and still respect the size constraint since a countable set of set of cardinal $\aleph_1$ is still of cardinal $\aleph_1$. It is however an inconsequential restriction which allows to associate one college with one quality since in equilibrium it is true that $card(S(q)) = \aleph_1$.

[40] Although this case might arise in some other version of the model, it doesn't happen in any equilibria analyzed in this paper.

Recall that the objective of the college is to deliver the highest quality possible. If they faced no constraint, they would all choose to deliver the highest quality. All colleges would like to be Harvard (or Princeton), but there is room for only one. This notion is captured by the size constraint: colleges can't be too small. Specifically, if at history $\mathscr{H}_j^+$ the set of available students at q is lower than the cardinality of the continuum, $card(S(q, \mathscr{H}_j^+)) < \aleph_1$, the payoff of college j if it chooses q is 0, and we say that college j is *not operating.*[41] If $card(S(q, \mathscr{H}_j^+)) \geq \aleph_1$, and college j chooses q, then its payoff is simply q and we say that it is *operating*.

This induces a preference relationship over the set of possible terminal histories. Consider any two terminal histories $\mathscr{H}_0, \mathscr{H}_0'$ in which college j is operating. College j prefers $\mathscr{H}_0$ to $\mathscr{H}_0'$, $\mathscr{H}_0 \succsim \mathscr{H}_0'$ if and only if $\mathscr{H}_0(j) = q_j \geq q_j' = \mathscr{H}_0'(j)$ with strict preference for strictly higher quality. A college always prefers a terminal history in which it is operating over one in which it is not.

Denote $q^*_{<j} = \{q^*_k(\mathscr{H}_k^+)\}_{k \in [0,j)}$ the strategy profile of colleges playing (strictly) after j and $\mathscr{H}_0\left(\mathscr{H}_j^+, q_j(\mathscr{H}_j^+), q_{<j}\right)$ the terminal history that follows history $\mathscr{H}_j^+$ and induced by the strategies of college j, $q_j(\mathscr{H}_j^+)$ and of the colleges playing afterwards $q_{<j}$. A subgame perfect Nash equilibrium of this game is a strategy profile $\{q^*_j(\mathscr{H}_j^+)\}_{j \in [0,1]}$ such that for all j, given the strategies of the colleges playing next $q^*_{<j}$

$$\mathscr{H}_0\left(\mathscr{H}_j^+, q^*_j(\mathscr{H}_j^+), q^*_{<j}\right) \succsim \mathscr{H}_0\left(\mathscr{H}_j^+, q, q^*_{<j}\right)$$

for all $q \in \mathbb{R}_+$.

Detail on the index set of households, $\mathscr{I}$. To be consistent with the notion that there is a continuum of colleges and a continuum of students within each college, it has to be the case that the cardinality of the set of students be strictly higher than the cardinality of the set of colleges, i.e. $card(\mathscr{I}) > card([0,1]) = \aleph_1$. It seems natural to consider the smallest such cardinal. Using the axiom of choice, such a cardinal is $\aleph_2$. To fix ideas, this corresponds for example to the index set $\mathscr{I} = [0,1]^{[0,1]}$.

Assumption 1. *The cardinal of the set of households is the same as the continuum of continua*

$$card(\mathscr{I}) = \aleph_2$$

[41] The size constraint is what makes the game strategic: the positioning decisions of higher-ranked colleges influence the payoffs of lower-ranked colleges.

Equilibrium Characterization. The following lemma says that the quality delivered by each college follows the same order as the order in which colleges play the game.

Lemma 2. *Assume the distribution of students over quality is continuous over* $\mathbb{R}_+$*. Then in equilibrium,*

$$q_j > q_{j'} \iff j > j'.$$

Proof. Since the distribution is continuous over $\mathbb{R}_+$ and there are a cardinal $\aleph_2$ of students, there must be a cardinal $\aleph_1$ of students demanding a given quality q, *i.e.* $card(S(q)) = \aleph_1$ for all $q \in \mathbb{R}_+$. (Otherwise there would be a mass point at some q, contradicting the assumption of a continuous distribution). Hence, by the assumption made earlier, whenever college j chooses a location q that is unoccupied $card(S(q, \mathscr{H}_j^+)) = \aleph_1$, it takes all of its students and no students is left for a college playing later, $card(S(q, \mathscr{H}_{j'})) = 0$ for all $j' \leq j$. This implies that if there exists $\underline{q}$ such that the history up to j is bounded on the left by $\underline{q}$: $\mathscr{H}_j^+((j,1]) = (\underline{q}, +\infty)$, then a college j's optimal location is $\underline{q}$: choosing strictly above $\underline{q}$ would mean not operating by the previous argument, and choosing exactly $\underline{q}$ rather than a strictly lower quality is strictly preferred. This shows that in any equilibrium in which the distribution for quality demanded is continuous over $\mathbb{R}_+$, for any $j > j'$, one has $q_{j'} < q_j$.

□

A.8. Existence and Uniqueness of Equilibrium Path

The set of equations defining an equilibrium path in proposition 4.3 is block-recursive. In particular, the law of motion of Σ_h, is independent and the path of all other variables are pinned-down by the path of Σ_h. It is therefore necessary and sufficient to focus on the existence and uniqueness of the path of Σ_h. We first define new notations:

$$\begin{aligned}
\Sigma_h'^2 &= f(\Sigma_h^2) \\
&= \left[\alpha_z^2 + \left(\frac{A}{1-\nu\omega_y}\right)^2 + \frac{2\alpha_z A}{1-\nu\omega_y}\right]\Sigma_h^2 + \sigma_y^2 + \left[\alpha_z^2 + \frac{B^2}{(1-\nu\omega_y)^2} + \frac{2B\alpha_z}{1-\nu\omega_y}\right]\sigma_z^2
\end{aligned}$$

with $A = \alpha_z\alpha_q(\omega_z + \tau_m(1-\tau_u)\omega_I) + \alpha_q\left(\omega_I(1-\tau_u)(1-\tau_n) - \omega_y\right)(1-\tau_y)\lambda$

$$B = \alpha_z\alpha_q\left(\omega_z + \tau_m(1-\tau_m)\omega_I\right) \qquad \nu = \frac{C}{E\Sigma_h^{-2} + (E+\omega_y)C}$$

$$C = \left(\frac{\omega_z}{\omega_I + \omega_z}\right)^{-2}\sigma_z^{-2} \qquad E = (\omega_I(1-\tau_u)(1-\tau_n) - \omega_y) + \frac{\omega_z}{(1-\tau_y)\lambda}$$

$f(.)$ is differentiable for $\Sigma_h^2 \in (0,\infty)$ and $\lim_{\Sigma_h^2\to 0} f(\Sigma_h^2) = \sigma_y^2 + [\alpha_z^2 + B^2 + 2B\alpha_z]\,\sigma_z^2 > 0$. The derivative $f'(.)$ is:

$$\begin{aligned}
&\left[\alpha_z^2 + \left(\frac{A}{1-\nu\omega_y}\right)^2 + \frac{2\alpha_z A}{1-\nu\omega_y}\right] \\
&+ \left[\left[\left(\frac{A}{1-\nu\omega_y}\right)^2 + \frac{\alpha_z A}{1-\nu\omega_y}\right]\Sigma_h^2 + \left[\frac{B^2}{(1-\nu\omega_y)^2} + \frac{B\alpha_z}{1-\nu\omega_y}\right]\sigma_z^2\right]\frac{2\omega_y}{1-\nu\omega_y}\frac{\partial\nu}{\partial\Sigma_h^2}
\end{aligned}$$

with $\dfrac{\partial\nu}{\partial\Sigma_h^2} = \dfrac{CE}{\left(E + \Sigma_h^2(E+\omega_y)C\right)^2}$

$$\begin{aligned}
\text{Hence} \quad \lim_{\Sigma_h^2\to\infty}\frac{\partial f}{\partial\Sigma_h^2} &= \left[\alpha_z^2 + \left(\frac{A}{1-\frac{\omega_y}{E+\omega_y}}\right)^2 + \frac{2\alpha_z A}{1-\frac{\omega_y}{E+\omega_y}}\right] \\
&= \left[\alpha_z + \alpha_z\alpha_q(\omega_z + \tau_m(1-\tau_m)\omega_I) + \alpha_q[\omega_I(1-\tau_u)(1-\tau_n) - \omega_y](1-\tau_y)\lambda\right]^2
\end{aligned}$$

Therefore if $\left[\alpha_z + \alpha_z\alpha_q(\omega_z + \tau_m(1-\tau_m)\omega_I) + \alpha_q[\omega_I(1-\tau_u)(1-\tau_n) - \omega_y](1-\tau_y)\lambda\right]^2 < 1$, the equation $\Sigma_h^2 = f(\Sigma_h^2)$ has at least one solution since f is continuous and $\lim f(0) > 0$. Moreover, it has to be that an odd number of these solutions are characterized by $f'(\Sigma_h) < 1$, which guarantees local stability of the equilibrium path around these solutions.

Let's now show that the equilibrium path is unique for ω_y small enough. A first order approximation of f in the neighborhood of $\omega_y = 0$ is

$$
\begin{aligned}
f(\Sigma_h^2) &\simeq \left[\alpha_z^2 + A^2 + 2\alpha_z A\right]\Sigma_h^2 \\
&\quad + \sigma_y^2 + \left[\alpha_z^2 + B^2 + 2B\alpha_z\right]\sigma_z^2 + \left[\left[A^2 + \alpha_z A\right]\Sigma_h^2 + \left[B^2 + \alpha_z B\right]\sigma_z^2\right] 2\nu\omega_y \\
f'(\Sigma_h^2) &\simeq \left[\alpha_z^2 + A^2 + 2\alpha_z A\right] \\
&\quad + \underbrace{\left(\left[\left[A^2 + \alpha_z A\right]\Sigma_h^2 + \left[B^2 + \alpha_z B\right]\sigma_z^2\right]\frac{E}{E + EC\Sigma_h^2} + \left[A^2 + \alpha_z A\right]\Sigma_h^2\right)\frac{C}{E + EC\Sigma_h^2} 2\,\omega_y}_{F(\Sigma_h^2)}
\end{aligned}
$$

with$\nu = \frac{C}{E\Sigma_h^{-2} + EC}$. Since we have assumed that $[\alpha_z^2 + A^2 + 2\alpha_z A] < 1$, and $F(\Sigma_h^2)$ is bounded for $\Sigma_h^2 \in (0, \infty)$, there exists an ω_y small enough such that for all Σ_h^2, $\frac{\partial f(\Sigma_h^2)}{\partial \Sigma_h^2} < 1$. This is sufficient for the existence and uniqueness of a globally stable steady-state.

A.9. Rise in returns to human capital

The total derivative of the IGE with respect to λ is given by

$$
\begin{aligned}
&\left[\frac{\partial \nu}{\partial \lambda} + \frac{\partial \nu}{\partial \Sigma_h^2}\frac{\partial \Sigma_h^2}{\partial \lambda}\right]\left[\alpha_z\alpha_q\left(\frac{\partial \varepsilon_z}{\partial \nu} + \tau_m\frac{\partial \varepsilon_I}{\partial \nu}\right) + \alpha_q\left(\frac{\partial \varepsilon_I}{\partial \nu}(1-\tau_n) - \frac{\partial \varepsilon_y}{\partial \nu}\right)(1-\tau_y)\lambda\right] \\
&+ \alpha_q(\varepsilon_I(1-\tau_n) - \varepsilon_y)(1-\tau_y)
\end{aligned}
$$

We then compute the derivatives:

$$
\frac{\partial \varepsilon_l}{\partial \nu} = \varepsilon_l\varepsilon_y > 0 \qquad \frac{\partial \nu}{\partial \Sigma_h^2} = \frac{CE}{(E + (E+\omega_y)C\Sigma_h^2)^2} > 0
$$

with C and E have been defined in the proof of existence and uniqueness.

$$
\begin{aligned}
\frac{\partial \nu}{\partial \lambda} &= \frac{2C\left(\frac{\omega_z}{\omega_z+\omega_I}\right)\frac{1}{\omega_I}\left[E\Sigma_h^{-2} + \omega_y C\right] + C\frac{\omega_z}{(1-\tau_y)\lambda^2}}{\left(E\Sigma_h^{-2} + (E+\omega_y)C\right)^2} > 0 \\
\frac{\partial X_2}{\partial \lambda} &= \sigma_z^2\alpha_z\left(1 + \alpha_q(\varepsilon_z + \tau_m\varepsilon_I)\right)\alpha_z\alpha_q\varepsilon_y(\varepsilon_z + \tau_m\varepsilon_I)\varepsilon_I\frac{\partial \nu}{\partial \lambda} > 0 \\
\frac{\partial \Sigma_h^2}{\partial \lambda} &= \frac{\frac{\partial X_2}{\partial \lambda} + \Sigma_h^2 2\frac{\partial \tilde{\rho}}{\partial \lambda}\tilde{\rho}}{1 - (\tilde{\rho})^2 - \Sigma_h^2 2\frac{\partial \tilde{\rho}}{\partial \Sigma_h^2}\tilde{\rho} - \frac{\partial X_2}{\partial \Sigma_h^2}} > 0
\end{aligned}
$$

where $\frac{\partial \tilde{\rho}}{\partial \lambda}$ has to be understood as the partial derivative of $\tilde{\rho}$ w.r.t. λ keeping Σ_h^2 constant. The last line stems from the fact that the steady-state is locally stable - which requires that $1-(\tilde{\rho})^2-\Sigma_h^2 2\frac{\partial \tilde{\rho}}{\partial \Sigma_h^2}\tilde{\rho}-\frac{\partial X_2}{\partial \Sigma_h^2}=\frac{\partial (\Sigma_h')^2}{\partial (\Sigma_h)^2}>0$. Hence, putting everything together yields

$$\begin{aligned}\frac{\partial \tilde{\rho}}{\partial \lambda} &= \left[\underbrace{\frac{\partial \nu}{\partial \lambda}+\frac{\partial \nu}{\partial \Sigma_h^2}\frac{\partial \Sigma_h^2}{\partial \lambda}}_{>0}\right]\varepsilon_y[\alpha_z\alpha_q\left(\varepsilon_z+\tau_m\varepsilon_I\right) \\ &\quad +\alpha_q\left(\varepsilon_I(1-\tau_n)-\varepsilon_y\right)(1-\tau_y)\lambda]+\alpha_q(\varepsilon_I(1-\tau_n)-\varepsilon_y)(1-\tau_y)>0.\end{aligned}$$

This proves not only that the steady-state IGE is increasing in λ but that the variance of human capital in the economy is as well. Given that the variance of market income is given by $\lambda^2\Sigma_h^2$ it is immediate that it increases too. Turning to the private spending on higher education, given by s, it is immediate to see from the expressions (14) and (16) that it is increasing in the future path of λ. Let's now turn to the ratio of within college variance of (log) parental income over economy-wide variance of (log) income:

$$\frac{V(\ln y|q)}{V(\ln y)}=\frac{1}{\lambda^2\Sigma_h^2}\lambda^2\frac{\Sigma_h^2\left(\frac{\theta_A}{\varepsilon_I+\theta_A}\right)^2\sigma_z^2}{\Sigma_h^2+\left(\frac{\theta_A}{\varepsilon_I+\theta_A}\right)^2\sigma_z^2}=\frac{\left(\frac{\theta_A}{\varepsilon_I+\theta_A}\right)^2\sigma_z^2}{\Sigma_h^2+\left(\frac{\theta_A}{\varepsilon_I+\theta_A}\right)^2\sigma_z^2}$$

$$\Rightarrow \frac{\partial\frac{V(\ln y|q)}{V(\ln y)}}{\partial\lambda}=\frac{\sigma_z^2\frac{\partial B}{\partial\lambda}\left[\Sigma_h^2+\left(\frac{\theta_A}{\varepsilon_I+\theta_A}\right)^2\sigma_z^2\right]-B\sigma_z^2\left[\frac{\partial\Sigma_h^2}{\partial\lambda}+\sigma_z^2\frac{\partial B}{\partial\lambda}\right]}{\left[\Sigma_h^2+\left(\frac{\theta_A}{\varepsilon_I+\theta_A}\right)^2\sigma_z^2\right]^2}=\frac{\sigma_z^2\frac{\partial B}{\partial\lambda}\Sigma_h^2-B\sigma_z^2\frac{\partial\Sigma_h^2}{\partial\lambda}}{\left[\Sigma_h^2+\left(\frac{\theta_A}{\varepsilon_I+\theta_A}\right)^2\sigma_z^2\right]^2}<0$$

with $B=\left(\frac{\theta_A}{\varepsilon_I+\theta_A}\right)^2$ and since $\frac{\partial\Sigma_h^2}{\partial\lambda}>0$ and $\frac{\partial B}{\partial\lambda}<0$.

Finally the variance of (log) college quality is given by $\theta_A^2\sigma_z^2+(\varepsilon_I+\theta_A)^2\Sigma_h^2$. It is immediate that it increases with λ since $\varepsilon_I,\theta_A,\Sigma_h^2$ increase with λ.

Monotonic transition path. From the law of motion of Σ_h^2, in the first period the initial increase in λ raises $\tilde{\rho}$ and triggers the initial increase in the dispersion of human capital. Since $X_2(\Sigma_h)$ and $\tilde{\rho}(\Sigma_h)$ are both increasing in Σ_h it further increases Σ_h^2 at the following period and so on... This establishes that Σ_h^2 is strictly increasing over the transition path. This also establishes the monotonic increase in $\tilde{\rho}$ and all ω's.

Turning to the private spending on higher education, given by s, it is easy to see that it is increasing in the future path of λ, $\tilde{\rho}$ and ε_I. Since these three variables are increasing over the transition path, s also increases. The variance of log college quality is also increasing because $\varepsilon_I, \theta_A, \Sigma_h^2$ are increasing over the transition path. The ratio of within college variance of (log) parental income over economy-wide variance of (log) income will decrease monotonically over the transition path because of the initial increase in λ, this is the first term in the derivative $\sigma_z^2 \frac{\partial B}{\partial \lambda} \Sigma_h^2$, and then decreases further as Σ_h increases, this is the second term $B\sigma_z^2 \frac{\partial \Sigma_h^2}{\partial \lambda}$.

A.10. The College Problem in the Quantitative Version

In order to keep the college problem tractable despite the loss of closed-form expressions for the distribution of students within the college and equilibrium tuition, we assume that the problem of the college is still given by (43), even if it is not possible to derive (43) from the primitive problem (9) since the within-college distribution of students isn't joint log-normal anymore.

The alternative way to microfound (43) is to assume that there is a loss in the efficiency with which resources are used when the inequality of tuition fees among students rises, i.e. that σ_u^2 is given by (42), a measure of the dispersion of tuition within the college, instead of the within-college heterogeneity in students. One can interpret it as a rise in human resources and administrative costs or as an increase in the sentiment of unfairness among students when tuition fees become more heterogeneous among students. The first order conditions for this problem are the same and the equilibrium tuition schedule is identical.