The Welfare Effects of Decentralizing Public Goods *

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Abstract

This paper analyzes the welfare impacts of fiscal decentralization. Using detailed municipal budget data in France, we uncover significant heterogeneity in both the level and the composition of public goods across localities. We show that localities with varying income levels and population sizes systematically allocate their budgets differently across varieties of public goods. To explain these patterns, we develop a quantitative spatial equilibrium model featuring households with non-homothetic preferences over multiple public goods, heterogeneous public good rivalry, local voting on tax rates and public goods bundles, and federal transfers. Our simulations reveal significant welfare gains from decentralization, primarily driven by the ability to tailor the mix of public goods to local preferences and costs. Household mobility significantly amplifies these gains.

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1 Introduction

The decentralization of public goods provision can in theory increase or decrease social welfare. One view holds that decentralization enhances welfare by better aligning provision with local preferences (Oates, 1972) and by promoting efficiency through inter-municipal competition (Tiebout, 1956). Another argues that decentralization exacerbates spatial household inequalities, generates fiscal externalities from migration, and introduces distortions via property taxation (Calabrese, Epple and Romano, 2012). Quantitative analyses of decentralization have supported both perspectives: while Oates (1972) finds positive welfare gains, Calabrese, Epple and Romano (2012) report welfare losses. Despite their opposite conclusions, these studies have in common a shared focus on the *overall level of local public spending*, which comes with an implicit assumption that the mix of public goods is uniform across locations.

In this paper, we hypothesize that considering the *mix* of public goods is crucial when evaluating the welfare effects of decentralization. This hypothesis is motivated by the substantial heterogeneity in local public goods composition that we uncover using micro-data on municipal spending in France. This observed dispersion suggests either varying household preferences for combinations of public goods or differing costs of provision across locations. Decentralization, by allowing local residents to tailor the mix of public goods to local preferences and costs, could therefore yield benefits greater than those suggested by total local spending alone. To test this hypothesis, we develop a spatial equilibrium model where local voting determines the provision of public goods. A key innovation of the model lies in incorporating different varieties of public goods, non-homothetic household preferences over these varieties, and varying degrees of rivalry across public goods. Simulations of our quantitative model show that the welfare gains of decentralization are large and primarily driven by the mix of public goods.

We begin by presenting evidence on the substantial diversity in the provision of local public services across municipalities. We first show that there are significant disparities in the total spending per capita at the local level, which is driven in part by different tax rates but to a greater extent by different tax bases, in line with previous evidence. More importantly, we find that the mix of public goods varies greatly across municipalities. Local choices play a crucial role in determining the mix of public goods: 76% of the variation in spending per capita across different public goods stem from different allocation choices while 24% stem from different total spending. Additionally, we show that this heterogeneity in the local property taxation and the mix of public goods is significantly related to local median household incomes and population sizes. For example, median households in high-income municipalities contribute a lower share of their budget to property taxes than median households in low-income municipalities. Regarding the mix of public goods, the budget share allocated to education is about 60% higher in the richest municipalities compared to the poorest ones. Similarly, the share of spending on social services nearly triples between the smallest and largest municipalities, whereas spending on general administrative services as a share of budgets declines.

These new facts leverage a granular administrative dataset on French municipalities' budgets, which to our knowledge, we are the first to use to analyze spending on different public goods. Municipalities are obligated to maintain detailed records of their finances, which they submit annually to higher levels of government. These accounts are highly disaggregated and break down municipal spending according to input (e.g., spending on personnel, furniture, building maintenance, etc.) and function (e.g., theaters, libraries, primary schools, police, etc.). The disaggregation by function allows us to characterize differences in the public service bundle across municipalities in terms of their relative and total spending. We complement these accounts with census data on household characteristics as well as rental price indices computed at the municipality level.

We rationalize these facts through the lens of a novel spatial equilibrium model. Households make private decisions about where to live and how much to spend on housing, and rents clear the local housing markets in each location. Importantly, households have non-homothetic preferences over several public goods, and they choose collectively and locally on the provision of public goods through voting. Utility over public goods follows a non-homothetic CES function with income elasticities that are specific to each public good. These non-homothetic preferences build on a stream of recent papers including those by Comin and Mestieri (2018), Borusyak and Jaravel (2018), Matsuyama (2019), and recently Hoelzlein (2021). These non-homotheticities rationalize the fact that municipalities with different median voters choose different tax rates and different bundles of public goods.

Another important novelty of the model is to allow for heterogeneous degrees of rivalry and congestibility across public goods. This feature helps rationalize why different municipalities of different sizes make different choices. These two features—non-homotheticities and varying degrees of rivalry—give rise to a "public good price index" which vary across communes depending on population size and composition of public goods. This new price index is a key determinant of sorting and welfare.

An important theoretical contribution of our model is to embed a tractable collective choice at the local level over multiple public goods and a tax rate in a framework with non-homothetic preferences. The collective decisions are taken through votes at the pairwise majority rule in each dimension *independently*, which we call a Kramer-Shepsle equilibrium following Kramer (1972), Shepsle (1979) and De Donder, Le Breton and Peluso (2012a). This equilibrium concept is appealing because it addresses several issues in settings with multidimensional voting. Instead, in our model, the resulting equilibrium is unique, independent of the ordering in which collective decisions are taken and implements the preferred policy of the median voter under some conditions.

We calibrate the model to target several important facts of the micro-data. More specifically, we estimate the non-homotheticity of preferences and varying degree of congestibility by matching the empirical cross-sectional relationship between municipal budget shares and the tax rate on the one hand, and median household welfare and population on the other. The results provide us with reasonable parameters that concord with our descriptive facts. Amenities, land supply and technology parameters are recovered by inverting the model to exactly match the spatial distribution of population, rents and average income across the full set of 35000 communes in France.

We use the estimated model to look at the aggregate and distributional effects of the decentralization of public goods provision on welfare. More specifically, we compare a counterfactual equilibrium in which the provision of public goods, both the property tax rates and the budget allocations, are centralized and common to all municipalities, to the actual decentralized equilibrium. We also provide additional counterfactuals that isolate the role of specific mechanisms, such as the role of property tax rates, of budget allocations, or of migration. Decentralization policies face trade-offs. On the one hand they allow local communities to align the level of property taxation and the bundle of public goods to local preferences and costs. On the other hand, decentralization policies entail several inefficiencies and they may increase inequality across people and places. These comparisons deliver five important findings.

First decentralization leads to substantial welfare gains. We find that the total effect on overall welfare is positive and economically large. Quantitatively, the welfare gains are equivalent to an increase in local municipal revenues per capita by 62%. Second, these gains are entirely driven by decentralizing the composition of public goods: the revenue equivalent of the increase in welfare coming from this margin alone is 45%. In comparison, the decentralization of property tax rates generates small aggregate welfare losses. This finding is novel relative to the literature, which typically focuses solely on the overall quantity of public goods. This highlights that the mix of public goods is a crucial margin to take into account when evaluating the welfare effects of decentralization. In particular, ignoring this margin and focusing solely on the overall quantity of public goods would lead to the opposite conclusion that decentralization provides no significant welfare improvements due to the distortions implied by property taxation, consistent with the literature (Calabrese, Epple and Romano, 2012). By explicitly modeling the heterogeneity in public goods composition, our analysis reveals welfare benefits that these previous studies have missed.

Third we find that these welfare gains are driven by large cities. These cities choose very different bundles of public goods, in particular they choose to spend relatively less on non-rival goods by taking advantage of the returns to scale. Heterogeneity in city size is a much more important driver of welfare gains than heterogeneity in household types. Fourth we find that the gains from better alignment with local preferences and costs are amplified by household mobility: the welfare gains of decentralization are three times larger when households are allowed to relocate than when they are not. Finally, welfare inequality increases with decentralization, mainly between small and large cities. We find very limited evidence of an increase in inequality across household types.

Our paper relates to the literature analyzing the implications of fiscal decentralization and federalism, which dates back to Oates (1969, 1972) and Tiebout (1956). We refer to Oates (1999), Scotchmer (2002) and Epple and Nechyba (2004) for comprehensive reviews. A stream of this literature endogenizes the provision of public goods, focusing either on the overall level or on education spending (Fernandez and Rogerson, 1996; Benabou, 1996; Calabrese, Epple and Romano, 2012; Epple, Romano and Sarpça, 2018). Another stream of papers builds rich quantitative spatial equilibrium models in which local policies are exogenous (Eckert and Kleineberg, 2019; Fajgelbaum et al., 2019). We contribute by incorporating into a quantitative spatial equilibrium model, endogenous provision of several local public goods through local voting, non-homothetic preferences and varying degrees of rivalry across public goods.

Closest to us, Calabrese, Epple and Romano (2012) develop and calibrate a quantitative model where households with different incomes and tastes sort into neighborhoods in a city and subsequently vote on a property tax rate to fund schooling. Our paper integrates similar mechanisms on property taxes, but accommodates several public goods, non-homothetic preferences and varying degree of congestibility. Our results show that it is crucial to consider the local mix of public goods to fully account for the welfare gains of decentralization.

Our paper also adds to a growing literature that studies household inequalities within cities and across regions using spatial equilibrium models. One strand of this literature has emphasized the role of endogenous adjustments of amenities to explain patterns of household sorting (e.g. Guerrieri, Hartley and Hurst, 2013; Ahlfeldt et al., 2015; Diamond, 2016). Our paper shows that public goods are an important endogenous amenity to understand the location decision of households. Recent papers by Almagro and Domínguez-Iino (2020) and Hoelzlein (2021) also consider heterogeneous demands for residential amenities in an urban context, with Hoelzlein (2021) proposing a non-homothetic demand framework for non-tradable services that is similar to our characterization of public services.

This paper contributes to the literature on estimating preferences for local public goods, which stems from Rosen (1974)'s work on capitalizing local spending and tax decisions into land markets. This approach has been used to estimate willingness-to-pay for public education (Black, 1999; Bayer, Ferreira and McMillan, 2007). An alternative method directly estimates demand functions for public goods, as in seminal works by Borcherding and Deacon (1972), Bergstrom and Goodman (1973), and Rubinfield, Shapiro and Roberts (1987), who link per-capita spending to local demographics and median incomes. Epple and Sieg (1999) further advance this by structurally estimating demand parameters using a complete residential choice model. Our paper extends these methodologies by estimating demand for diverse public goods while incorporating non-homothetic preferences and varying degrees of congestibility.

The paper is organized as follows. Section 2 introduces our data on French municipalities' budgets and establishes new facts on revenue sources, and allocations of spending. Section 3 presents our model. Section 4 explains properties of the model, focusing on the endogenous local collective choice of public goods. Section 5 presents our estimation strategy. Section 6 uses the estimated model to understand the implications of decentralization policies in France. Section 8 concludes.

2 Facts about the Public Finances of Municipalities

This section presents key empirical patterns in French municipal public finance. Importantly, we document substantial heterogeneity in both the level and composition of public goods provision across municipalities. These motivate our hypothesis that the mix of public goods is important to consider when evaluating the welfare impact of decentralization. We then show that this heterogeneity is significantly related to the local median income and population size.

2.1 Institutional Setting

France's local government structure is an ideal environment for studying fiscal decentralization due to its extensive network of municipalities (*communes*) with considerable fiscal autonomy. France is divided into nearly 35,000 municipalities, which are the smallest administrative units in the country and vary significantly in size, from Paris with over 2 million inhabitants to rural communes with fewer than 100 residents.

Municipalities in France have substantial fiscal independence, including the authority to set local tax rates and influence the allocation of public budgets. This independence is especially evident in sectors like primary education, local public infrastructure, and social services, where municipalities can determine the level of local spending on goods such as primary school infrastructure, road maintenance, public transportation, housing assistance, and community health services. Similarly, decisions regarding cultural and recreational facilities—such as funding for libraries, sports complexes, and parks—are all made locally. This autonomy is balanced by the central control of public services that are relevant to national priorities. For instance, major public services like healthcare or secondary and higher education are predominantly funded and regulated by higher levels of government.

Municipalities in France make use of several fiscal instruments to collect revenue. The property tax (*taxe foncière*) and housing tax (*taxe d'habitation*) are historically the primary sources of local revenue, although the latter was abolished in 2023 for primary residences. Both of these taxes are levied against property valuations that were last assessed in the 1970s. Most municipalities also levy a business tax (*taxe professionnelle*) on firms operating within their jurisdiction. In addition to local revenue sources, federal support plays an important role in municipal finances, primarily through a system called the *dotation globale de fonctionnement* (DGF). This grant system includes various components, such as a population-based allocation and a progressive element designed to address fiscal disparities.

2.2 Data

To establish key facts about local public finances in municipalities and to later estimate our model, we draw on data from multiple sources.

Municipal finances. We use a novel administrative dataset on municipal public finances in France, spanning from 2012 to 2022. The data, obtained from the Ministry of Economy and Finance (DGFiP), are derived from mandatory financial reports that municipalities submit to higher governmental authorities. These reports offer granular information about municipal fiscal activities, including revenue streams and spending patterns. To our knowledge, this paper is the first to leverage this comprehensive dataset on public finances.

For municipalities with populations exceeding 3,500 residents, expenditures are disaggregated by "function" (e.g., theaters, libraries, primary schools, police) using a multi-digit classification system. This detailed disaggregation enables us to identify heterogeneity in public service bundles across municipalities. We consolidate these functions into ten broader, one-digit categories: administrative services, planning (including transportation), parks, health and safety, education, recreation, arts and culture, youth services, social services, and economic services.

Detailed descriptions of these categories are provided in Appendix A.

To smooth over annual volatility in spending, we aggregate municipal variables over the 2012–2022 sample period. We then focus on typical spending patterns by keeping only those municipalities with disaggregated spending data ("full sample") where the share spent on each public service is neither extremely high nor low. Specifically, we exclude municipalities in the top and bottom 5% of spenders across public goods according to expenditure share. We also include only municipalities with spending data across all ten public goods. After this selection process, we end up with a sample that includes 43% of the initial municipalities, or 54% of the population. Table C.9 shows that the selected sample and full sample of municipalities have similar characteristics, with only slight differences in population size. This suggests that our selected sample is broadly representative of the full sample, though both are more affluent, have higher tax rates, and are by construction larger and more populous than the average municipality in France.

Household variables. The third data source is household income at the municipal level, which was collected for 2018 from the Filosofi database. This data provides information on the income distribution for households within each municipality, which we use to characterize the median household. Additionally, we use data on household populations and their socio-professional categories from the 2018 population census.

Rents. We obtain gross rental prices per square meter (including utilities) at the municipality level from Breuillé, Grivault and Le Gallo (2020). Their methodology aggregates rental listings from three major online platforms in France (leboncoin, SeLoger, and pap.fr) covering the period 2015-2019. Rental prices are hedonically adjusted for quality attributes (i.e., surface area, average surface area per room) and deseasonalized to construct a standardized rental price index for each municipality.

2.3 Facts

2.3.1 Heterogeneity in Revenues, Tax Bases, and Tax Rates

Table 1 reveals significant variability in municipal revenues and tax bases across municipalities. The median total revenue per capita is €1,196, with a standard deviation of €477. Local property tax revenue, accounting for around 40% of total revenue, shows considerable dispersion, with a median of €495 per capita and a standard deviation of €210. Notably, municipalities' choices regarding property tax rates account for 28% of the variation in property tax revenue, while the remaining variation is due to differences in the tax base. This highlights the significance of the property tax rate as a discretionary tool for municipalities to raise revenue.

To examine the distribution of resources across different household income levels, we calculate the P90/P10 income ratio, which compares municipalities in the top and bottom deciles based on median household income. Federal transfers appear to substantially equalize per capita resources in this dimension, as evidenced by the near-parity P90/P10 ratio of 1.04 for revenue per capita. However, municipalities in the top income decile benefit from a significantly larger property tax base (P90/P10 ratio of 1.94) while simultaneously setting lower tax rates (P90/P10 ratio of 0.72) compared to those in the bottom decile. Despite the system of progressive transfers from the central government, residents of lower-income municipalities face disproportionately higher property tax rates.

2.3.2 Heterogeneity in Public Goods Provision

Turning to the composition of municipal spending, Table 2 demonstrates significant heterogeneity in municipal budget allocations across locations. For example, education, the second-largest spending category, has a median per capita expenditure of €246 with a standard deviation of €116. Across public goods, the standard deviation of expenditure per capita is relatively high compared to its median.

To quantify the sources of this variation, we perform a variance decomposition of per capita expenditure for each public good category. Let e_{ip} denote the per capita expenditure on public good p in municipality i. We can express this as:

$$e_{ip} = s_{ip} r_i$$

where s_{ip} is the share of total revenue allocated to good p in municipality i, and r_i is the total per capita revenue of municipality i. Taking the variance of the log of

Table 1: Municipal Revenue and Tax Base Characteristics

Characteristic	Median (€/cap)	SD (€/cap)	P90/P10 Ratio
Total Revenue	1,196.05	476.99	1.04
- Property Tax Revenue	495.09	209.97	1.38
- Transfers and Other Revenue	699.54	359.02	0.86
Property Tax Rate	0.38	0.10	0.72
Property Tax Base	1,289.28	620.64	1.94
Variance Decomposition of Property Tax Revenue:			
- Due to Tax Rate			27.5%
- Due to Property Tax Base			72.5%

Notes: All monetary values are in 2018 euros per capita. SD represents standard deviation. P90/P10 Ratio is the ratio of the respective variable for municipalities above the 90th percentile of median income to those below the 10th percentile.

Table 2: Breakdown of Municipal Budgets

				% of Variance in Spending per Capita	
Spending Category	Median (€/cap)	SD (€/cap)	P90/P10 Income Ratio	Due to Rev./cap	Due to Exp. Share
Administrative	377.87	184.99	0.95	0.53	0.47
Arts & Culture	87.56	64.29	0.95	0.24	0.76
Economic	7.67	22.96	0.67	0.07	0.94
Education	245.57	116.25	1.50	0.39	0.61
Health & Safety	37.89	31.96	1.01	0.19	0.84
Parks	45.89	29.56	0.77	0.18	0.82
Planning	181.31	126.05	0.98	0.28	0.72
Recreation	82.34	56.80	0.96	0.28	0.71
Social	44.58	42.79	0.67	0.17	0.82
Youth	49.55	45.05	1.36	0.10	0.91
Mean				0.24	0.76

Notes: All monetary values are in 2018 euros. P90/P10 Income Ratio represents the ratio of the respective variable for municipalities above the 90th percentile of median income to those below the 10th percentile.

this expression and re-arranging, we obtain:

$$1 = \frac{\text{Var}(\log s_{ip})}{\text{Var}(\log e_{ip})} + \frac{\text{Var}(\log r_i)}{\text{Var}(\log e_{ip})} + \frac{2\text{Cov}(\log s_{ip}, \log r_i)}{\text{Var}(\log e_{ip})}$$

This decomposition allows us to separate the contribution of variation in total revenue per capita and variation in expenditure shares, where we split the covariance equally between the two.

The results of this decomposition, presented in Table 2, reveal that choices in expenditure shares account for 76% of the variation in allocations on average, compared to 24% attributed to revenue differences. This suggests that local choices with respect to the allocation of resources play an important role in shaping public goods provision under decentralization.

2.3.3 Income and Population Effects on Budget Allocations

After documenting significant heterogeneity in public goods provision across municipalities, we turn our attention to understanding the underlying characteristics driving these differences. Figure B.4 presents municipal "Engel curves," illustrating how budget shares for different public goods vary across quantiles of median household income. We focus on median income because it serves as a proxy for the income of the pivotal voter in many political economy models.[1] Higher-income municipalities consistently allocate larger budget shares to education and youth programs, while lower-income municipalities prioritize social services. These findings provide further suggestive evidence that household income levels may play an allocative role in shaping the composition of public goods provision under decentralization. If preferences for public goods vary systematically with income, then allowing local choice through decentralization may lead to different bundles of public services being provided in rich and poor communities despite equalization on average of overall resources by transfers.

Population size also plays a role in shaping budget allocations, as shown in Figure B.5. Larger municipalities allocate relatively more to cultural goods, social services, recreational activities, and youth services, suggesting either potential economies of scale or relatively stronger congestion forces in these areas. Conversely, administrative services exhibit a negative population elasticity, indicating efficiency gains in governance as population increases.

To disentangle the associations of income and population size with public good expenditures, we estimate the following equation for each public good category:

$$\log(s_{jp}) = \alpha + \beta_1 \log(y_j^{p50}) + \beta_2 \log(pop_j) + X_j'\gamma + \varepsilon_{jp},$$

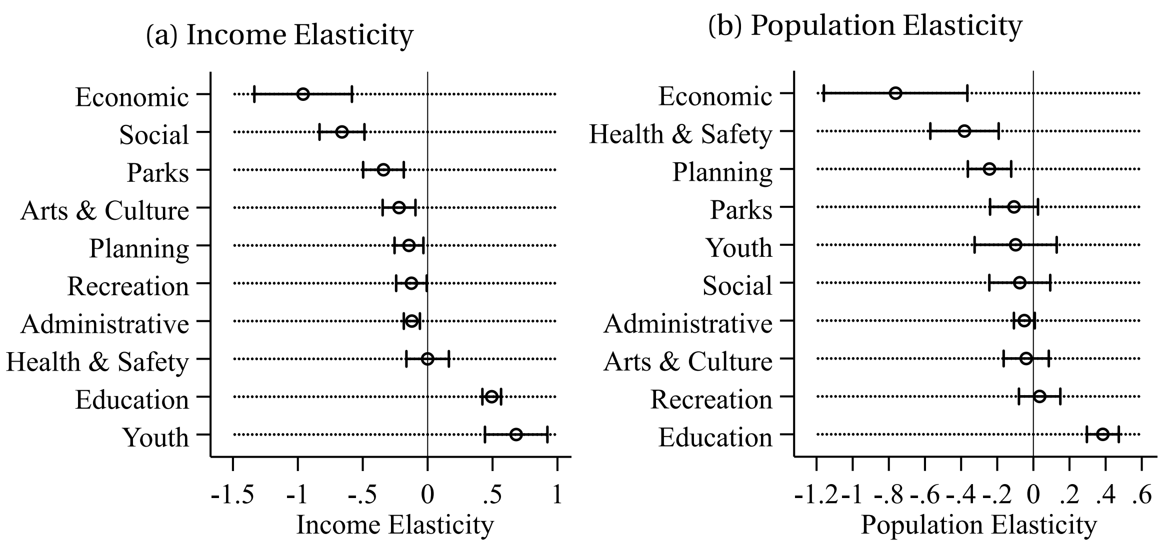
where s_{jp} is the expenditure share of public good p in municipality j, y_j^{p50} is the median income, pop_j is population, and X_j is a vector of additional covariates including log density and log total revenue. The coefficient β_1 represents the income elasticity, and β_2 the population elasticity. We control for density to account for possible urban-rural preference differences, and for total revenue to isolate income and population effects from the effect of the municipal budget constraint.

Figure 1 presents these estimates. The patterns observed in the municipal En-

[1] These results are robust to using mean income instead.

gel curves largely persist, and most of the estimated elasticities are statistically significant at the 5% level. For example, education shows the highest positive income elasticity, around 0.5. This suggests that every 1% increase in income is associated with a 0.5% increase in spending share.

Figure 1: Elasticity of Public Goods Spending Share



Note: These figures illustrate the estimated elasticities of the share of expenditure with respect to income and population across various local public goods, controlling for log of density and log of total revenue. Confidence intervals are calculated at the 95% level based on robust standard errors.

3 Spatial Equilibrium Model of Sorting and Voting on Local Public Services

To rationalize these facts and analyze their implications for the welfare effects of decentralization policies, we now introduce a model in which heterogeneous households decide individually where to live and in which they collectively vote on the local tax rate and the composition of public goods in each municipality.

3.1 Environment and Technology

There is a mass L of households located in J locations. L_j denotes the mass of households living in j. There are I types of households which differ in their labor productivity, x_i for $i \in I$. The mass of type i is denoted L_i, and the mass of type i living in j is denoted L_{ij}.

There is one final good which is freely traded across locations and whose price is normalized to one. Locations differ in their exogenous productivity to produce the final good, z_j. The technology is linear and the production of final goods by a household of type i located in j supplying one unit of labor is given by y_{ij}. We further assume that the firms producing the final good behave competitively, so that y_{ij} is also the income received by a household of type i.

Locations also differ in their exogenous supply of land, b_j. This will shape the endogenous local supply of housing and its price, which we describe in more details in the housing supply section 3.4.

Finally, locations endogenously differ in the supply of public services and the tax rate. In each location, there are P public goods, indexed by p and a local government chooses how much to supply of each public good $g_j = \{g_{pj}\}_{p=1}^{P}$ and funds these expenses with transfers from the central government T_j and the receipts from a local property tax, t_j. The only input in the production of public goods is the final good and for simplicity we assume that one unit of a public good requires one unit of the final good.

3.2 Households Preferences

Households value a basket of public goods and a basket of private goods, which we denote g and u respectively. In the rest of the paper, we will refer to the former as public utility and the latter as private utility. The elasticity of substitution between the public and the private utility is constant and denoted γ. Public utility comprises P publicly provided goods and the elasticity of substitution between these public goods is also constant and denoted γ_g. Private utility is made of housing services and the tradable final good and we assume a unitary elasticity of substitution between the two.[2]

$$v(g,u) = \left(\omega_g(v)^{\frac{1}{\gamma}} g^{1-\frac{1}{\gamma}} + (1-\omega_g(v))^{\frac{1}{\gamma}} u^{1-\frac{1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \tag{1}$$

$$g(\tilde{g}_1, ...\tilde{g}_P) = \left(\sum_{p=1}^{P} \omega_p(v)^{\frac{1}{\gamma_g}} \tilde{g}_p^{1-\frac{1}{\gamma_g}}\right)^{\frac{\gamma_g}{\gamma_g-1}} \quad \text{with} \quad \sum_{p=1}^{P} \omega_p(v) = 1 \tag{2}$$

$$u(h,c) = h^{\omega_h(v)} c^{1-\omega_h(v)} \tag{3}$$

Motivated by the fact that the composition of public services and the local tax rates systematically differ with household income, we allow households to value the bundles of goods with non-homothetic preferences in a very flexible way. More specifically, our specification allows for non-homothetic preferences within private consumption, within public consumption and across public and private housing. The weight associated with public utility in the upper-tier CES function ($\omega_g(v)$ in equation (1)), the weight on public good p in public utility ($\omega_p(v)$ in equation (2)) and the weight on housing in private utility ($\omega_h(v)$ in equation (3)), all vary with the level of utility v.

Building on a stream of recent papers including Comin and Mestieri (2018), Borusyak and Jaravel (2018), Matsuyama (2019), and more recently Hoelzlein (2021), we assume that the weights take the following simple functional form:

$$\omega_h(v) = \frac{\alpha_h v^{\nu_h}}{\alpha_h v^{\nu_h} + 1} \tag{4}$$

$$\omega_g(v) = \frac{\alpha_g v^{\nu_g}}{\alpha_g v^{\nu_g} + 1} \tag{5}$$

$$\omega_p(v) = \frac{\alpha_p v^{\nu_p}}{\sum_{p'=1}^{P} \alpha_{p'} v^{\nu_{p'}}}. \tag{6}$$

and we further normalize $\nu_1 = 0$ and $\alpha_1 = 1$ since only $P-1$ parameters are separately identifiable. We will say that good n is relatively more basic than good n' if and only if $\nu_n < \nu_{n'}$, that housing is more basic than consumption if and only if $\nu_h < 0$ and that public consumption is more basic than private consumption if and only if $\nu_g < 0$.

While private goods are completely rival, public goods are, to varying degrees,

[2]The assumption of a unitary elasticity of substitution is made for simplicity, since this is not the focus of our paper.

non-rival. To capture the degree of non-rivalry of public good p, we assume that the utility perceived by a household $\tilde{g}_{jp}$, decreases at the rate $\rho_p \in [0,1]$ with the number of households located in the same municipality:

$$\tilde{g}_{jp} = g_{pj} L_j^{-\rho_p} \tag{7}$$

where g_{pj} is the supply of public good p. The parameter ρ_{jp} captures the degree of rivalry, or equivalently the degree of congestion, where $\rho_{jp} = 0$ corresponds to a perfectly non-rival good and $\rho_{jp} = 1$ to a perfectly rival good. In addition, this parameter also captures the extent of returns to scale in the provision of public goods: the same utility from public services can be generated with lower spending per capita in large cities than in small cities. When $\rho_p = 0$, there are infinitely large returns to scale and when $\rho_p = 1$, there are none. Allowing for heterogeneity in the degrees of rivalry and congestibility across public goods is motivated by the facts that cities of different sizes spend different amounts on each public good.

Households' decisions. Every period, households make three decisions. First, they choose where to locate, then they vote on the local bundle of public goods and tax rate and finally they decide how to allocate their net-of-tax income between housing services and final goods. We start with the last of the three decisions.

Taking the supply of public goods, $\{g_{pj}\}_{p=1}^{P}$, the tax rate t_j and rents r_j as given, households choose how much to consume and to spend on housing to maximize their private utility

$$u_{ij} = \max_{h,c} u(h,c) \tag{8}$$

subject to a budget constraint

$$y_{ij}(1-\tau) = c + r_j(1+t_j)h \tag{9}$$

where y_{ij} is given by the local technology, τ is a central government income tax rate, and the local property taxes are proportional to the value of spending on housing.

The value of a household of type i located in j is denoted V_{ij}. The location decision solves the following maximization problem:

$$\max_{j \in \{1,\ldots,J\}} \{V_{ij} + \varepsilon_j\} \quad \text{with} \quad V_{ij} = \ln(a_j v_{ij}) \tag{10}$$

$$\varepsilon_j \sim \text{i.i.d. Gumbel}(0, \mu) \tag{11}$$

where the location parameter is normalized to 0 and μ is the scale parameter.

Before voting over the public goods, households decide where to locate. The attractiveness of a location j to a household of type i depends on the local income, hence the local productivity, z_j, the local cost of housing which includes both the net of tax rent r_j, and the local tax rate t_j and the composition of public goods $\{g_{pj}\}_{p=1}^{P}$, whose congestion also depends on the size of the local population, L_j, and the other local amenities a_j. These variables are all captured in the indirect utility V_{ijt} for a household of type i.

In addition, households have an idiosyncratic preference for each location. Following the literature, we assume that these preference shocks are identically and independently distributed over individuals and locations and follow a Gumbel distribution.

3.3 Collective Decision over Public Goods and Tax Rates

After they have moved and before they make their private consumption decisions, households located in the same municipality vote to collectively decide on the allocation of local revenues across public goods to maximize their public utility (2) and on the tax rate to maximize their upper-tier utility (1).

As in the literature on decentralized choices of local taxes, households disagree on the optimal level of the proportional tax rate because they have different tastes and different income and therefore different levels of contribution to the local budget. One novelty of our framework is to link the heterogeneity in tastes to the non-homotheticity of preferences. Individuals disagree on the optimal tax rate because the valuation of public goods relative to private goods, captured by $\omega_g(v)$, varies with the level of utility, and they also disagree on the optimal composition of public goods, because the valuation of different public goods vary with utility, as captured by $\omega_p(v)$.

Voters understand that their collective decisions about the expenses on public goods and about the tax rate should be consistent with a balanced budget.

The municipality funds spending on public services with two sources of revenues: taxes collected on housing consumption and transfers received from the central government, T_j. The budget constraint of the local government is given by

$$t_j r_j H_j + T_j = \sum_{p=1}^{P} g_{pj}. \tag{12}$$

Defining $s_{pj} = \frac{g_{pj}}{t_j r_j H_j + T_j}$ the share of public revenues spent on public good g_p in j, the budget constraint (12) can be rewritten as $\sum_{p=1}^{P} s_{pj} = 1$, or

$$s_{1j} = 1 - \sum_{p=2}^{P} s_{pj}. \tag{13}$$

Substituting the budget constraint (13) into the public utility (2), it is clear that households only need to vote on the tax rate and the $P-1$ expenditure shares on public goods $\left(t_j, \{s_{pj}\}_{p=2}^{P}\right)$.

Collective decisions are taken through votes at the pairwise majority rule in each dimension *independently*. More specifically, our concept of equilibrium for the local voting game builds on Kramer (1972) and Shepsle (1979), and following De Donder, Le Breton and Peluso (2012a), we refer to it as a Kramer-Shepsle equilibrium.

Definition 1 (Kramer-Shepsle equilibrium)**.** *A tax rate and a vector of public goods* $\left(t_j, \{s_{pj}\}_{p=2}^{P}\right)$ *is a Kramer-Shepsle equilibrium if*

- *the tax rate* t_j *coincides with the pairwise majority choice given the expenditure shares* $\{s_{pj}\}_{p=2}^{P}$
- *if each expenditure share* s_{pj} *coincides with the pairwise majority choice given the other* $P-2$ *expenditure shares and the tax rate* t_j*.*

This equilibrium definition has been analyzed and used in the local political economy literature, for example in De Donder and Hindriks (1998), Diba and Feldman (1984), Nechyba (1994) and Sadanand and Williamson (1991). It also shares some features with sequential voting procedures found in related papers such as Alesina, Baqir and Easterly (1999), Alesina, Baqir and Hoxby (2004) and De Donder, Le Breton and Peluso (2012b).

This equilibrium concept is appealing because it addresses several issues in settings with multidimensional voting. Typically voting on several dimensions simultaneously may lead to multiple equilibria and unstable political coalitions. It may also imply that the order in which public goods are voted upon matters for the outcome or that there are no Condorcet winners (De Donder, Le Breton and Peluso, 2012a).[3] By contrast, in our setting a voting equilibrium exists, is unique and independent of the ordering in which collective decisions are taken.

3.4 Technology for Housing and the Landlords' Problem

A continuum of landlords rents housing services to households at price r_j per unit. We denote the quantity of housing supplied in location j, B_j. The technology for housing services is conventional: to construct B_j units of housing, landlords need $B_j^{\eta_j}/(\gamma_j b_j^{\eta_j - 1})$ units of final good, where $\eta_j > 0$ governs the convexity of the cost function.

Landlords are absentee and consume only final goods. Landlords therefore seek to maximize their income at each period and taking rents r_j as given, they solve the following problem

$$\max_{B_j} r_j B_j - \frac{B_j^{\gamma_j}}{\gamma_j b_j^{\eta_j - 1}}. \tag{14}$$

3.5 Market Clearing and Equilibrium

An equilibrium is a sequence of public good vectors $\{g_{jpt}\}_{p=1}^{P}$, tax rates t_{jt}, rents r_{jt}, supplies of housing B_{jt}, population vectors $\{L_{ijt}\}_{i=1}^{I}$, consumption and housing decisions $\{c_{ijt}, h_{ijt}\}_{i=1}^{I}$ for each location $j = 1...J$, such that:

- Taking rents, the tax rate and public goods as given households maximize their private utility (3) subject to the budget constraint (9).
- The tax rate t_{jt} and the bundle of public services $\{g_{pjt}\}_{p=1}^{P}$ is a Kramer-Shepsle equilibrium of the voting game in each location j.
- Households choose their location to maximize (10).

[3] For example, with multidimensional voting it would be possible for a rich municipality to choose high social spending to get the poorest to vote for a bundle that is otherwise preferred by the richest households, at the expense of the middle income households.

- Taking rents as given, landlords maximize their profits (14).
- All housing markets clear

$$B_{jt} = \sum_{i=1}^{I} L_{jit} h_{jit} \quad \forall\, j, t \tag{15}$$

- The final goods market clears

$$\sum_{j=1}^{J} \sum_{i=1}^{I} y_{ijt} L_{ijt} = \sum_{j=1}^{J} \left(\sum_{i=1}^{I} c_{ijt} L_{ijt} + \sum_{p=1}^{P} g_{pjt} + r_{jt} B_{jt} \right) \tag{16}$$

- The central government's budget is balanced

$$\sum_{j=1}^{J} T_j = \sum_{j=1}^{J} \sum_{i=1}^{I} y_{ijt} L_{ijt} \tau \tag{17}$$

4 Properties of the Decentralized Equilibrium

In this section, we characterize the households' optimal private consumption given prices and the collective decision about the provision of local public goods. Finally, we derive the optimal migration decision of households.

4.1 Private Consumption of Housing and Final Goods

Consistent with the assumption of a unitary elasticity of substitution between housing and final goods, at the optimum, households allocate a share $\omega_h(v)$ of their private income on housing. This result is formalized in the following proposition.

Lemma 1 (Private Goods)**.** *The optimal spending on housing is given by*

$$(1 + t_j) r_j h = \omega_h(v) y_{ij} (1 - \tau) \quad \textit{with} \quad \omega_h(v) = \frac{\alpha_h v^{\nu_h}}{\alpha_h v^{\nu_h} + 1} \tag{18}$$

$$\textit{and} \quad u = \frac{y(1 - \tau)}{p_j(v)} \quad \textit{with} \quad p_j(v) = (r_j(1 + t_j))^{\omega_h(v)} \tag{19}$$

where the second equation defines u, the private indirect utility, only implicitly as the solution to a non-linear equation. In the special case with non-homothetic housing $\nu_h = 0$, we can solve for the indirect private utility explicitly:

$$(1+t_j)r_j h = \frac{\alpha_h}{\alpha_h+1} y(1-\tau) \quad \text{and} \quad u = \frac{y(1-\tau)}{(r(1+t))^{\frac{\alpha_h}{\alpha_h+1}}}.$$

4.2 Equilibrium Provision of Public Goods and Taxes

We now turn to the characterization of the political equilibrium bundle of public goods and the tax rate, the most novel aspect of our model. We proceed in two steps. We first derive expressions for the optimal allocation of public revenues across different public services and for the optimal tax rate for each type of households. We then show that under some conditions, the political equilibrium bundle of public good and taxes is the median voter's ideal bundle.

What is the optimal allocation of public revenues across different public goods from the perspective of households of type i in municipality j? To answer this question, we solve for the optimal expenditures shares on each public good $\{s_{pj}\}_p$ from the perspective of households i. Recall from the definition of the public utility (2) and the budget constraint (13) that their objective is

$$\max_{s_{pj}} \left(\sum_{p=1}^{P} \omega_p(v)^{\frac{1}{\gamma_g}} \left(L_j^{-\rho_p} s_p \right)^{1-\frac{1}{\gamma_g}} \right)^{\frac{\gamma_g}{\gamma_g-1}} \quad \text{with} \quad s_{1j} = 1 - \sum_{p=2}^{P} s_{pj} \tag{20}$$

The following lemma shows the first order condition associated with this problem.

Lemma 2 (Expenditure Shares on Public Goods)**.** *Define the function*

$$\bar{g}(s_{2j}, ...s_{Pj}) = \left(\omega_1(v)^{\frac{1}{\gamma_g}} \left(L_j^{-\rho_1} \left(1 - \sum_{p=2}^{P} s_{pj} \right) \right)^{1-\frac{1}{\gamma_g}} + \sum_{p=2}^{P} \omega_p(v)^{\frac{1}{\gamma_g}} \left(L_j^{-\rho_p} s_{pj} \right)^{1-\frac{1}{\gamma_g}} \right)^{\frac{\gamma_g}{\gamma_g-1}}$$

1. *The function $\bar{g}(s_{2j}, ...s_{Pj})$ is strictly concave in each component s_{pj} for $p = 2...P$.*

2. *The optimal allocation of public spending for a household of type i is given by*

$$s_{pj} = \left(\frac{\alpha_p}{\alpha_1} v_{ij}^{\nu_p - \nu_1} \right) L_j^{(\rho_p - \rho_1)(1-\gamma_g)} s_{j1} \quad \textit{with} \quad s_{1j} = 1 - \sum_{p=2}^{P} s_{pj} \tag{21}$$

The expression (21) gives important intuition about how households would like to allocate municipal revenues across different public goods. The first term, $\frac{\alpha_p}{\alpha_1}$ captures the preference for good p relative to public good 1 that is common across all households. The second term, $v_{ij}^{\nu_p-\nu_1}$, captures the non-homotheticity of preferences: if good p is less basic than good 1 $\nu_p > \nu_1$ and as a household's welfare improves, it is willing to shift more spending to good p. The third term, $L_j^{\rho_p-\rho_1}$, captures the congestion forces. When $\gamma_g < 1$, an increase in population makes it more expensive to provide a given level of enjoyment of the public good. Suppose that good p is more rival than good 1 such that $\rho_p > \rho_1$, then households would optimally spend relatively more on good p as population increases.

In the perspective of the proof of the median voter's theorem, the first bullet point of the lemma is important because the fact that the public utility (20) is strictly concave in each of its component means that households preferences are single-peaked in each dimension of the allocation of local revenues on public goods.

Given an allocation of local revenues across public goods, we now turn to the optimal level of local tax rate from the perspective of households i. Using the notation introduced in lemma 2, the utility derived by household i from public spending in j can be written as $g_{ij} = \frac{t_j r_j H_j + T_j}{q_{ij}}$ where $q_{ij} = 1/\bar{g}_i(s_{2j}, ...s_{Pj})$ should be interpreted as the price index for public goods, which varies across households. This price index is a generalized mean of the prices of each public good, which reflect essentially the degrees of congestion, $L_j^{\rho_p}$, across public goods. The stronger the congestion for a specific public good, the higher its price and the lower the public utility g_{ij} for a given level of municipal revenues. In addition, the weights in the price index are a combination of the shares of municipal revenues spent on good p, s_{pj}, and of a household's specific tastes, $\omega_p(v_i)$. The closer the effective weights are to the households' tastes, the lower the price index. In proposition 2, we give an explicit expression of the price index when the local composition of public goods perfectly coincides with the preference of the median voter.

From the definition of the upper-tier utility (1) and the indirect private utility

(19), the problem for a voter is thus to choose t_j to maximize

$$\bar{v}_i(t_j) = \left(\omega_{gj}(v)^{\frac{1}{\gamma}} \left(\frac{t_j r_j H_j + T_j}{q_{ij}}\right)^{1-\frac{1}{\gamma}} + (1-\omega_{gj}(v))^{\frac{1}{\gamma}} \left(\frac{y(1-\tau)}{(r(1+t))^{\omega_h(v)}}\right)^{1-\frac{1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \tag{22}$$

The following lemma shows the first order condition associated with this problem.

Lemma 3 (Tax Rate)**.**

1. *The function $\bar{v}_i(t_j)$ is strictly concave.*

2. *The optimal tax rate for a household of type i is implicitly given by*

$$\left(t_j + \frac{T_j}{r_j H_j}\right) = \frac{\alpha_g v_{ij}^{\nu_g}}{\omega_h(v_{ij})} \left(\frac{q_{ij}(v_{ij})}{p_j(v_{ij})} \frac{\omega_h(v_{ij}) y_{ij}(1-\tau)}{r_j H_j}\right)^{1-\gamma} \tag{23}$$

where $r_j H_j = \sum_i L_{ij} \omega_h(v_{ij}) y_{ij} \frac{(1-\tau)}{1+t_j}$ denotes total spending on housing in location j.

The proof is in Appendix D.1. The formula for the optimal tax rate (23) is intuitive. The optimal tax rate increases with the voter's general preference for public goods $\frac{\omega_g}{1-\omega_g} = \alpha_g v^{\nu_g}$, but decreases with its preference for housing $\omega_h(v) = \frac{\alpha_h v^{\nu_h}}{\alpha_h v^{\nu_h}+1}$. The terms α_g and α_h capture the preference for public goods relative to housing that is common across households. But the terms v^{ν_g} and v^{ν_h} capture the non-homotheticity of preferences. Since there are good reasons to think that both housing and public goods are basic goods, whether the tax rate increases or decreases with v will depend on which one is more basic than the other.

Looking at the rightmost term of the optimal tax formula, the one into bracket, we see that when $\gamma < 1$, the tax rate is decreasing in the total spending on housing $r_j H_j$. High spending on housing, either because households are rich, numerous or spend a large share of their income on housing because they value it a lot, implies that the tax base is large and that the tax rate doesn't need to be as high to fund local public goods.

Under the same condition, $\gamma < 1$, the tax rate increases with the relative price of public goods q_{ij}/p_{ij}. Given that the local population L_j is an important driver of the price index q_{ij} through the congestion effect, the total effect of the size of the population on the tax rate is ambiguous and results from two forces. On the one hand the increase in population implies stronger congestion which increases the price of public services q_{ij}, which tends to increase the tax rate. On the other, higher population also means that the tax burden can be spread among more households, as explained in the previous paragraph. Overall, it seems reasonable to expect that for a reasonable parametrization of the congestion forces, the former effect should prevail, and the tax rate should decrease with population size.

In the same way that we underlined that preferences were single-peaked in the expenditure shares on each good in lemma 2, lemma 3 establishes that the upper-tier utility function is strictly concave in the tax rate t_j and that preferences are also single-peaked in the tax rate dimension. This leads us directly to the most important result of this section: the median voter's theorem.

Proposition 1 (Median Voter)**.**

1. *The equilibrium expenditure shares* $\{s_{pj}\}_p$ *are the ideal bundle of public goods for the household with the median utility* v_{Mj} *in* j*.*

2. *The equilibrium tax rate* t_j *is the ideal tax rate of the median voter.*

   - *The median voter is also the household with median utility* v_{Mj} *in* j *if* $\gamma = 1$ *and* $\nu_h = 0$*.*

Proof. Lemma 2 and 3 establish that the preferences of each household are single-peaked in each separate dimension of the vote, the $P-1$ expenditure shares and the tax rate t_j, respectively. From the traditional median voter theorem, we know that the median voter's ideal expenditure shares on public goods and tax rate is the unique equilibrium of the pairwise majority vote in each dimension. In addition, it is obvious from the expression in lemma (2) that the median voter is also the voter with the median v.

A sufficient condition to ensure that the median voter in the tax dimension is also the household with median utility v is that the optimal tax rate, solution to equation (23), is monotonic in v. This is true if $\gamma = 1$ and $\nu_h = 0$ since the ratio $\frac{\omega_g}{\omega_h(1-\omega_g)} = \frac{\alpha_g}{\alpha_h^{\gamma}} v^{\nu_g - \gamma\nu_h}(1+\alpha_h v^{\nu_h})^{\gamma}$ simplifies to $\frac{\alpha_g}{\alpha_h} v^{\nu_g}(1+\alpha_h)$ which is monotonic in v. □

This sufficient condition guarantees that both the tax rate and the allocation of spending reflect the preferences of a single type of households, the one with median utility. Without this condition, the tax rate could reflect the preference of one type of household and the composition of public goods could reflect the preference of another type. However, while this condition is sufficient, it is far from necessary. Intuitively, what needs to be true is simply that the right-hand side of equation (23) be monotonic in v which is likely to be true if non-homotheticicities in the preference for housing ω_h are not too strong and the elasticity of substitution γ is close to 1, thereby ensuring that the households' heterogeneous price indices for the public goods q_{ij} and private goods p_{ij} do not play an important role in determining their favorite tax rate.

Given that the bundle of public goods is the ideal bundle of the household with median v we solve more explicitly for its indirect public utility and the associated price index for public goods in the following proposition.

Proposition 2 (Public Utility for the Median Voter)**.** *The public utility of the median voter is*

$$g_{ij} = \frac{t_j r_j H_j + T_j}{q_{ij}} \quad \textit{with} \quad q_{ij} = \left(\sum_{p=1}^{P} \omega_p(v_{Mj}) \left(L_j^{-\rho_p} \right)^{\gamma_g - 1} \right)^{\frac{1}{1-\gamma_g}} \tag{24}$$

This price index is the optimal price index for this household type in the sense that it maximizes the utility of this household for a given level of municipal revenues. This formula will be useful when estimating the model, in section 5.

4.3 Migration and Housing Supply

We conclude this section with two important equilibrium conditions governing the migration of households and the supply of housing.

From the assumption that preference shocks are EV1 distributed, the probability that a household of type i locates in j is given by

$$\pi_{ij} = \frac{\exp\left(V_{ij}\right)^{1/\mu}}{\sum_{l=1}^{J} \exp\left(V_{il}\right)^{1/\mu}} \tag{25}$$

Finally, after taking the first order condition of landlords with respect to the supply of housing B_j, we obtain the following upward-sloping supply curve:

$$B_j = b_j r_j^{\frac{1}{\eta_j - 1}} \tag{26}$$

where $\frac{1}{\eta_j - 1}$ should be interpreted as the local elasticity of housing.

4.4 Efficiency considerations

There are several sources of inefficiency in the decentralized economy. They create a meaningful trade-off for decentralization policies. Most of these inefficiencies stem from the determination of the tax rate.[4]

First, the property tax rate distorts the optimal allocation of private spending between the final good and housing, leading to under-consumption of the latter. Second, there is a free-riding problem associated with the setting of the tax rate since the median voter understands that a marginal increase in the tax rate will apply to everyone's property values, not only theirs. This problem is particularly acute in neighborhoods where the median voter is low skilled, since it is tempting for them to free ride on the larger property values of high-skilled, thus richer, households.

Third, the median voter doesn't consider the negative general equilibrium effect of the tax rate on rents, which in general leads to tax rates that are "too low." Finally, the property tax and the composition of the public goods bundle are chosen through voting which may not result in a local welfare maximizing outcome. Instead, it maximizes the median voter welfare.

5 Calibration

We now turn to the calibration of the model, which we will use to quantify the welfare and economic impact of decentralization. A small subset of the parameters is calibrated externally and the rest are estimated internally to match important moments from the data.

[4] See, for instance, Calabrese, Epple and Romano (2012) for a detailed analysis.

5.1 First-order approximation of welfare effects and parameters

We start by deriving an analytical expression of the first-order change in social welfare following an arbitrary perturbation in the local spending shares and tax rates around a centralized allocation. Our goal is to highlight the role of the elasticities we calibrate in this section in shaping the welfare effects we compute in the non-linear version of the model in Section 6.

The utilitarian social welfare function is given by the weighted sum of individual values:

$$W = \sum_j \sum_i \pi_{ij} V_{ij}$$

We denote $\mathbf{x}_j$ the $(P \times 1)$ vector collecting the spending shares and the tax rate in location j and $\mathbf{x}$ the vector collecting these location-specific vectors x_j for all locations j:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_J \end{bmatrix} \quad \text{with} \quad \mathbf{x}_j = \begin{bmatrix} t_j \\ s_{2j} \\ \vdots \\ s_{Pj} \end{bmatrix} \tag{27}$$

where we omit the first share in each location, s_{1j}, since $s_{1j} = 1 - \sum_{p=2}^{P} s_{pj}$.

Proposition 3. *Consider an arbitrary change in the local spending shares and tax rate, $d\mathbf{x}$ around a centralized allocation in which the tax rate $\bar{t}$ and spending shares $\bar{s}_p$ are solution to the median individual maximization problem. Denote $\overline{T/rH}$ the transfers to housing spending ratio in the median individual's location. At first order, the change in the social welfare is given by*

$$dW = \sum_j \sum_i \pi_{ij} \left(1 + \frac{V_{ij}}{\mu}\right) \nabla_{\mathbf{x}_j} V_{ij} \;\cdot\; d\mathbf{x} \tag{28}$$

where the change in an individual's welfare of type i in location j, $\nabla_{\mathbf{x}} V_{ij}$, is given by

$$\nabla_{\mathbf{x}} V_{ij} d\mathbf{x} = \Gamma_{ij} \Bigg[\omega_g(v_{ij}) A_{ij} \left[\frac{1}{\gamma_g} \ln \frac{v_{ij}}{\bar{v}} \sum_{1=2}^{P} \left(\nu_p - E_{\omega_p(\bar{v})} [\nu_p] \right) ds_{pj} + \frac{L_j - \bar{L}}{\bar{L}} \sum_{p=2}^{P} \rho_p \left(\frac{1}{\gamma_g} - 1 \right) ds_{pj} \right] \tag{29}$$

$$+ (1 - \omega_g(\bar{v_{ij}})) \omega_h \left(\nu_g \ln \frac{v_{ij}}{\bar{v}} - \frac{\left(\frac{T_j}{r_j H_j} - \overline{\left(\frac{T}{rH} \right)} \right)}{\bar{t} + \overline{\left(\frac{T}{rH} \right)}} \right) dt_j + GE_{ij} \Bigg] \tag{30}$$

where $\Gamma_{ij}, A_{ij}, GE_{ij}$ are defined in Appendix D.3.

Equations (28) and (29) clarify how the key parameters in the model shape the change in social welfare due to decentralization. First, the migration elasticity μ is a key determinant of the indirect effect through the migration of individuals to places that see the greater improvements in local welfare. This amplifies the direct increase in the values of individuals living in different location $\nabla_{\mathbf{x}} V_{ij}$. This amplifying force will turn out to be important in our quantitative results.

Second, the degree of non-homotheticities of public goods $\sum_{p=2}^{P} \left(\nu_p - E_{\omega_p(\bar{v})} [\nu_p] \right) ds_{pj}$ shapes the increase in values coming from better tailoring in the local mix of public goods to local preferences, as shown in the first line of equation (29). It is intuitive that its strength depends on the distance of an individual's value to the reference household $\ln \frac{v_{ij}}{\bar{v}}$. Third, the degree of non-rivalry of public goods $\sum_{p=2}^{P} \rho_p \left(\frac{1}{\gamma_g} - 1 \right) ds_{pj}$ shapes the increase in values coming from better tailoring the local mix of public goods to local population sizes. Its strength depends, also intuitively, on the distance of a municipality's population size to the median $L_j - \bar{L}$. Both effects are also stronger when public goods are important in the individuals' overall utility ω_g, and when public goods are not easily substitutable with one another, γ_g.

Fourth the degree of non-homotheticities between public and private goods, ν_g, shapes the increase in values coming from changes in the local tax rate, as shown in the second line of equation (29). Like for spending shares, it is stronger when an individual's value is far from the reference $\ln \frac{v_{ij}}{\bar{v}}$ but also when the transfers to housing consumption ratio is far from the reference $\frac{\left(\frac{T_j}{r_j H_j} - \overline{\left(\frac{T}{rH} \right)} \right)}{\bar{t} + \overline{\left(\frac{T}{rH} \right)}}$. This effect is stronger when housing is important for the individuals' overall utility$(1 - \omega_g)\omega_h$.

Finally, the general equilibrium effects gathered in the GE_j term and reported in Appendix D.3 include the change in the income tax rate τ required to balance the general government's budget and the effects of the local tax rates t_j through rents r_j.

5.2 Externally calibrated parameters

Table 3 provides a full list of our externally calibrated parameters. We calibrate the migration elasticity to 3, which is in the range of estimates from other static models incorporating residential choice (Diamond, 2016; Monte, Redding and Rossi-Hansberg, 2018).

To calibrate the elasticity of substitution across public goods, we follow the literature on substitution across sectors and set $\gamma_g = 0.9$ (Herrendorf, Rogerson and Valentinyi, 2014; Atalay, 2017; Farhi and Baqaee, 2019).[5] We set the share of housing expenditure to $.19$ consistent with the average households in the Family Budget Survey (INSEE). For simplicity, we abstract from non-homotheticity in housing, $\nu_h = 0$, and given that $\omega_h = \frac{\alpha_h}{1+\alpha_h}$ we obtain $\alpha_h = .19$. We assume that the elasticity of substitution between public and private goods is one, $\gamma = 1$.

Finally, we calibrate the housing supply elasticity to 0.5 such that $\eta = 3$, which aligns with the mean elasticity across French metropolitan areas as estimated by Chapelle, Eyméoud and Wolf (2023).

5.3 Internally calibrated parameters

Our internal estimation proceeds in several steps: first we estimate the preferences parameters, including those related to the private goods, the public goods, and the upper-tier utility $\alpha_h, \alpha_p, \nu_p, \rho_p, \alpha_h, \nu_h$. Then we estimate the household income by type and by location y_{ij}. Finally, we estimate the location-specific intercepts in the housing supply functions, b_j and the amenities a_{ij}. Table 4 provides a full list of these parameters.

[5] Herrendorf, Rogerson and Valentinyi (2014) consider long-run changes in broad sectors' relative prices and final consumption expenditure shares. Their benchmark estimate of the preference elasticity of substitution between expenditures on agricultural products, manufactured goods, and services is 0.9.

Table 3: Externally Calibrated Model Parameters

Parameters	Description	Value	Source
γ_p	Elasticity of substitution across public goods	0.90	Herrendorf, Rogerson and Valentinyi (2014); Atalay (2017); Farhi and Baqaee (2019)
$1/\mu$	Migration elasticity	3	Diamond (2016); Monte, Redding and Rossi-Hansberg (2018)
$1/(\eta-1)$	Housing elasticity	0.50	Chapelle, Eyméoud and Wolf (2023)
α_h	Preference for housing	0.19	Family Budget Survey (INSEE)
ν_h	Preference for housing	0	Authors' assumption
γ	Elasticity of substitution between private and public goods	1	Authors' assumption

Table 4: Internally Calibrated Model Parameters

Parameters	Description	Value
$\{\alpha_1, \alpha_2, ...\alpha_P\}$	Preference for different public goods	-
$\{\nu_1, \nu_2, ...\nu_P\}$	Non-homotheticity	See Figure E.7a
$\{\rho_1, \rho_2, ...\rho_P\}$	Congestion of public goods	See Figure E.7b
$\ln \alpha_g$	Preference for public goods (rel. to private)	13.231
ν_g	Non-homotheticity (rel. to private)	-1.483
$\mathbf{b_j}$	Housing supply by location	-
$\mathbf{a_{ij}}$	Amenities by type $\times$ location	-
$\mathbf{y_{ij}}$	Income by type $\times$ location	-

5.3.1 Preference for public goods

To estimate the parameters of the public utility, α_p, ν_p and ρ_p we start from the optimality condition (20) and take the logs which gives:

$$\ln\left(\frac{s_{pj}}{s_{1j}}\right) = \ln\frac{\alpha_p}{\alpha_1} + (\nu_p - \nu_1)\ln v_{jM} + (1-\gamma_g)(\rho_p - \rho_1)\ln L_j. \qquad (31)$$

where v_{Mj} denotes the indirect utility of the median income in location j. To identify all the coefficients, we normalize $\alpha_1 = 1, \nu_1 = 0$ and we set $\rho_1 = .02$ equal to the elasticity of the share of spending on Administrative—our reference category corresponding to $p = 1$—with respect to population, as displayed in chart B.5.

To estimate the parameter of the upper-tier utility, α_g and ν_g, our strategy is similar to the previous step. We start with the first order condition with respect to taxes (23) and use the calibrated value $\gamma = 1$ and $\nu_h = 0$, which gives $\hat{\omega}_{gj} = \frac{\omega_h(t_j r_j H_j + T_j)}{r_j H_j + \omega_h(t_j r_j H_j + T_j)}$. From this expression, we get $\frac{\hat{\omega}_{gj}}{1-\hat{\omega}_{gj}} = \omega_h \left(t_j + \frac{T_j}{r_j H_j}\right)$. We then solve for the equivalent model-based expression for ω_g, which is equal to $\frac{\omega_{gj}}{1-\omega_{gj}} = \alpha_g v_{Mj}^{\nu_g}$. Equalizing the two gives the following regression which we run in the data

$$\ln\left(t_j + \frac{T_j}{r_j H_j}\right) = \ln\left[\frac{\alpha_g(1+\alpha_h)}{\alpha_h}\right] + \nu_g \ln v_{Mj}. \tag{32}$$

Model-consistent indirect utilities, v_{Mj}. To bring these regressions to the data, we need to identify the households with median utility in each commune and then construct its indirect utility, v_{Mj}. A natural candidate for the median voter is the household with median income. This is however true only if the function $v_{jM}(y)$ is strictly monotonic in y. A sufficient condition would for example be that the weight on the public goods is small $\omega_g \simeq 0$ so that private utility and hence income dominates over the utility for public goods. Our approach is to guess that this condition holds and to verify that it is true in the estimated model.

Guess 1. *The household with median indirect utility v_j is also the household with median income.*

We then construct v_{jM} in a model-consistent way. Using the expression of the utility given by (22) and the optimal price index for the public good of the median household given by (24), the following lemma reports the expression for the median indirect utility in terms of observable variables and parameters.

Lemma 4. *In equilibrium, the indirect utility of the median income in location j,*

v_{Mj}, *is given by*

$$v_{Mj} = \left(\frac{t_j r_j H_j + T_j}{q_{Mj}}\right)^{\hat{\omega}_g} \left(\frac{y_{Mj}(1-\tau)}{((1+t_j)r_j)^{\omega_h}}\right)^{1-\hat{\omega}_g} \tag{33}$$

$$\textit{with} \quad \hat{\omega}_{gj} = \frac{\omega_h(t_j r_j H_j + T_j)}{r_j H_j + \omega_h(t_j r_j H_j + T_j)} \tag{34}$$

$$q_{Mj} = \left(\sum_{p=1}^{P} \omega_p(v_{Mj}) \left(L_j^{-\rho_p}\right)^{\gamma_g - 1}\right)^{\frac{1}{1-\gamma_g}} \tag{35}$$

where our estimate for $\omega_g(v_{Mj})$, given by equation (34), comes from the optimality condition of the median voter.

To address the issue that the expression for v_{Mj} also depends on the parameters we seek to estimate α_p, ν_p and ρ_p, our estimation strategy is akin to GMM. We start with a guess for these parameters. We construct q_{Mj} and v_{Mj} accordingly, and we run the two regressions (31) and (32). We use the estimates of these regressions to update our guess and we iterate until convergence. In practice, convergence is very fast since these parameters only affect the price index.

Model-consistent tax rates. When mapping property tax rates in France to our model, two key challenges arise. The first challenge stems from the existence of two separate tax instruments: a residential tax that applies to all households and a property tax that exempts renters. While both effectively tax housing, our model, for the sake of tractability, assumes a single instrument and ignores the household's decision to rent or own. By assuming that the property tax burden is entirely passed on to renters, we can consolidate these two instruments into a single tax, consistent with our model's assumptions.

The second challenge is that the tax base in the model is the spending on housing (a flow) while it is the value of the property in the data (a stock). An additional complication is that the property value considered for tax purposes is not the actual market value but property assessments dating back from the 1970s. These assessments have not been updated since, except for municipality-level and nationwide adjustments. As a result, the tax base is likely to be unrepresentative of current housing values, leading to a discrepancy between statutory tax rates and the effective tax rates experienced by households.

To address these challenges, we construct a model-consistent tax rate which ensures that tax revenues in the model are equal to those in the data: Tax Revenues$_j$ $=$ $t_j r_j H_j = \frac{t_j}{1+t_j}[(1+t_j)r_j H_j] = \frac{t_j}{1+t_j}\omega_h \times$ Total Household Income$_j$. This gives

$$t_j = \frac{\text{Tax Revenues}_j}{\omega_h \text{Total Household Income}_j - \text{Tax Revenues}_j} \tag{36}$$

and the model-consistent tax bases is given by

$$r_j H_j = \frac{\omega_h \times \text{Total Household Income}_j}{1+t_j}$$

Results. We estimate equation (31) in the cross-section of municipalities with over 3,500 inhabitants for which we have detailed data on municipal expenditures. Figure E.7a presents our estimates for $\{\nu_p\}_p$, which govern the degrees of non-homotheticity over components of the public goods basket. These parameters are estimated relative to the elasticity for administrative services. These estimates largely corroborate our stylized findings presented in Figure B.4. Estimates are highest for spending on youth programs and education. A 1% increase in median household welfare is associated with a 1.1% increase in relative spending on youth programs and a 0.88% increase in relative spending on education. On the other hand, the same increase is associated with a 0.52% decline in relative spending on social services and a 0.35% decline in relative spending on economic initiatives.

Our estimates for the population elasticities $\{\rho_p\}_p$ are presented in Figure E.7b. These estimates are also relative to the elasticity for administrative services and are scaled by one minus the elasticity of substitution across sectors $(1-\gamma_g) = .1$. The elasticity of administrative services to population is assumed to be zero, consistent with the intuition that municipal administration is a non-rival public good with a fixed cost that can be spread across households. Economic initiatives exhibit the highest elasticity, where a 1% increase in population is associated with a 5.8% increase in relative spending. This is followed by social services, youth services, and art & culture.

Finally, Table E.10 present estimates for parameters in the upper-tier of utility. We find a ν_g of -1.48, suggesting that tastes for public consumption are basic relative to private consumption.

5.3.2 Income Processes

Next, we estimate the income in each location by household type. If we observed average income by type in each location, we could estimate y_{ij} directly. Unfortunately, we observe only a average and median income. Given this data, we assume that income of type i in location j is given by the product of a type (x_i) and location-specific (z_j) effects:

$$y_{ij} = x_i z_j. \tag{37}$$

Our approach to estimate this set of $I + J$ parameters is to express the average and median income as a function of the average and median individual type in each place:

$$y_j^M = z_j x_j^M \quad \text{with} \quad x_j^M(\mathbf{x_j}) = \sum_{i \in I} \mathbf{1}_{\text{i is median in j}} x_i$$
$$y_j^A = z_j x_j^A \quad \text{with} \quad x_j^A(\mathbf{x_j}) = \sum_{i \in I} \mu_{ij} x_i$$

where μ_{ij} and whether type i is median in j are observable. We then eliminate z_j by taking the ratio and estimate the following linear relation relation with non-linear least square; $\mathbf{x} = \text{argmin}_{x_H} \sum_{j=1}^J w_j \left(\ln \left(\frac{y_j^A}{y_j^M} \right) - \ln \left(\frac{x_j^A(\mathbf{x})}{x_j^M(\mathbf{x})} \right) \right)^2$ where w_j are the population weights. Given that there is one degree of freedom, we normalize the low-education type, $x_L = 1$. We then estimate z_j by matching the median income

$$z_j = \frac{y_j^M}{x_j^M}.$$

In the data, we categorize individuals into high and low types based on their socio-professional categories. The high type includes individuals in managerial, professional, and higher-skilled occupations, who generally have higher income and education levels. Conversely, the low type comprises individuals in lower-skilled, manual, or service-related occupations with relatively lower income and education levels.

5.3.3 Housing Supply Parameters, Amenities and Income Tax Rate

The remaining parameters—the housing supply parameters b_j, amenities a_{ij} and the income tax rate τ— require simulating the model. Although these parameters are jointly estimated, we highlight which moment matters most for each parameter.

Housing-supply parameters. We estimate the housing supply parameters b_j by inverting the housing market clearing condition:

$$b_j = \frac{\sum_{i=1}^{I} L_{ijt}\omega_h(v_{ij})y_{ij}(1-\tau)}{(1+t)r_j^{1+\frac{1}{\eta_j - 1}}}$$

Intuitively, a municipality with higher average income, or lower rent r_j, should have a larger supply of land b_j.

A key input in the previous expression is the rent in each city, r_j. We use gross rents per square meter (including utilities) at the municipality level developed by Breuillé, Grivault and Le Gallo (2020). These rental prices are derived from listings on three major online platforms in France, covering the period 2015-2019. Rents are standardized using a hedonic model estimated for spatially clustered zones across France, consider various measures of housing quality (e.g., surface area, number of rooms, type of housing).

Amenities. We estimate amenities in each location to match the spatial distribution of both types across locations. Intuitively places with more people should have higher amenities, everything else equal. To estimate amenities, we thus need to loop over guesses of vectors of amenities and compute the model's equilibrium until we find vectors consistent with the empirical distribution of population.

Income tax rate. Finally, the income tax rate τ is chosen to balance the central government's budget constraint.

6 A Quantitative Analysis of Decentralization

We now use our calibrated model to evaluate the aggregate and distributional effects of decentralizing the choice of local public goods on social welfare. To do so, we compare the decentralized equilibrium to a counterfactual in which both the tax rate and the mix of public goods is centralized and identical across municipalities. We then provide a series of additional counterfactual exercises to isolate separate mechanisms.

Decentralization policies face trade-offs. On the one hand they allow local communities to align the level of property taxation and the bundle of public goods to local preferences and costs. On the other hand, decentralization policies entail several inefficiencies, which we explained in section 4.4, and they may increase inequality across people and places.

6.1 Welfare Gains from Decentralization

Centralized benchmark. We start by simulating a counterfactual centralized allocation, in which the choice of the property tax rate and the composition of local public goods is common to all municipalities in France. The property tax rate and the expenditure shares are set to their median values across municipalities in the decentralized equilibrium.

All exogenous variables of the economy remain otherwise unchanged. Specifically, the distribution of amenities $\mathbf{a}$, transfers $\mathbf{T}$, productivities $\mathbf{z}$ and $\mathbf{x}$, and housing supply parameters $\mathbf{b}$ are the same as in the decentralized equilibrium. The model endogenously generates a new distribution of populations $\mathbf{L}$, rents $\mathbf{r}$ and welfare $\mathbf{V}$.

Main results. Column (2) in Table 5 shows the change implied by decentralization relative to the centralized counterfactual allocation. We find that decentralization has a positive and large effect on overall welfare of about +1.39 units. To get a sense of the magnitude of this effect, we also compute a revenue equivalent, which is defined as the increase in local public revenue per capita required to raise welfare in the centralized counterfactual to the level observed in the actual decentralized equilibrium. In Table 5, we report this value as a percentage of the median municipal revenue per capita in France. We find that the welfare gain from decentralization is equivalent to a 62% increase in local revenues, or around 750 euros per capita.

Table 5: Effects of Decentralization Scenarios from Centralization

Counterfactual:	Decentralize Tax & Public Goods		Decentralize Tax Only		Decentralize Public Goods Only	
	No Mobility	With Mobility	No Mobility	With Mobility	No Mobility	With Mobility
	(1)	(2)	(3)	(4)	(5)	(6)
Welfare						
All	0.56	1.65	0.05	-0.04	0.51	1.28
Revenue Equivalence	17.44%	61.58%	1.41%	-1.01%	15.80%	44.76%
Welfare by Type						
High-skilled	0.57	1.39	0.09	-0.11	0.48	1.15
Low-skilled	0.54	1.95	0.01	0.05	0.54	1.42
Inequality (Theil Index)						
Welfare	0.49	0.45	0.00	-0.02	0.49	0.40
Income	0.00	0.01	0.00	0.00	0.00	0.01
Public spending	-0.22	-0.98	-0.20	-0.25	0.00	0.00
Exposure Index of Sorting	0.00	-0.03	0.00	0.07	0.00	-0.03

6.2 The Important Role of Mix of Public Goods

Partial decentralization scenarios. Our key hypothesis posits that considering the composition of the public goods basket is important when assessing the welfare impact of decentralization. To test this hypothesis, we now propose two additional partial decentralization scenarios that isolate the gains from decentralizing property taxes, and those from decentralizing the composition of local public goods. In the first counterfactual, we let municipalities choose their tax rate, but not the composition of public goods which remains identical across municipalities and equal to the its median value in the centralized allocation. In the second, we do the opposite: we let municipalities choose the composition of public goods, but not their tax rate.

Results. As shown in columns (4) and (6), our results clearly support the view that the welfare gains stem entirely from the decentralization of public goods composition. The decentralization of the composition of public goods has a positive and large effect on overall welfare, increasing it by approximately 1.28 units. This amounts to an equivalent increase in local revenues by 45%, or around 540 euros per capita.

By contrast, the decentralization of tax rates decreases welfare by 0.04 units or 1.01% in municipal revenue per capita terms. This finding aligns with Calabrese, Epple and Romano (2012), who similarly observed no significant welfare improvements from local tax decentralization. As discussed earlier in Section 4.4 and analyzed in the above-mentioned paper, the limited impact of tax decentralization is attributable to the distortions inherent to property taxation as well as to the free riding of low-skilled individuals migrating to richer neighborhoods. This latter effect is visible in the higher increase in welfare by low-skilled relative to high-skilled individuals in column (4). These distortions counteract the potential gains from aligning local tax rates with the local median voter's preferences.

6.3 Amplification Through Migration

No mobility scenarios. As shown in equation (28), the primary benefit of decentralization — allowing localities to tailor public goods to local preferences and other local characteristics — is amplified by the individuals' ability to relocate. This mobility enables people to select municipalities offering public goods bundles that best align with their preferences and with the lowest cost.

To separately identify the welfare gains stemming from the mobility of people across locations and those from the flexibility of municipalities to choose policies locally, we compute a counterfactual allocation in which, starting from the centralized allocation, municipalities are allowed to choose the mix of public goods and the local tax rate but individuals face prohibitively high migration costs. This counterfactual effectively shuts down mobility. The results of this counterfactual simulation are presented in the column (1) labeled "No mobility." We conduct the same exercise for the scenario in which municipalities may only choose their tax rate and for the one in which they may only choose their spending shares. These results are shown in columns (3) and (5), respectively.

Results. In the absence of mobility, we find that decentralization is equivalent to an increase of 17% in municipal revenue per capita, which is already sizable. But comparing column (1) and (2), we find that allowing individuals to move across municipalities multiplies these welfare gains by a factor of almost 4, from 17 to 62%. This underscores the crucial role of population mobility in amplifying the benefits of decentralization.

Consistent with our key hypothesis that the composition of the public goods basket is important, we find that this strong amplifying role of migration is driven by the decentralization of the mix of public goods, not by the decentralization of the tax rate. Comparing columns (5) and (6), we see that the welfare gains are multiplied by a factor of 3 when individuals are allowed to move across municipalities. By contrast, comparing columns (3) and (4), there are no gains from mobility when only the tax rate is decentralized. If anything, there are losses, also consistent with the free riding of low-skilled individuals migrating to richer neighborhoods emphasized in Calabrese, Epple and Romano (2012).

6.4 Better Tailoring of Public Goods in Large Cities

What are the sources of heterogeneity across cities underlying these welfare gains from decentralizing the composition of local public goods? There are two potential sources, as indicated by the public goods price index in equation (24) and the optimal local policy rule for spending shares in equation (21): municipalities may choose different compositions of public goods because their median voter's value, v_j^M, differs and preferences are non-homothetic, or because the size of their population, L_j, differs and public goods have different degree of non-rivalry.

It turns out that most of the gains are coming from the ability of municipalities to tailor the composition of public goods to the size of their local population. To see this, we conduct two additional counterfactuals. In the first one, we assume that when municipalities set their optimal spending shares, they don't take into account the specific size of their population. Formally, in the formula (21), we replace L_j by the median across municipalities. This counterfactual isolates the role of the first source. In the second counterfactual, we do the opposite: when municipalities set their optimal spending shares, they don't take into account the specific value of their local median voter and use instead the the median across municipalities. Both counterfactuals abstract from mobility. The revenue-equivalent welfare impact are given in the Table 6 below. It is worth noting that because it is not an exact decomposition, the sum of both figures don't sum up to the total effect in column 1 of Table 5.

Table 6: Sources of Gains: Median Voter (v_j^M) *vs.* Population (L_j) Heterogeneity

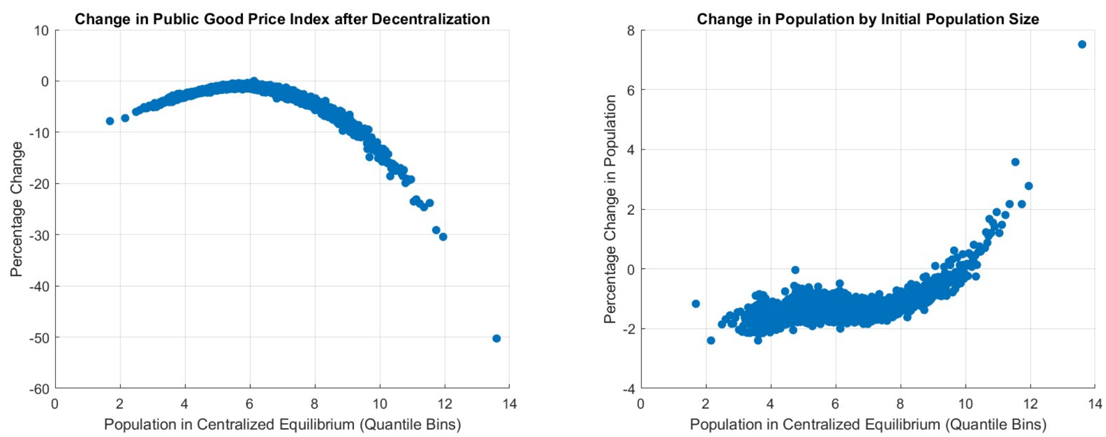
	Median Voter Heterogeneity (v_j^M)	Population Heterogeneity (L_j)
Revenue-Equivalent Gains (in %)	1.90	17.25

The intuition for these results is as follows. In the centralized counterfactual allocation, bigger cities are forced to allocate the same share of their income to low-ρ_p goods—those that are less congestible or less rivalrous—as smaller municipalities do. Given the returns to scale associated with these goods, this allocation exceeds what larger cities would optimally choose. Following decentralization, larger cities reduce their provision of low-ρ_p goods, redirecting a greater share of resources towards more congestible public goods. This reallocation is reflected in the large decrease in the "public goods price index." As illustrated in Figure 2, this index drops by up to 50% in larger municipalities.

By the same mechanism, municipalities that are significantly smaller than the median also reduce their public goods price index. But they do so by doing the exact opposite of bigger cities: they re-allocate a greater share of their budget to relatively less congestible public goods. However, the welfare gains for smaller cities are not as large, consistent with the fact that the empirical distribution of municipality sizes is very skewed.

The large decline in the public good price index in large cities in turn attracts more people, which amplifies the initial increase in welfare, echoing the previous result in Section 6.3 that migration magnifies the gains from decentralization. The right panel of Figure 2 shows that bigger cities grow by up to 8%, while many small towns shrink by 1 to 2%.

Figure 2: Changes in Population & Public Goods Price Index after Decentralization



6.5 Implications for Inequality

A common concern with decentralization policies is that they increase spatial disparities. We find support for the idea that decentralization leads to an increase in welfare inequality. However, it is driven entirely by the increase in inequality between small and large cities. We find little evidence for an increase in inequality across household types, nor do we find evidence of increased sorting by types.

Across municipalities. The benefits derived from decentralization are unevenly distributed among municipalities. Generally, the further away is a municipality from the median in terms of median income and population size, the more substantial the welfare gains. To see this, Figure 3a plots the welfare gains as a function of population size and median income and a quadratic fit . Consistent with this intuition, the gains are U-shaped in household median income income and population size, indicating that cities that are farthest from the median experience the most significant benefits from decentralization. Consistent with our previous findings, we find that larger cities gain the most in terms of both welfare and population, as shown in Figure 2. One implication is that the spatial distribution of population is more concentrated in the decentralized allocation. As a result inequality of population size across locations widen. To quantify the effects of spatial welfare inequality, we compute the Theil index. Recall that the Theil index for a measure x

with mean $\overline{x}$ is defined as:

$$T_x = \frac{1}{\sum_{i=1}^{n} L_i} \sum_{i=1}^{n} L_i \left(\frac{x_i}{\overline{x}} \cdot \log \left(\frac{x_i}{\overline{x}} \right) \right)$$

where values closer to zero indicate greater equality. Table 3b shows that the Theil index for population across municipalities increase by X. This is a sizable amount.

Because population size and welfare are tightly linked in the model, another important implication is that welfare inequality widens across locations. This implies that decentralization may exacerbate spatial disparities.

Interestingly, public spending inequality decreases (with the Theil index declining by 0.98), as wealthier individuals opt for lower tax rates in the decentralized system, resulting in a more evenly distributed allocation of public expenditures across municipalities.

Across individuals. The rise in inequality across municipalities is reflected in the rise in the Theil index of welfare across individuals, which is also reported in Table 3b. Because welfare gains tend to accrue primarily to areas that already had higher welfare levels, the Theil index for welfare across individuals increase by X.

One concern with decentralization is that it exacerbates inequality across individuals with different skills, favoring richer individuals at the expense of poorer ones. The mechanism underlying this intuition is that decentralization exacerbates the sorting of individuals by skills. To investigate this question, we look at the chance in two statistics: welfare by skill level and the exposure index for sorting, which is defined as follows Diamond and Gaubert (2022):

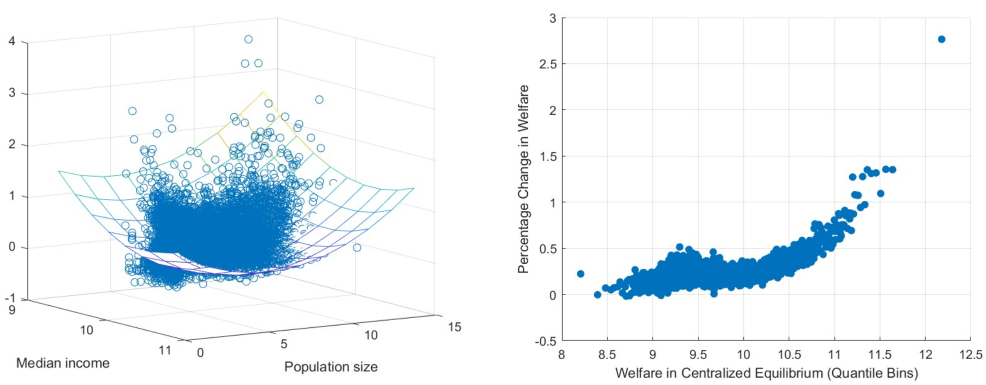
$$\text{Exposure} = \sum_j \frac{L_j^H \mu_j^H}{\sum_j L_j^H} - \sum_j \frac{L_j^L \mu_j^H}{\sum_j L_j^L}, \tag{38}$$

where L_j^t represents the population of type t in j, with H and L denoting high and low types, and μ_j^H is the high-type share in location j. Specifically, this measure quantifies the extent to which high-type individuals are more or less likely to be exposed to their own type compared to low-type individuals.

As shown in Table 3b, it is not the case that inequality across skills rise in response to decentralization. While it is true that high-skill individuals gain slightly more (.57 units) than low-skill individuals (.54 units) in the “no mobility” scenario, the results are completely offset once mobility is allowed (1.39 vs. 1.95), implying a reduction in inequality on average between the two groups. Similarly, income inequality remains stable.

Consistent with this finding, the exposure index demonstrates that population movements following decentralization appear to mitigate, rather than exacerbate, sorting effects. The exposure index decreases slightly following decentralization (-.02). The reason is that low-skilled individuals migrate to richer neighborhoods attracted by lower tax rates and better public goods, even if the mix of public goods and the tax rate are further away from their private optimum.

Figure 3: Welfare Gains



(a) by Population and Income

(b) by Initial Welfare

7 Robustness Analysis and the Role of Transfers

One important way in which the central government intervenes in local public finance is through transfers. As shown in section XX above transfers account for XX % of municipal budgets on average. Our analysis so far has kept transfers fixed. In practice they are endogenous to city characteristics such as population size and average local income. In this section, we allow transfers from the central government to adjust endogenously and investigate the robustness of our previous results to this extension of the model. In addition, on-going reforms in France have changed the design of these transfers and we use our calibrated model to analyze their impact on welfare.

7.1 A Reduced-Form Transfer Rule.

While the French municipal transfer system involves multiple components and complex eligibility criteria, our approach is to adopt a parsimonious reduced-form transfer rule that captures two essential forces of the allocation mechanism: population size and the local population mean income, as follows

$$\ln T_j = \ln \bar{T}_j + \alpha_P \ln L_j + \alpha_M \ln y_j^A \tag{39}$$

where α_P, α_M are two parameters and $\bar{T}_j$ is a location-specific exogenous variable capturing other determinants of transfers in practice from which we abstract.

To estimate the two key parameters α_P, α_M, we bring the following econometric model to the data [6]

$$\ln T_{j,t} = \alpha_0 + \alpha_P \ln(\text{Population}_{j,t}) + \alpha_M \ln(\text{Mean Income}_{j,t}) + \gamma_j + \gamma_t + \varepsilon_{j,t} \tag{40}$$

where $T_{j,t}$ denotes the total transfers to municipality j at time t, $\text{Population}_{j,t}$ represents the population, and $\text{Mean Income}_{j,t}$ is the mean household income. We include municipality and year fixed effects, γ_j and γ_t, respectively, to account for time-invariant determinants of transfers (e.g., components based on a municipality's geographic area or long-term guarantees in municipal transfer levels) and the aggregate level of transfers over time. We are not concerned about reverse causality because the transfer formula codified into law depends mechanically on contemporaneous municipal characteristics.

We estimate the transfer rule using a panel of municipal transfers covering the years 2018 and 2024. Results are presented in Table 7.[7] We find a population elasticity of 1.5, meaning that a 10% increase in population raises transfers by 15.3%. Similarly, we find a mean income elasticity of -0.39, implying that a 10% increase in mean income reduces transfers by 3.9%. As expected, this suggests that municipal

[6]See Table F.11 for a breakdown of the institutional framework governing transfers.

[7]Figure E.8 assesses the fit of the log-log specification by examining partial regression plots. The figures show that despite the highly non-linear and discontinuous nature of the transfer rules, the relationship between transfers, population, and income is well-approximated by a log-log specification.

transfers grow with population and that the system is redistributive in the sense that higher-income municipalities receive comparatively less funding.

Table 7: Reduced-form Transfer Rule

	(1)
	Log Transfer
Log Population	1.533***
	(0.0277)
Log Mean Income	-0.393***
	(0.0174)
Constant	4.253***
	(0.276)
Municipality FE	✓
Year FE	✓
Observations	66,506

* p<0.10, ** p<0.05, *** p<0.01. Robust standard errors (clustered at the municipality level) in parentheses. Sample period: 2018–2024.

7.2 Robustness Check: Endogenous Transfers

We now investigate whether the results derived in the previous section are robust to allowing transfers $\mathbf{T}$ to adjust endogenously to changes in population sizes $\mathbf{L}$ and average income $\mathbf{y}^{\mathbf{M}}$. Results are displayed in XX.

We find that our main results are robust to allowing transfers to respond endogenously to changes in population size and average income. More specifically, we find that XX

7.3 Reforms to Transfers and Local Taxation

Context. The abolition of the taxe d'habitation in France was a major fiscal reform introduced in France by President Emmanuel Macron as part of his broader tax policy agenda aimed at increasing purchasing power and simplifying local taxation. The reform gradually phased out the taxe d'habitation.[8] To offset the loss in revenues for municipalities, the government reallocated a share of the property tax (taxe foncière) collected by departments to municipalities and provided additional state transfers to ensure local government budgets remained stable.

Counterfactuals. We now run counterfactuals to evaluate the strength of the difference forces. Although our goal is not to provide a detailed evaluation of the policy change, we believe that our results help understand some of its implications. We do two policy counterfactuals.

First, we impose an upper bound on the local tax rate $t < \bar{t}$, which is the same for all municipalities, and we consider a range of values of $\bar{t}$ from 0 (completely abolishing local taxation) to .5, at which point no municipalities is constrained. In this counterfactual, we also change the government transfers to offset the loss in municipal revenues. More specifically, we increase transfers so that budget remain unchanged, *i.e.* in the transfer rule we re-calibrate the location-specific constant $\bar{T}_j$ so that transfers T_j remain unchanged, before any changes in population size and average income. In the counterfactual equilibrium, individuals move across cities, which implies that local budgets and transfers will endogenously change. The key aspect that this counterfactual captures is the loss in autonomy to set the tax and the substitution of local revenues from local property taxes to transfers.

In the second counterfactual we allow municipalities to set their local property tax freely but we vary the generosity of government's transfers. More specifically, we decrease the local-specific parameter $\bar{T}_j$ by the same multiplicative factor, which varies from 0 (no transfers) to 1 (existing transfers).

Each counterfactual captures different dimensions of the reform. The second one captures the increase in government transfers and the fact that municipalities can increase other property taxes even if the taxe d'habitation is abolished. The first counterfactual captures the fact that at least in the short run the effective local

[8] Initially, in 2018, about 80% of households saw a reduction and eventual elimination of the tax based on income thresholds. By 2023, the tax was fully abolished for all households.

tax rate decreases and that in the long run, municipalities face regulatory limits to further increases in other local property taxes.

Results. We find that the decrease in the tax rate ceiling can increase social welfare up to a point. After a threshold, further decreases leads to losses in social welfare. This result suggests that a moderate decrease in local property tax could address the distortions from property taxation—the distortion of the housing consumption choice and the externality on local costs of housing. More aggressive limits to local property tax rates would however further disconnect the provision of local public goods from its funding which exacerbates the free-riding problem—too many people moving to cities with generous public goods funded by taxpayers living elsewhere.

Turning to the second counterfactual, we find that scaling down transfers and allowing municipalities to adjust their local property tax could increase social welfare up to a point. More aggressive cuts would lead to losses in social welfare. This result is driven by a similar combination of mechanisms as the previous one. Starting from high levels, moderate decreases in transfers, which are potentially offset by endogenous adjustments in local property tax could alleviate the free-riding problem by connecting more tightly the provision of local public goods and its funding. Aggressive cuts would however lead to over-reliance of municipal budgets on property taxation, exacerbating the distortion of the housing consumption choice and the externality on local costs of housing.

8 Conclusion

The provision of local public goods and the sorting of households with diverse preferences are closely intertwined. We present new facts that shed light on the relationships between household preferences, public goods provision, and population size. We then build a model where households' residential choices are influenced by the quantity and quality of local public goods, which in turn can affect their provision. Policies aimed at equalizing local revenues or centralizing the provision of public goods may not necessarily lead to desirable outcomes, given the heterogeneity in demand for public services. Next, we will look at counterfactuals designed to study the potential distributional effects of equalization policies in the presence.

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Appendix

A Taxonomy of Public Goods

Our classification of public goods is closely tied to public accounting rules for municipalities known as M14. The M14 is a system of classifying public expenditures that a municipality uses to categorize and report budgetary operations. The M14 requires municipalities above 3,500 inhabitants to classify spending and revenue according to type of activity. The purpose of this classification is to provide elected officials with knowledge about the amount of financial resources devoted annually to different municipal services. This classification is done based on a "functional nomenclature" that uses a three-digit code to standardize the classification of activities.

We aggregate these classifications into ten broad categories of public spending based on the first digit of this code. We sometimes split this category when it encompasses distinct municipal services that each have significant budget shares. Table A.8 presents our classification. The first column are the public good categories used in the paper. The second and third columns provide sub-functions that these categories are based on. The final column titles "Details" provides the fully disaggregated functions that make up the category.

In analyzing the fiscal data of local governments, we must distinguish between budgetary transactions that reflect actual economic activity and those that reflect accounting adjustments. The dataset includes variables that capture both types of operations. We calculate real spending by subtracting non-cash budgetary operations (`OOBDEB`) from net budgetary debits (`OBNETDEB`). We do the same for revenue. This approach aligns with the methodology adopted by the Direction de l'Évaluation de la Statistique et des Études Locales (DESL) since 2012.

Table A.8: Taxonomy of Public Goods

Function	Code	Sub-function	Details
Administrative	02	General Administration	General administration of the community; local assemblies; general administration on behalf of the State; information, communication, advertising; festivals and ceremonies; aid to unclassified associations; cemeteries and funeral homes; decentralized projects with foreign authorities.
	03	Justice	Courts and courthouses; municipal jails; penitentiary services; youth detention centers; legal counsel and victim support.
Health & Safety	11	Security	National policy; municipal police; firefighters and rescue services; other ther civil security services.
	12	Hygiene and sanitation	This sub-function includes specific actions related to public safety, such as rat extermination or emergency interventions on buildings in danger.
Education	21	Primary Education	Preschool; primary schools. Excluding Teachers.
	22	Secondary Education	Middle schools; technical education; apprenticeships; high schools; maritime schools. Excluding Teachers.
	23	Higher Education	Higher education establishments; fine arts schools; architecture schools; teacher trainer colleges; nursing schools; other professional schools. Excluding Teachers.
	24	Continuing Education	Vocational training centers; continuing education centers.
	25	Additional Services	Accommodations and cafeterias; school transport; school sports; school nurses; other ancillary services.
Arts & Culture	31	Arts	Musical, lyrics, choreographic activities (e.g., orchestras); plastic arts; expositions; theaters; cinemas and performance halls.
	32	Cultural Institutions	Libraries and media centers; museums; archives; maintenance of cultural heritage (e.g., restoration of monuments); other cultural activities.

Recreation	41	Sports and Recreation	Sports centers and gymnasiums; stadiums; pools; other sports and leisure sports equipment; sporting events.
Youth	42	Youth Services	Youth leisure clubs; youth centers; camps; other youth services (e.g., playgrounds).
Social	51	Health Services	Clinics and other health establishments; other health services (e.g., family planning centers).
	52	Social Services	Social services for the disabled; social services for children and adolescents; assistance for individuals in need; other services (e.g., services in favor of refugees).
	61	Elder Services	Retirement homes; centers for the elderly; home care; senior citizens clubs.
	62	Maternity Services	Miscellaneous benefits related to maternity.
	63	Family Support	Miscellaneous benefits for families.
	64	Nurseries and Daycares	Nurseries; kindergartens; support for childcare at home.
Planning	70-73	Housing	General housing services; private sector housing services; rental assistance; construction assistance.
	81	Urban Services	Water and sewage; sanitation; waste management; hygiene; public lighting; transit;
	82	Urban Planning	Road equipment (e.g., traffic lights); municipal road infrastructure; other urban development projects (e.g., renovation of neighborhoods).
	83	Environment	Water systems (e.g., dams, management of rivers); control of pollution; conservation of natural areas.
Parks	823	Outdoor Spaces	Public parks and gardens; urban green spaces; public squares; landscaping (*villes fleuris*).
Economic	91	Markets	Covered markets; spaces for wholesale trade.
	92	Assistance for the Agricultural Sector	Land development; agro-food industries (e.g., fisheries, slaughterhouses).
	93	Assistance for Manufacturing and Energy Sectors	Power stations, gas pipelines, electricity transmission.
	94	Assistance for Commerce	Retailers; measurement of commercial activities.

95	Assistance for Tourism	Winter resorts; collection of tourism tax; hotels; holiday villages.
96	Assistance for Public Services	Maintenance of miscellaneous public services (e.g., post office).

B Municipal Engel Curves

Figure B.4: Municipal Engel Curves

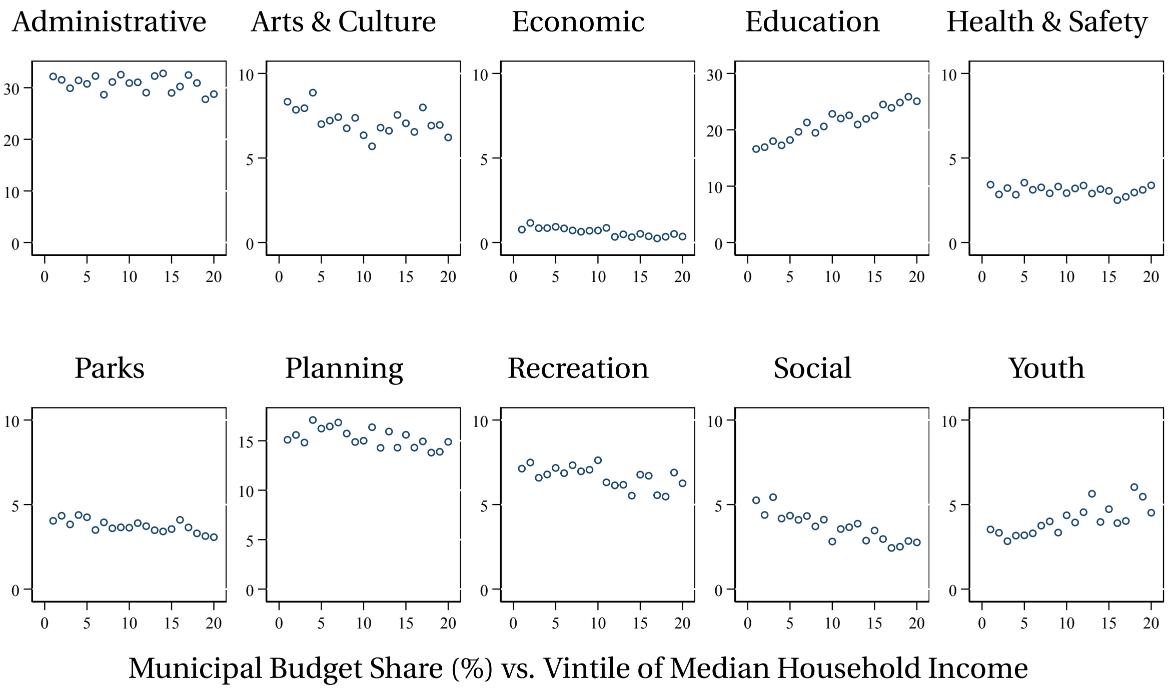
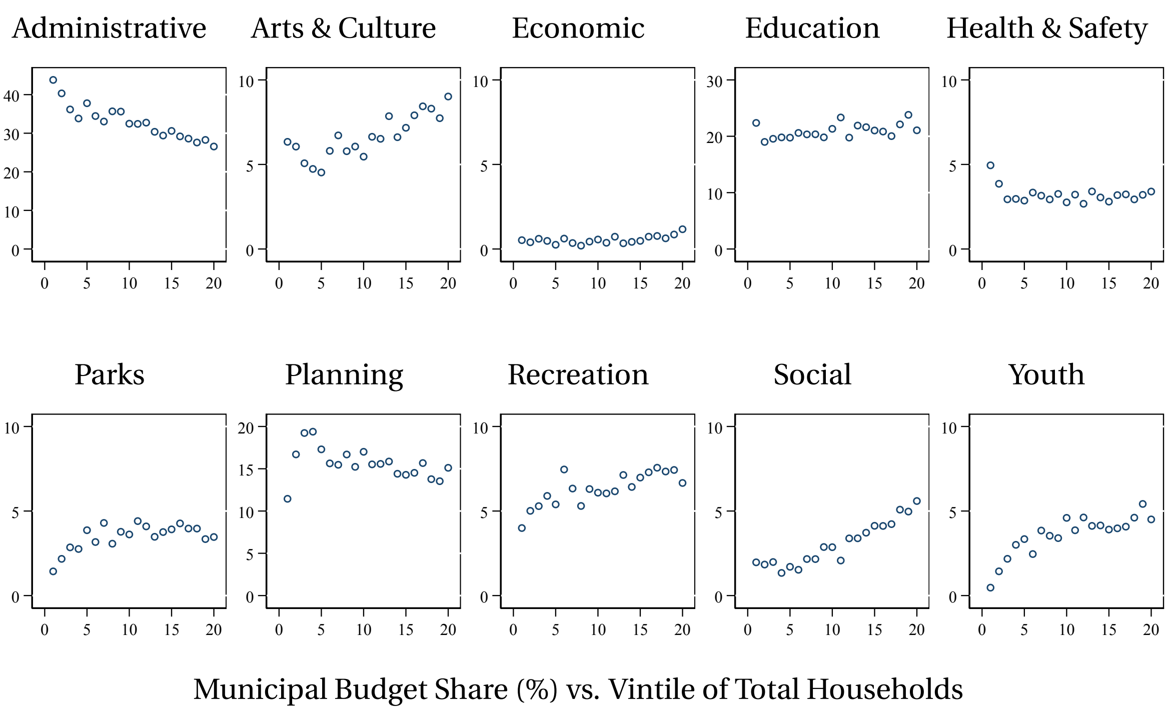


Figure B.5: Municipal Budget Shares vs. Total Households



C Sample Summary Statistics

Table C.9: Mean Characteristics of Municipalities Across Samples

	Sample of Municipalities		
	Selected	Full	Universe
Revenue per capita (€)	1,276.74	1,229.20	928.48
Mean income (€)	28,120.44	27,943.14	26,359.47
Median income (€)	21,125.07	21,202.29	21,530.10
Population	14,904	12,256	1,894
Area (km^2)	2,643.88	2,532.37	1,562.28
Rent index	11.62	11.13	9.00
Mean property tax rate	0.39	0.37	0.27
Number of municipalities	1,542	3,464	34,928
Total population (millions)	23.98	42.45	66.07

D Model: Proofs

D.1 Optimal tax rate

The first order condition is given by

$$\omega_{gij}^{\frac{1}{\gamma}}\frac{1}{q_{ij}}\left(r_jH_j-\frac{t_jr_jH_j}{1+t_j}\right)\left(\frac{t_jr_jH_j+T_j}{q_{ij}}\right)^{-\frac{1}{\gamma}}v_{ij}^{\frac{1}{\gamma}}=\left(\frac{y_{ji}(1-\tau)}{p_{ij}}\right)^{-\frac{1}{\gamma}}\frac{y_{ji}(1-\tau)}{p_{ij}^2}\frac{\partial p_{ji}}{\partial t_j}\left(1-\omega_{gij}\right)^{\frac{1}{\gamma}}v_{ij}^{\frac{1}{\gamma}}$$

where we used the fact that when choosing the tax rate, the government understands that it will also affect the base, the household's spending on housing, equal to $rH=\frac{\sum_i\omega_h(v_i)y_{ji}(1-\tau)}{1+t_j}$ which gives $\frac{\partial rH}{\partial t}=-\frac{rH}{1+t_j}$. Using the fact that $\frac{\partial p_{ji}}{\partial t_j}=\omega_h(v)(1+t_j)^{-1}p_{ji}$. Substituting this expression and simplifying gives

$$\omega_{gij}^{\frac{1}{\gamma}}\left(r_jH_j-\frac{t_jr_jH_j}{1+t_j}\right)(t_jr_jH_j+T_j)^{-\frac{1}{\gamma}}=\left(\frac{q_{ij}y_{ji}(1-\tau)}{p_{ij}}\right)^{1-\frac{1}{\gamma}}\omega_h(v)(1+t_j)^{-1}(1-\omega_{gij})^{\frac{1}{\gamma}}$$
$$\left(\frac{\omega_{gij}}{1-\omega_{gij}}\right)^{\frac{1}{\gamma}}(r_jH_j)^{1-\frac{1}{\gamma}}\left(t_j+\frac{T_j}{r_jH_j}\right)^{-\frac{1}{\gamma}}=\left(\frac{q_{ij}y_{ji}(1-\tau)}{p_{ij}}\right)^{1-\frac{1}{\gamma}}\omega_h(v)$$

Using the fact that $\omega_{gij}/(1-\omega_{gij})=\alpha_g v_{ij}^{\nu_g}$, further simplifying and raising both sides by the exponent $-\gamma$ gives

$$\left(\alpha_g v_{ij}^{\nu_g}\right)^{-1}\left(t_j+\frac{T_j}{r_jH_j}\right)=\left(\frac{q_{ij}y_{ji}(1-\tau)}{r_jH_jp_{ij}}\right)^{1-\gamma}\omega_h(v)^{-\gamma}$$
$$\left(t_j+\frac{T_j}{r_jH_j}\right)=\frac{\alpha_g v_{ij}^{\nu_g}}{\omega_h(v)}\left(\frac{q_{ij}\omega_h(v)y_{ji}(1-\tau)}{r_jH_jp_{ij}}\right)^{1-\gamma}$$

which gives equation (23).

D.2 Public good CES price index

We first show that $g=L^{-\rho_p}\left(\frac{\omega_p}{g_p}(trH+T)^{\gamma_g}\right)^{\frac{1}{\gamma_g-1}}$ for any p.

Proof. We seek to minimize $\sum_p g_p$ subject to the constraint $g=\left(\sum_{p=1}^{P}\omega_p(v)^{\frac{1}{\gamma_g}}\left[g_{pj}L_j^{-\rho_p}\right]^{1-\frac{1}{\gamma_g}}\right)^{\frac{\gamma_g}{\gamma_g-1}}$. Denote the Lagrange multiplier associated with this constraint λ. The F.O.C. is given by $1=\lambda\left(L^{-\rho_p}\right)^{1-\frac{1}{\gamma_g}}\omega_p(v)^{\frac{1}{\gamma_g}}\left(\frac{g}{g_p}\right)^{\frac{1}{\gamma_g}}$. Multiplying both sides by g_p and summing over p allows to solve for λ, which is equal to: $\lambda=\frac{\sum_p g_p}{g}=q_{ij}$. We can thus solve for q by raising $1=\lambda\left(L^{-\rho_p}\right)^{1-\frac{1}{\gamma_g}}\omega_p(v)^{\frac{1}{\gamma_g}}\left(\frac{g}{g_p}\right)^{\frac{1}{\gamma_g}}$ to the γ_g and summing over p, which gives

$$q_{ij}=\left(\sum_p\omega_p\left(L^{-\rho_p}\right)^{\gamma_g-1}\right)^{\frac{1}{1-\gamma_g}}$$

□

D.3 Proof of Proposition 3

We start with differentiating the social welfare function with respect to an arbitrary change in local spending shares and tax rates x:

$$dW = \sum_j \sum_i \left(\pi_{ij} \times \nabla_{\mathbf{x}} V_{ij} \ \cdot \ d\mathbf{x} + V_{ij} \times \nabla_{\mathbf{x}} \pi_{ij} \ \cdot \ d\mathbf{x} \right)$$

We now find an expression for $\nabla_{\mathbf{x}} \pi_{ij}$. Given that $\pi_{ij} = \frac{\exp(V_{ij})^{1/\mu}}{\sum_{l=1}^{J} \exp(V_{il})^{1/\mu}}$ we get

$$\nabla_{\mathbf{x}} \pi_{ij} = \frac{1}{\mu} \left(\pi_{ij}(1 - \pi_{ij}) \nabla_{\mathbf{x}} V_{ij} - \sum_{j' \neq j} \pi_{ij} \pi_{ij'} \nabla_{\mathbf{x}} V_{ij'} \right)$$

Consistent with a first-order approximation and the fact that there are a very large number of locations in France, we can ignore the second-order terms in π_{ij}^2 and $\pi_{ij}\pi_{ij'}$. This gives

$$\nabla_{\mathbf{x}} \pi_{ij} = \frac{1}{\mu} \pi_{ij} \nabla_{\mathbf{x}} V_{ij}$$

Replacing into the expression for dW we get

$$dW = \sum_j \sum_i \pi_{ij} \left(1 + \frac{V_{ij}}{\mu} \right) \nabla_{\mathbf{x}_j} V_{ij} \ \cdot \ d\mathbf{x}$$

We now turn to the computation of $\nabla_{\mathbf{x}} V_{ij}$. The value V_{ij} depends on the vector of local populations π through three channels: changes in the aggregate income tax rate τ through the government budget constraint, changes in local rents through the local housing market clearing condition and the local public goods price index due to the non-rivalry of public goods. This gives:

$$\nabla_{\mathbf{x}} V_{ij} = \nabla_{\mathbf{x}} V_{ij}\big|_{\pi} + \frac{\partial V_{ij}}{\partial \tau} \nabla_{\mathbf{x}} \tau + \frac{\partial V_{ij}}{\partial r_{ij}} \frac{\partial r_j}{\partial \pi_j} \nabla_{\mathbf{x}} \pi_{ij} + \frac{\partial V_{ij}}{\partial q_{ij}} \frac{\partial q_j}{\partial \pi_j} \nabla_{\mathbf{x}} \pi_{ij}$$

We can ignore the last two terms which are second order since $\nabla_{\mathbf{x}} \pi_{ij} = \frac{1}{\mu} \pi_{ij} \nabla_{\mathbf{x}} V_{ij}$ is linear in π_{ij} and would then multiply π_{ij} in the expression for dW.

When computing $\nabla_{\mathbf{x}_j} V_{ij}$, we can thus only consider the direct effect through the tax rates and spending shares as well as τ, but we can ignore the second-order feedback effects of changes in populations. It is worth noting that we do account for the first-order effects of changes in population on social welfare.

Recall that $V_{ij} = \ln(a_{ij}v_{ij})$. Together with the assumption that $\gamma = 1$, we get

$$\begin{aligned}\nabla_{\mathbf{x}} V_{ij} &= \nabla_{\mathbf{x}} \ln v_{ij} = \nabla_{\mathbf{x}} \left(\omega_g(v_{ij}) \ln g_{ij} + (1 - \omega_g(v_{ij})) \ln u_{ij}\right) \\ &= \omega_g(v_{ij}) \nabla_{\mathbf{x}} \ln g_{ij} + (1 - \omega_g(v_{ij}))\, \nabla_{\mathbf{x}} \ln u_{ij} + \nabla_{\mathbf{x}} \omega_g(v_{ij}) \ln\left(\frac{g_{ij}}{u_{ij}}\right)\end{aligned}$$

We compute each of the three derivatives separately, starting with $\nabla_{\mathbf{x}_j} \omega_g(v_{ij})$. Given that $\omega_g(v) = \frac{\alpha_g v^{\nu_g}}{\alpha_g v^{\nu_g}+1}$, we have

$$\nabla_{\mathbf{x}} \omega_g(v_{ij}) = \nu_g \omega_g(v_{ij})(1 - \omega_g(v_{ij})) \nabla_{\mathbf{x}} \ln v_{ij}$$

Given that $u = \frac{y(1-\tau)}{(r(1+t))^{\omega_h}}$, we have

$$\begin{aligned}\nabla_{\mathbf{x}} \ln u_{ij} d\mathbf{x} &= \frac{\partial \ln(1-\tau)}{\partial L_j} \nabla_{\mathbf{x}} L_j d\mathbf{x} - \omega_h \left(\frac{\partial \ln r_j}{\partial t_j}\Big|_{L_j} dt_j + \frac{\partial \ln r_j}{\partial L_j} \nabla_{\mathbf{x}} L_j d\mathbf{x} + \frac{1}{1+t_j} dt_j \right) \\ &= \frac{\partial \ln(1-\tau)}{\partial L_j} \nabla_{\mathbf{x}} L_j d\mathbf{x} - \omega_h \left(\frac{\partial \ln r_j}{\partial t_j}\Big|_{L_j} + \frac{1}{1+t_j} \right) dt_j\end{aligned}$$

where we can ignore the second-order effect of population on rents. The remaining effects are the change in the income tax rate, the indirect effect of the tax rate through rents, and the direct effect of the tax rate on the cost of housing.

We next compute $\nabla_{\mathbf{x}_j} \ln g_{ij}$:

$$\begin{aligned}\nabla_{\mathbf{x}_j} \ln g_{ij} d\mathbf{x} &= \nabla_{\mathbf{x}_j} \ln(t_j r_j H_j + T_j) d\mathbf{x} - \nabla_{\mathbf{x}_j} \ln q_{ij} d\mathbf{x} \\ &= \frac{r_j H_j + t_j \frac{\partial r_j H_j}{\partial t_j}}{t_j r_j H_j + T_j} dt_j - \frac{1}{q_{ij}} \frac{\partial q_{ij}}{\partial t_j} dt_j - \sum_{p=2}^{P} \frac{1}{q_{ij}} \frac{\partial q_{ij}}{\partial s_{pj}} ds_{pj}\end{aligned}$$

We start with grouping the first term in that last expression in this last equation with the term $\omega_h \left(\frac{\partial \ln r_j}{\partial t_j}\Big|_{L_j} + \frac{1}{1+t_j} \right)$ in the derivative of u since we recognize the first-order condition of the median voter in the decentralized economy. This gives

$$\omega_g(v_{ij})\frac{r_jH_j + t_j\frac{\partial r_jH_j}{\partial t_j}}{t_jr_jH_j + T_j} - (1-\omega_g(v_{ij}))\,\omega_h\frac{1}{1+t_j}$$

Given that $\frac{\partial r_jH_j}{\partial t_j} = \frac{r_jH_j}{1+t_j}$ we then get

$$\omega_g(v_{ij})\frac{r_jH_j - t_j\frac{r_jH_j}{1+t_j}}{t_jr_jH_j + T_j} - (1-\omega_g(v_{ij}))\,\omega_h\frac{1}{1+t_j} = \omega_g(v_{ij})\frac{1}{t_j + \frac{T_j}{r_jH_j}} - (1-\omega_g(v_{ij}))\,\omega_h$$

To make progress, we then consider a first order approximation of $\omega_g(v)$ around $\omega_g(\bar{v})$, i.e. around the median individual's value $\ln v = \ln \bar{v}$. Given that $\omega_g(v) = \frac{\alpha_p v^{\nu_p}}{1+\alpha_g v^{\nu_g}}$, we obtain

$$\omega_g(v) = \omega_g(\bar{v}) + \omega_g(\bar{v})(1-\omega_g(\bar{v}))\nu_g \ln\frac{v}{\bar{v}}.$$

We also consider an approximation of the transfers over housing spending ratio $T_j/(r_jH_j)$ around the median individual level $\overline{T/(r_jH_j)}$. Replacing in the equation above and given the assumption that the tax rate in the centralized allocation $\bar{t}$ are solution to the median individual's local maximization problem, we obtain

$$\left(\omega_g(\bar{v}) + \omega_g(\bar{v})(1-\omega_g(\bar{v}))\nu_g \ln\frac{v}{\bar{v}}\right)\frac{1}{\bar{t}+\overline{\left(\frac{T}{rH}\right)}} - \frac{\omega_g(\bar{v})}{\left(\bar{t}+\overline{\left(\frac{T}{rH}\right)}\right)^2}\left(\frac{T}{rH} - \overline{\left(\frac{T}{rH}\right)}\right)$$

$$-\left(1-\omega_g(\bar{v}) - \omega_g(\bar{v})(1-\omega_g(\bar{v}))\nu_g \ln\frac{v}{\bar{v}}\right)\omega_h\frac{1}{1+\bar{t}}$$

$$\left(\omega_g(\bar{v})(1-\omega_g(\bar{v}))\nu_g \ln\frac{v}{\bar{v}}\right)\left(\frac{1}{\bar{t}+\overline{\left(\frac{T}{rH}\right)}} + \omega_h\frac{1}{1+\bar{t}}\right) - \frac{\omega_g(\bar{v})}{\left(\bar{t}+\overline{\left(\frac{T}{rH}\right)}\right)^2}\left(\frac{T}{rH} - \overline{\left(\frac{T}{rH}\right)}\right)$$

$$\left(\omega_g(\bar{v})(1-\omega_g(\bar{v}))\nu_g \ln\frac{v}{\bar{v}}\right)\left(\frac{1}{\bar{t}+\overline{\left(\frac{T}{rH}\right)}} + \frac{\omega_g(\bar{v})}{1-\omega_g(\bar{v})}\frac{1}{\bar{t}+\overline{\left(\frac{T}{rH}\right)}}\right) - \frac{\omega_g(\bar{v})}{\left(\bar{t}+\overline{\left(\frac{T}{rH}\right)}\right)^2}\left(\frac{T}{rH} - \overline{\left(\frac{T}{rH}\right)}\right)$$

$$\frac{\omega_g(\bar{v})}{\bar{t}+\left(\frac{\bar{T}}{rH}\right)}(1-\omega_g(\bar{v}))\nu_g \ln\frac{v}{\bar{v}}\left(1+\frac{\omega_g(\bar{v})}{1-\omega_g(\bar{v})}\right) - \frac{\omega_g(\bar{v})}{\left(\bar{t}+\overline{\left(\frac{T}{rH}\right)}\right)^2}\left(\frac{T}{rH} - \overline{\left(\frac{T}{rH}\right)}\right)$$

$$(1-\omega_g(\bar{v}))\omega_h\left[\nu_g \ln\frac{v}{\bar{v}} - \frac{1}{\left(\bar{t}+\overline{\left(\frac{T}{rH}\right)}\right)}\left(\frac{T}{rH} - \overline{\left(\frac{T}{rH}\right)}\right)\right]$$

We next compute $\sum_{p=2}^{P}\frac{1}{q_{ij}}\frac{\partial q_{ij}}{\partial s_{pj}}$. We start by noting that $q_{ij} = 1/\bar{g}_{ij}$ hence

$$\frac{1}{q_{ij}}\frac{\partial q_{ij}}{\partial s_{pj}} = \frac{1}{\bar{g}_{ij}}\frac{\partial \bar{g}_{ij}}{\partial s_{pj}}$$

with

$$\bar{g}(s_{2j},...s_{Pj}) = \left(\omega_1(v)^{\frac{1}{\gamma_g}}\left(L_j^{-\rho_1}\left(1-\sum_{p=2}^{P}s_{pj}\right)\right)^{1-\frac{1}{\gamma_g}} + \sum_{p=2}^{P}\omega_p(v)^{\frac{1}{\gamma_g}}\left(L_j^{-\rho_p}s_{pj}\right)^{1-\frac{1}{\gamma_g}}\right)^{\frac{\gamma_g}{\gamma_g-1}}$$

We can thus compute $\frac{\partial \bar{g}_{ij}}{\partial s_{pj}}$.

$$\frac{\partial \bar{g}_{ij}}{\partial s_{pj}} = \bar{g}_{ij}^{1/\gamma_g}\left(s_{pj}^{-\frac{1}{\gamma_g}}\omega_p(v)^{\frac{1}{\gamma_g}}\left(L_j^{-\rho_p}\right)^{1-\frac{1}{\gamma_g}} - s_{1j}^{-\frac{1}{\gamma_g}}\omega_1(v)^{\frac{1}{\gamma_g}}\left(L_j^{-\rho_1}\right)^{1-\frac{1}{\gamma_g}}\right) + \frac{\partial \bar{g}_{ij}}{\partial \ln(v_{ij})}\frac{\partial \ln(v_{ij})}{\partial s_{pj}}$$

We start with the derivative holding v constant.

To make progress, we then consider a first order approximation of $\omega_p(v)^{1/\gamma_g}$ around $\omega_p(\bar{v})^{1/\gamma_g}$, i.e. around the median individual's value $\ln v = \ln \bar{v}$. Given that

$\omega_p(v) = \frac{\alpha_p v^{\nu_p}}{1+\sum_{p'=2}^P \alpha_{p'} v^{\nu_{p'}}}$, that is given by

$$
\begin{aligned}
\omega_p(v)^{1/\gamma_g} \left(L_j^{-\rho_p}\right)^{1-\frac{1}{\gamma_g}} &= \omega_p(\bar{v})^{1/\gamma_g} \left(\bar{L}^{-\rho_p}\right)^{1-\frac{1}{\gamma_g}} + \frac{\alpha_p^{1/\gamma_g} \frac{\nu_p}{\gamma_g} \ln \frac{v}{\bar{v}} v^{\nu_p/\gamma_g} \left(\sum_{p'=1}^P \alpha_{p'} v^{\nu_{p'}}\right)^{1/\gamma_g}}{\left(1+\sum_{p'=2}^P \alpha_{p'} v^{\nu_{p'}}\right)^{2/\gamma_g}} \left(\bar{L}^{-\rho_p}\right)^{1-\frac{1}{\gamma_g}} \\
&- \frac{\alpha_p^{1/\gamma_g} v^{\nu_p/\gamma_g} \left(\sum_{p'=2}^P \alpha_{p'} \ln \frac{v}{\bar{v}} \frac{\nu_{p'}}{\gamma_g} v^{\nu_{p'}}\right) \left(1+\sum_{p'=1}^P \alpha_{p'} v^{\nu_{p'}}\right)^{1/\gamma_g - 1}}{\left(1+\sum_{p'=2}^P \alpha_{p'} v^{\nu_{p'}}\right)^{2/\gamma_g}} \left(\bar{L}^{-\rho_p}\right)^{1-\frac{1}{\gamma_g}} \\
&- \rho_p \omega_p(\bar{v})^{1/\gamma_g} \left(1 - \frac{1}{\gamma_g}\right) \bar{L}^{-\rho_p\left(1-\frac{1}{\gamma_g}\right)} \frac{L_j - \bar{L}}{\bar{L}} \\
&= \omega_p(\bar{v})^{1/\gamma_g} + \frac{\nu_p}{\gamma_g} \omega_p(\bar{v})^{1/\gamma_g} \ln \frac{v}{\bar{v}} - \omega_p(\bar{v})^{1/\gamma_g} \sum_{p=2} \frac{\nu_{p'}}{\gamma_g} \omega_{p'}(\bar{v})^{1/\gamma_g} \ln \frac{v}{\bar{v}} \left(\bar{L}^{-\rho_p}\right)^{1-\frac{1}{\gamma_g}} \\
&- \rho_p \omega_p(\bar{v})^{1/\gamma_g} \left(1 - \frac{1}{\gamma_g}\right) \bar{L}^{-\rho_p\left(1-\frac{1}{\gamma_g}\right)} \frac{L_j - \bar{L}}{\bar{L}} \\
&= \omega_p(\bar{v})^{1/\gamma_g} + \frac{\nu_p}{\gamma_g} \omega_p(\bar{v})^{1/\gamma_g} \left[1 - \sum_{p=2} \frac{\nu_{p'}}{\nu_p} \omega_{p'}(\bar{v})^{1/\gamma_g}\right] \ln \frac{v}{\bar{v}} \left(\bar{L}^{-\rho_p}\right)^{1-\frac{1}{\gamma_g}} \\
&- \rho_p \omega_p(\bar{v})^{1/\gamma_g} \left(1 - \frac{1}{\gamma_g}\right) \bar{L}^{-\rho_p\left(1-\frac{1}{\gamma_g}\right)} \frac{L_j - \bar{L}}{\bar{L}}
\end{aligned}
$$

Replacing in the equation above and given the assumption that the spending shares in the centralized allocation are solutions to median individual's local maximization problem, we obtain

$$
\begin{aligned}
\frac{\partial \bar{g}_{ij}}{\partial s_{pj}} &= \frac{\nu_p}{\gamma_g} \ln \frac{v}{\bar{v}} \bar{g}_{ij}^{1/\gamma_g} \omega_p(\bar{v})^{1/\gamma_g} \left[1 - \sum_{p=2} \frac{\nu_{p'}}{\nu_p} \omega_{p'}(\bar{v})^{1/\gamma_g}\right] s_{pj}^{-\frac{1}{\gamma_g}} \left(\bar{L}^{-\rho_p}\right)^{1-\frac{1}{\gamma_g}} \\
&- \rho_p \bar{g}_{ij}^{1/\gamma_g} \omega_p(\bar{v})^{1/\gamma_g} s_{pj}^{-\frac{1}{\gamma_g}} \left(1 - \frac{1}{\gamma_g}\right) \bar{L}^{-\rho_p\left(1-\frac{1}{\gamma_g}\right)} \frac{L_j - \bar{L}}{\bar{L}} \\
&+ \frac{\partial \bar{g}_{ij}}{\partial \ln(v_{ij})} \frac{\partial \ln(v_{ij})}{\partial s_{pj}}
\end{aligned}
$$

Using the FOC of the individual with $\ln \bar{v}$ gives

$$\begin{aligned}
\frac{\partial \bar{g}_{ij}}{\partial s_{pj}} &= \frac{\nu_p}{\gamma_g} \ln \frac{v}{\bar{v}} \bar{g}_{ij}^{1/\gamma_g} \omega_1(\bar{v})^{1/\gamma_g} \left[1 - \sum_{p=2} \frac{\nu_{p'}}{\nu_p} \omega_{p'}(\bar{v})^{1/\gamma_g} \right] s_{1j}^{-\frac{1}{\gamma_g}} \left(\bar{L}^{-\rho_1} \right)^{1-\frac{1}{\gamma_g}} \\
&\quad - \rho_p \bar{g}_{ij}^{1/\gamma_g} \omega_1(\bar{v})^{1/\gamma_g} \left(1 - \frac{1}{\gamma_g} \right) s_{1j}^{-\frac{1}{\gamma_g}} \bar{L}^{-\rho_1 \left(1-\frac{1}{\gamma_g}\right)} \frac{L_j - \bar{L}}{\bar{L}} \\
&\quad + \frac{\partial \bar{g}_{ij}}{\partial \ln(v_{ij})} \frac{\partial \ln(v_{ij})}{\partial s_{pj}}
\end{aligned}$$

Summing over all $P-1$ goods, we obtain

$$\begin{aligned}
\sum_{p=2}^{P} \frac{1}{q_{ij}} \frac{\partial q_{ij}}{\partial s_{pj}} ds_p &= \ln \frac{v}{\bar{v}} \bar{g}_{ij}^{1/\gamma_g - 1} \sum_{p=2}^{P} \frac{\nu_p}{\gamma_g} \omega_1(\bar{v})^{1/\gamma_g} \left[1 - \sum_{p=2} \frac{\nu_{p'}}{\nu_p} \omega_{p'}(\bar{v}) \right] s_{1j}^{-\frac{1}{\gamma_g}} \left(\bar{L}^{-\rho_1} \right)^{1-\frac{1}{\gamma_g}} ds_p \\
&\quad - \frac{L_j - \bar{L}}{\bar{L}} \bar{g}_{ij}^{1/\gamma_g - 1} \sum_{p=2}^{P} \rho_p \left(1 - \frac{1}{\gamma_g} \right) \omega_1(\bar{v})^{1/\gamma_g} s_{1j}^{-\frac{1}{\gamma_g}} \left(\bar{L}^{-\rho_1} \right)^{1-\frac{1}{\gamma_g}} ds_p \\
&\quad + \sum_{p=2}^{P} \frac{\partial \bar{g}_{ij}}{\partial \ln(v_{ij})} \frac{\partial \ln(v_{ij})}{\partial s_{pj}} ds_p \\
\sum_{p=2}^{P} \frac{1}{q_{ij}} \frac{\partial q_{ij}}{\partial s_{pj}} ds_p &= \ln \frac{v}{\bar{v}} \bar{g}_{ij}^{1/\gamma_g - 1} s_{1j}^{-\frac{1}{\gamma_g}} \left(\bar{L}^{-\rho_1} \right)^{1-\frac{1}{\gamma_g}} \omega_1(\bar{v})^{1/\gamma_g} \sum_{p=2}^{P} \frac{\nu_p}{\gamma_g} \left[1 - \sum_{p=2} \frac{\nu_{p'}}{\nu_p} \omega_{p'}(\bar{v}) \right] ds_p \\
&\quad - \frac{L_j - \bar{L}}{\bar{L}} \bar{g}_{ij}^{1/\gamma_g - 1} \sum_{p=2}^{P} \rho_p \left(1 - \frac{1}{\gamma_g} \right) \omega_1(\bar{v})^{1/\gamma_g} s_{1j}^{-\frac{1}{\gamma_g}} \left(\bar{L}^{-\rho_1} \right)^{1-\frac{1}{\gamma_g}} ds_p \\
&\quad + \sum_{p=2}^{P} \frac{\partial \bar{g}_{ij}}{\partial \ln(v_{ij})} \frac{\partial \ln(v_{ij})}{\partial s_{pj}} ds_p \\
\sum_{p=2}^{P} \frac{1}{q_{ij}} \frac{\partial q_{ij}}{\partial s_{pj}} ds_p &= A_{ij} \left[\frac{1}{\gamma_g} \ln \frac{v}{\bar{v}} \sum_{1=2}^{P} \left(\nu_p - E_{\omega_p(\bar{v})} \left[\nu_p \right] \right) + \frac{L_j - \bar{L}}{\bar{L}} \sum_{p=2}^{P} \rho_p \left(\frac{1}{\gamma_g} - 1 \right) \right] ds_p \\
&\quad + \sum_{p=2}^{P} \frac{\partial \bar{g}_{ij}}{\partial \ln(v_{ij})} \frac{\partial \ln(v_{ij})}{\partial s_{pj}} ds_p \\
\text{with} \quad A_{ij} &= \bar{g}_{ij}^{1/\gamma_g - 1} s_{1j}^{-\frac{1}{\gamma_g}} \left(\bar{L}^{-\rho_1} \right)^{1-\frac{1}{\gamma_g}} \omega_1(\bar{v})^{1/\gamma_g} \\
E_{\omega_p(\bar{v})} \left[\nu_p \right] &= \sum_{p'=1} \nu_{p'} \omega_{p'}(\bar{v})
\end{aligned}$$

We next solve for the derivative of $\frac{\partial \bar{g}_{ij}}{\partial s_{pj}}$ through v, which corresponds to the term $\frac{\partial \bar{g}_{ij}}{\partial \ln(v_{ij})} \frac{\partial \ln(v_{ij})}{\partial \mathbf{x}_j} d\mathbf{x}_j$.

$$
\begin{aligned}
\frac{\partial \bar{g}_{ij}}{\partial \ln(v_{ij})}\frac{\partial \ln(v_{ij})}{\partial s_{pj}} &= \frac{\partial \ln(v_{ij})}{\partial s_{pj}} \sum_{p=2}^{P} \frac{\nu_p}{\gamma_g - 1} \bar{g}_{ij}^{1/\gamma_g} \omega_p(\bar{v})^{1/\gamma_g} \left[1 - \sum_{p=2} \frac{\nu_{p'}}{\nu_p} \omega_{p'}(\bar{v})\right] s_{pj}^{1-\frac{1}{\gamma_g}} \left(\bar{L}^{-\rho_p}\right)^{1-\frac{1}{\gamma_g}} \\
&= \frac{\partial \ln(v_{ij})}{\partial s_{pj}} \sum_{p=2}^{P} \frac{\nu_p}{\gamma_g - 1} \bar{g}_{ij}^{1/\gamma_g} \omega_1(\bar{v})^{1/\gamma_g} \left[1 - \sum_{p=2} \frac{\nu_{p'}}{\nu_p} \omega_{p'}(\bar{v})\right] s_{pj} s_{1j}^{-\frac{1}{\gamma_g}} \left(\bar{L}^{-\rho_1}\right)^{1-\frac{1}{\gamma_g}} \\
&= \frac{\partial \ln(v_{ij})}{\partial s_{pj}} \frac{\bar{g}_{ij}^{1/\gamma_g}}{\gamma_g - 1} \omega_1(\bar{v})^{1/\gamma_g} s_{1j}^{-\frac{1}{\gamma_g}} \left(L_j^{-\rho_1}\right)^{1-\frac{1}{\gamma_g}} \sum_{p=2}^{P} \nu_p \left[1 - \sum_{p=2} \frac{\nu_{p'}}{\nu_p} \omega_{p'}(\bar{v})\right] s_{pj} \\
&= \frac{\partial \ln(v_{ij})}{\partial s_{pj}} \frac{\bar{g}_{ij}^{1/\gamma_g}}{\gamma_g - 1} \omega_1(\bar{v})^{1/\gamma_g} s_{1j}^{-\frac{1}{\gamma_g}} \left(\bar{L}^{-\rho_1}\right)^{1-\frac{1}{\gamma_g}} \sum_{p=2}^{P} \nu_p (s_{pj} - \omega_p(\bar{v}))
\end{aligned}
$$

We obtain a similar expression for the derivative with respect to t_j :

$$
\frac{\partial \bar{g}_{ij}}{\partial \ln(v_{ij})}\frac{\partial \ln(v_{ij})}{\partial t_{pj}} = \frac{\partial \ln(v_{ij})}{\partial t_{pj}} \frac{\bar{g}_{ij}^{1/\gamma_g}}{\gamma_g - 1} \omega_1(\bar{v})^{1/\gamma_g} s_{1j}^{-\frac{1}{\gamma_g}} \left(\bar{L}^{-\rho_1}\right)^{1-\frac{1}{\gamma_g}} \sum_{p=2}^{P} \nu_p (s_{pj} - \omega_p(\bar{v}))
$$

Hence

$$
\begin{aligned}
\frac{\partial \bar{g}_{ij}}{\partial \ln(v_{ij})}\frac{\partial \ln(v_{ij})}{\partial \mathbf{x}_j} &= B_{ij} \nabla_{\mathbf{x}_j} \ln(v_{ij}) \\
\text{with} \quad B_{ij} &= \frac{\bar{g}_{ij}^{1/\gamma_g}}{\gamma_g - 1} \omega_1(\bar{v})^{1/\gamma_g} s_{1j}^{-\frac{1}{\gamma_g}} \left(\bar{L}^{-\rho_1}\right)^{1-\frac{1}{\gamma_g}} \sum_{p=1}^{P} \nu_p (s_{pj} - \omega_p(\bar{v}))
\end{aligned}
$$

We are now ready to put everything together.

$$
\begin{aligned}
\nabla_{\mathbf{x}} V_{ij} &= \Gamma_{ij} \left[\omega_g(v_{ij}) \nabla_{\mathbf{x}} \ln g_{ij} + (1 - \omega_g(v_{ij})) \nabla_{\mathbf{x}} \ln u_{ij}\right] \\
\text{with} \quad \Gamma_{ij} &= \frac{1}{1 - \nu_g \omega_g(v_{ij})(1 - \omega_g(v_{ij})) \ln\left(\frac{g_{ij}}{u_{ij}}\right) - B_{ij} \omega_g(v_{ij})}
\end{aligned}
$$

$$\nabla_{\mathbf{x}} V_{ij} d\mathbf{x} = \Gamma_{ij} \left[\omega_g(v_{ij}) \nabla_{\mathbf{x}} \ln g_{ij} + (1-\omega_g(v_{ij}))\, \nabla_{\mathbf{x}} \ln u_{ij}\right] d\mathbf{x}$$

$$\begin{aligned}
\nabla_{\mathbf{x}} V_{ij} d\mathbf{x} = \Gamma_{ij} \Bigg[& \omega_g(v_{ij}) A_{ij} \left[\frac{1}{\gamma_g} \ln \frac{v}{\bar{v}} \sum_{1=2}^{P} \left(\nu_p - E_{\omega_p(\bar{v})}\left[\nu_p\right]\right) + \frac{L_j - \bar{L}}{\bar{L}} \sum_{p=2}^{P} \rho_p \left(\frac{1}{\gamma_g} - 1\right)\right] ds_p \\
& + (1-\omega_g(\bar{v}))\omega_h \left[\nu_g \ln \frac{v_{ij}}{\bar{v}} - \frac{1}{\left(\bar{t} + \overline{\left(\frac{T}{rH}\right)}\right)} \left(\frac{T_j}{r_j H_j} - \overline{\left(\frac{T}{rH}\right)}\right)\right] dt_j \\
& + \underbrace{(1-\omega_g(v_{ij})) \left(\frac{\partial \ln(1-\tau)}{\partial L_j} \nabla_{\mathbf{x}} L_j d\mathbf{x} - \omega_h \frac{\partial \ln r_j}{\partial t_j}\Big|_{L_j} dt_j\right) \Bigg]}_{GE_j}
\end{aligned}$$

with $GE_j = (1-\omega_g(v_{ij})) \left(\frac{\partial \ln(1-\tau)}{\partial L_j} \nabla_{\mathbf{x}} L_j d\mathbf{x} - \omega_h \frac{\partial \ln r_j}{\partial t_j}\Big|_{L_j} dt_j\right)$.

E Estimates

Table E.10: Estimates for α_g and ν_g

	$\ln\left(t_j + \frac{T_j}{r_j H_j}\right)$
$\ln v^{median}$	-1.483***
	(0.061)
(Constant)	13.231***
	(0.581)
N	2,232

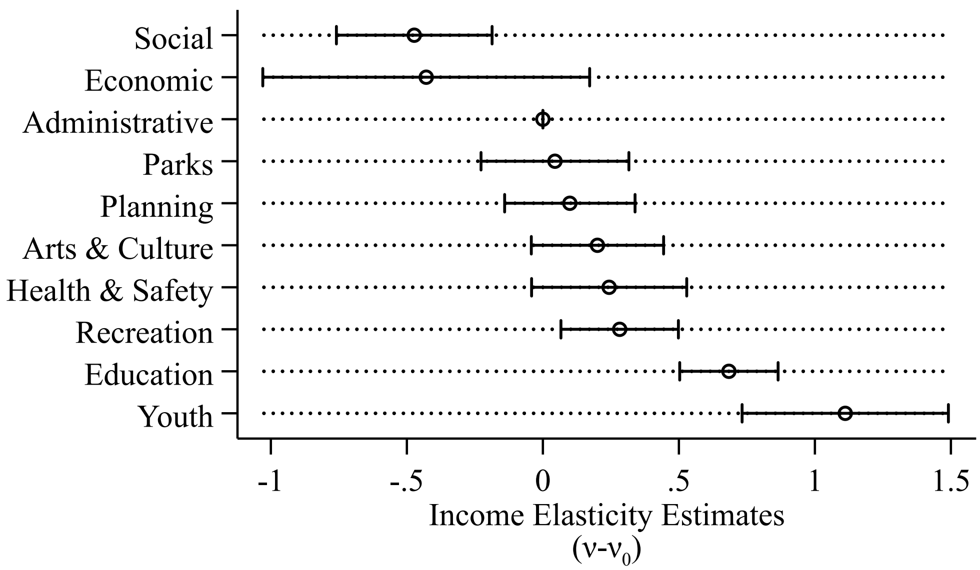
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Robust standard errors in parentheses.

Sample: urban municipalities, 2018.

Figure E.7: Elasticity Estimates

(a) Income Elasticities



(b) Population Elasticities

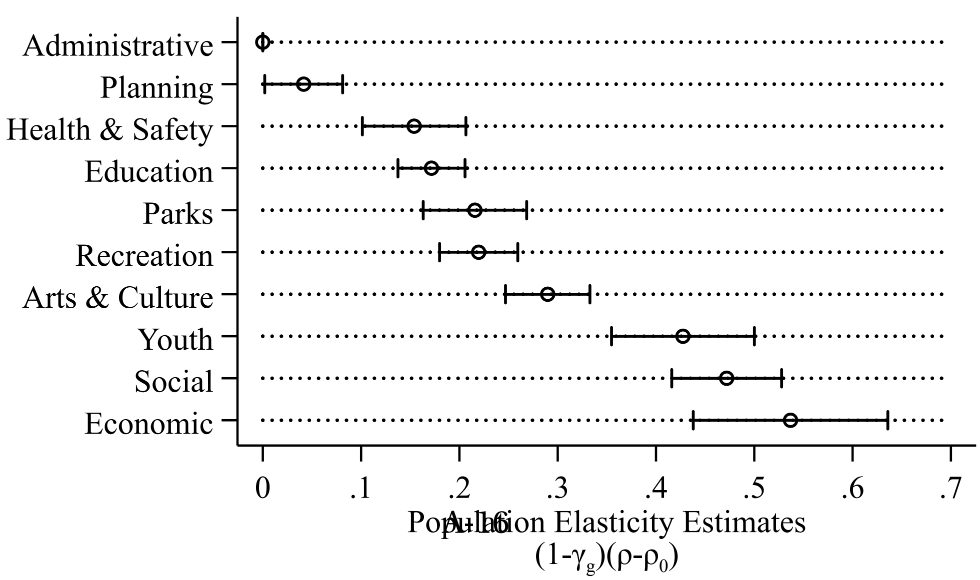
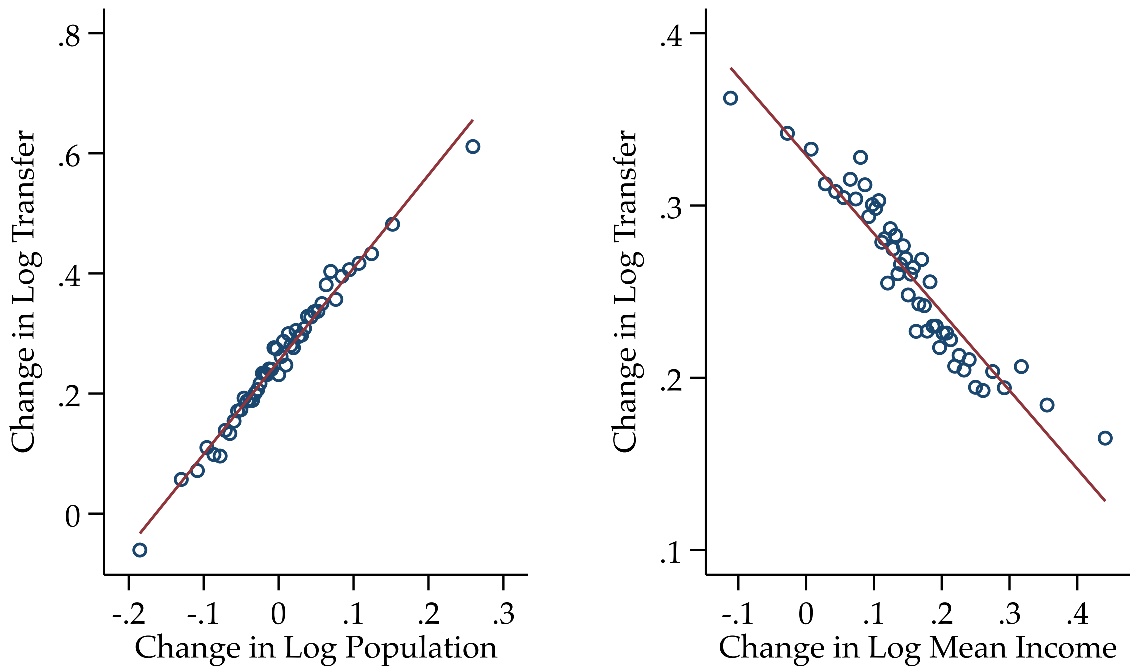


Figure E.8: Partial Regression Plots for Transfer Rule



Note: These figures illustrate the relationship between changes in municipal transfers and changes in population (left) and mean income (right). Each plot presents a binned scatterplot, where both axes reflect residualized values after controlling for year fixed effects, municipality fixed effects, and the excluded explanatory variable.

F Allocation of Municipal Transfers in France

Table F.11: Structural Components of France's Municipal Transfer System (DGF)

Component	Allocation Rules	Measurement	Reforms/Exclusions
Flat-Rate Endowment	- Universal entitlement - Progressive reduction for FP >85% national avg - Population weighting: DGF pop = Census + 2nd homes	- Annual population updates (INSEE + cadastral data) - FP = 3-yr avg taxable resources per capita - Curtailing: Logarithmic reduction above threshold	2015: Component consolidation 2023: CPS transfer to EPCI Annual indexation formula
Urban Solidarity (DSU)	- 5k-9,999 pop: Top 10% SI - ≥10k pop: Top 67% SI - Exclusion: RPU >2.5× stratum avg - SI = 0.3FP + 0.25Inc + 0.3HA + 0.15SH	- FP: Rolling 3-yr fiscal capacity - Inc: Median taxable household income - HA: Recipients/Total population - SH: Social units/Housing stock	2017: SI reweighting (Inc↑) 2016: Threshold tightening Progression guarantees
Rural Solidarity (DSR)	- Town-centers - Equalization: FP <2x stratum avg - Target: Synthetic index (0.7FP + 0.3Inc) - <10k pop (exceptions per L.2334-22-2)	- Service centrality: 15% canton pop - FP: Departmental comparisons - 3-yr income mobility adjustment	2011: Target fraction added 2022: Rural exception for >10k 90-120% allocation bounds
National Equalization (DNP)	- Standard: FP ≤105% + TE ≥100% - Cities >10k: FP ≤85% + TE ≥85% - Derogations: Max tax rates - Main vs. increase shares	- FP: Post-reform tax capacity - TE = (Collections/Potential)/Avg - Progressivity: 0.5-1.0 multipliers	2004: DNP integration 2010: Post-TP reform adjustments Dual allocation rates

Key: FP = Financial Potential, Inc = Mean Household Income, HA = Housing Assistance, SH = Social Housing, SI = Synthetic Index, TE = Tax Effort, RPU = Own Resources, EPCI = Intercommunal Entities.
Sources: CGCT Articles L.2334-20 to L.2334-22-2; Finance Laws 2010-2023; DGCL technical guides