Optimal Interest Rate Tightening with Financial Fragility*

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Abstract

Recent evidence highlights that interest rate tightening has important financial stability implications. We develop a tractable model in which financial intermediaries face occasionally binding leverage constraints and endogenous run risks, while producers are subject to price adjustment frictions. When intermediaries perform maturity transformation and their equity is low, higher interest rates depress asset prices and amplify financial distortions. We characterize the optimal (Ramsey) mix of interest rate policy, credit policy and equity injections in periods of fragility. If non-interest-rate tools were costless, the right combination could fully separate financial stability from price stability objectives. When these tools are costly, optimal interest rate policy is less aggressive and, in the presence of run risk, incorporates risk-management considerations. The optimal policy mix depends on the extent of fragility, and the cost and effectiveness of non-interest-rate instruments.

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1 Introduction

Recent financial market turmoil—including the collapse of Silicon Valley Bank and other U.S. regional banks, as well as the liability-driven investment (LDI) crisis in the UK—has revived concerns that monetary tightening aimed at fighting inflation can exacerbate financial instability. These episodes highlighted the importance of valuation effects and interest rate risk: rising rates reduce the value of long-duration assets, weakening intermediaries' balance sheets. This in turn may trigger disruptions to intermediation when institutions are forced to scale back their investments and depositor runs. Historical evidence confirms that interest rate tightening has important financial stability implications (Schularick et al., 2021; Boissay et al., 2023; Grimm, 2024). Such episodes raise important theoretical questions. When and how does monetary tightening trigger or amplify financial instability? How should monetary policy respond to inflation when financial stability is at risk? How can other tools be used to better separate financial stability from price stability concerns?

In this paper, we develop a two-period New Keynesian (NK) model with a financial sector that issues short-term deposits and buys long-term financial claims subject to a leverage constraint and to a risk of depositors run. The model can rationalize why monetary policy tightening in times of high inflation and financial fragility may lead to a rise in financial instability, through a drop in asset prices, a decline in bank equity, tightening balance sheet constraints and more banking panics, all consistent with empirical evidence. We then use the model to characterize the optimal (Ramsey) combination of interest rate and other tools, including credit policy, equity injections and deposit insurance. The tractability of our model allows us to derive simple intuitive formulas for optimal policies, clarifying the determinants of the optimal policy mix.

The model incorporates essential ingredients needed to analyze the trade-offs between price and financial instability in times of interest rate tightening and financial fragility. Firms and workers face adjustment costs to price changes. This implies that prices are sticky in the short run, giving rise to an *aggregate demand externality*. Sticky prices generate a meaningful role for monetary policy in stabilizing prices. Motivated by their empirical importance, we introduce two financial frictions, which we analyze independently. First financial intermediaries, which issue short-term deposits and purchase long-term bonds and real assets, face an occasionally binding leverage constraint due to moral hazard in the spirit of Gertler et al. (2020) (henceforth, GKP) or due to macroprudential regulation. A binding constraint limits the financial sector's ability to arbitrage and opens up *a wedge between the deposit and the lending rate* (the return on real assets). Second intermediaries face a *risk of systemic bank runs* due to coordination failures among depositors. Coordination failures arise from the inefficient drop in asset prices when intermediaries are forced to quickly liquidate their assets in a situation of run. Consistent with our global games microfoundation, the likelihood of runs increases with fundamental fragilities (Morris and Shin, 2003; Goldstein and Pauzner, 2005).

We show how interest rate tightening interact with each financial friction and amplify financial distortions by depressing asset prices and reducing banks' returns on assets.[1] These negative valuation effects erode intermediaries' net worth. In the case of the leverage constraint, this erosion tightens intermediaries' constraints, raising the spread between lending and deposit rates. In the case of depositors' run, it increases the risk of runs by making banks more vulnerable to insolvency in the event of a widespread panic. The resulting increase in financial distortions—reflected in wider spreads and heightened run risk—entails welfare costs that policymakers should seek to mitigate.

Before turning to the normative analysis, we show that the model's core mechanisms are consistent with empirical evidence. We use a historical panel of financial crises and monetary policy, which covers 18 advanced economies and spans the period 1870–2016, by combining the global historical datasets by Jordà et al. (2017) and by Baron et al. (2021). Building on the recent evidence by Schularick et al. (2021) and Boissay et al. (2023), we test the model's core mechanisms by looking at the impact of interest rate tightening events on the likelihood of banking panics and bank equity crashes, asset prices and bank loans and at the differential effect of tightening in supply-driven and demand-driven inflationary environment. Our identification strategy relies on a rich set of macro controls to address the issue of omitted variables and on instrumenting changes in the nominal rates with the Trilemma instrument. Our empirical results are all qualitatively consistent with the model's predictions: interest rate tightening significantly increases the likelihood of bank panics and crashes, decreases asset prices and leads banks to lend less, with stronger effects in times of supply-driven inflation.

An important contribution of the paper is to characterize the optimal (Ramsey) combination of interest rate policy and other tools in both cases of financial fragility: when balance sheet constraints bind or when there is a risk of a systemic run. While our baseline approach assumes that the planner maximizes households welfare, we show that our results are robust to imposing a strict inflation targeting mandate for interest

[1] Loan delinquencies are another important channel in practice. While not modeled explicitly, they are captured by the decline in asset returns.

rate policy. Crucially, we allow for the possibility that non-interest-rate tools—such as equity injections, credit policy, and deposit insurance—are costly, in line with recent work. These tools may entail fiscal burdens, distort risk-taking incentives, or hinder financial market development. We don't take a specific stance of which of these costs is more important and we model them instead in a reduced-form but comprehensive way. We then analyze how these costs affect the optimal policy mix.

We start with deriving a first best result with complete separation under the extreme assumption that other tools have no costs. In this case, policymakers can implement the first best allocation by addressing the coordination failures among depositors or the financial distortions implied by binding balance sheet constraints. They do so by using non-interest-rate tools while interest rate policy can ignore its impact on financial distortions and focus on inflation and output stabilization. This result is a version of the Tinbergen principle according to which each instrument should specialize in their area of relative effectiveness.

In the more general case in which other tools are costly, however, the optimal monetary policy should respond less aggressively to inflation. This result holds across both sources of financial fragility. The rationale is that higher interest rates depress asset prices and returns, eroding intermediaries' net worth. When the leverage constraint binds, we derive a simple formula showing that the optimal policy rate decreases with both the *level of spreads and their sensitivity to interest rate* changes—reflecting the banks' diminished intermediation capacity. Similarly, when there is a risk of a run, the optimal interest rate declines with both the *level of run risk and its sensitivity to interest rate changes*. In this case, tightening exacerbates coordination failures among depositors by reducing the value of bank assets and increasing insolvency risk.

The prescriptions for interest rate policy are thus qualitatively robust to the source of financial instability—the leverage constraint and the coordination failures. But there are also important differences. A distinctive feature of systemic bank runs is that they can trigger severe financial disruptions and very large costs. Our closed-form solution shows that the optimal interest rate must incorporate run risk, even when this probability is unaffected by marginal changes in the rate. By contrast, in the case of binding collateral constraints, the optimal interest rate must incorporate financial distortions only when interest rate spreads respond to changes in the policy rate. In other words, the existence of a probable high-cost state of the world calls for interest rate policy to adopt a risk-management approach.

Non-interest policy tools should be used to alleviate these trade-offs and better separate financial stability from price and output stability objectives. They should be deployed to the extent they are effective at reducing financial distortions and their costs remain limited. Asset purchases can improve the overall scale of intermediation and boost asset prices which increases banks' net worth. This relaxes the collateral constraint of intermediaries which increases their ability to intermediate assets and further raises asset prices. Maybe more surprisingly, asset purchases also decrease the likelihood of runs by raising the banks' liquidation value. Equity injections have a more direct impact on banks' intermediation capacity and also help with run risk by changing the banks' liability structure. Macroprudential limits should be released when the key distortion comes from the intermediaries' collateral constraint but doing so wouldn't help mitigate the risk of runs. By contrast, deposit insurance and lender of last resort facilities are important instruments to address risks of bank run but they can't address the other distortion.

We then use a calibrated version of the model to illustrate the optimal mix of policies as a function of the economy's primitives. We derive how the optimal mix of policy tools depends on the vulnerability of the financial system—the initial level of banks' net worth—and the cost of other tools. The more fragile the financial system is ex ante, and the larger the cost of other tools, the more interest rate policy should internalize its effect on financial instability by deviating from its conventional stance and accept a deviation of inflation from its medium-term target. Taken together, these findings suggest that when other tools are costly, the perfect specialization of instruments—or the Tinbergen principle—doesn't apply and optimal monetary tightening should take into account its effect on financial distortions, including limited intermediation capacity and coordination failures among investors.

Related literature. This paper contributes to a long-standing literature analyzing macroeconomic fluctuations in an environment with financial frictions and their implications for the optimal design of policies (Bernanke and Gertler, 1989; Bernanke et al., 1999; Kiyotaki and Moore, 1997; Gertler and Kiyotaki, 2010; Cúrdia and Woodford, 2011; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Boissay et al., 2016; Cúrdia and Woodford, 2016; Collard et al., 2017; Drechsler et al., 2018; Di Tella and Kurlat, 2021). The structure of the model most closely relates to Gertler and Kiyotaki (2015) and Gertler et al. (2020).[2] Like Cúrdia and Woodford (2010) we find that interest

[2]We also build on the literature looking at the interest rate exposure of banks and the transmission of monetary policy. This literature emphasizes the role of the cash-flow exposure of banks to interest rate risk (Gomez et al., 2021), of uninsured deposits (Drechsler et al., 2023), of profit margins and net worth (Abadi et al., 2023) and balance sheet and interest rate risk management (Di Tella and Kurlat, 2021).

rate policy should take into account credit spreads when the balance sheet constraint of intermediaries binds. Like Gertler and Karadi (2011) and Karadi and Nakov (2021) the use of additional tools such as credit policy can improve welfare, even outside of the ZLB. We build on Gertler et al. (2020) by analyzing the normative question of how monetary policy should optimally respond to inflation when intermediation constraint and run risk are endogenous to the policy rate, clarifying when and how interest rate and non-interest rate policy should internalize their impact on financial stability.

Our approach complements three recent papers by Adrian and Duarte (2020), Akinci et al. (2021), Caballero et al. (2025) and Boissay et al. (2022). The key novelty of our paper is to incorporate a microfounded risk of run into a NK model with endogenous asset prices, to focus on interest rate risk and the transmission of monetary policy through valuation effects, and to analytically characterize the optimal (Ramsey) combination of interest rate policy and other financial policies. While Akinci et al. (2021) characterize r**, the real interest rate consistent with financial stability, and Caballero et al. (2025) develop FCI*, the level of financial conditions that closes the output gap, we focus on the normative question of how monetary policy should respond to financial fragility. Relative to Adrian and Duarte (2020), who look at the implications of value-at-risk constraint for interest rate policy and to Boissay et al. (2022) which highlight how monetary policy affects the accumulation of capital, we consider instead an environment with more traditional collateral constraints, endogenous runs and we focus on how interest rate policy and other tools affect balance sheets through valuation effects.

Relative to the lean-against-the-wind literature, we highlight new trade-offs between price and financial stability. This literature is concerned with the build-up of financial imbalances in times of low inflation and low interest rate and focuses on the risk-taking channel of monetary policy, the development of bubbles and the search for yield (Svensson, 2014; Gerdrup et al., 2017; Ajello et al., 2019; Bauer and Granziera, 2017; Abbate and Thaler, 2023). We are instead motivated by financial instability triggered or exacerbated by interest rate tightening. In this situation, the relevant frictions and mechanisms are different: asset price drops, leverage constraints bind and run risks rise. We argue that when non-interest-rate tools are costly, the optimal policy is to implement less aggressive rate hikes. When they are not costly, full separation of objectives is implementable and optimal.[3]

Recent empirical work has also shed light on the impact of monetary policy on macro-level financial risk (Schularick et al., 2021; Jiménez et al., 2022; Greenwood et al., 2022; Boissay et al., 2023; Grimm et al., 2023).

[3]The papers considering the coordination between monetary policy and other tools usually abstract

We contribute to the literature on the optimal use of policies to address financial distortions stemming from coordination failures. The literature has looked at the optimal use of macroprudential tools and bank regulation, public liquidity provision, deposit insurance, and bank resolution in models with bank runs (Vives, 2014; Phelan, 2016; Tella, 2019; Dávila and Goldstein, 2023; Schiling, 2023; Ikeda, 2024; Kashyap et al., 2024; Porcellacchia and Sheedy, 2024).[4] We draw on this literature by considering a large set of financial policies. The novelty of our approach is to consider the optimal combination of these tools with monetary policy in a setting with nominal frictions. Our findings highlight the need for monetary policy to adopt a risk-management approach in the face of coordination failures.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the equilibrium path of the economy and show how interest rate tightening gives rise to price and financial trade-offs. Section 4 validates empirically the model's core mechanisms and Section 5 explains the calibration. Section 6 examines the optimal mix of interest rate policy and other tools. Section 7 concludes.

2 A Model with Sticky Prices, Constraints to Intermediation and Panics

In this section, we develop a two-period model with nominal rigidities. Motivated by their empirical importance, we introduce two key financial frictions: (i) a balance sheet constraint that limits bank intermediation capacity, and (ii) run-prone deposits. Our aim is to characterize optimal policies—combining interest rate policy and other tools—in each type of financial fragility.

2.1 Environment and Sequence of Events

There are 2 periods indexed by $t = 1, 2$ and the economy is populated by five agents: households, financial intermediaries, final good firms, intermediate good producers and the government which includes a central bank.

from their costs (Paoli and Paustian, 2017; Martinez-Miera and Repullo, 2019; Carrillo et al., 2021; Van der Ghote, 2021). In addition, they focus on macroprudential tools while we consider a broader set of other tools, such as credit policy, equity injection and deposit insurance, and analyze the implications of the risk of run, which is empirically important.

[4]These papers and ours build on a long-standing literature analyzing bank runs with a global game approach (Diamond and Dybvig, 1983; Carlsson and van Damme, 1993; Morris and Shin, 2003; Rochet and Vives, 2004; Goldstein and Pauzner, 2005; Bebchuk and Goldstein, 2011).

At the beginning of the first period, shocks are realized, then the government announces its policies. Knowing the shock and the policies, depositors decide to withdraw their deposits or not. After the outcome of the run game is realized, there is no more uncertainty. At this point, consumers decide how much to consume and work, and firms choose their level of production and prices. Subsequently households and banks decide their portfolio allocations. In the second period, firms produce, households supply labor and consume their earnings (Figure 1).

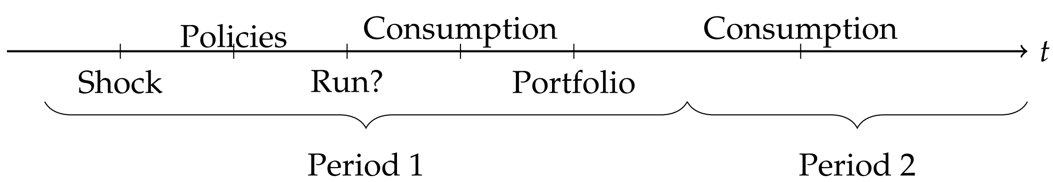

Figure 1: Timeline

In the first period, prices are sticky because firms face adjustment costs to changing their goods' prices and nominal wages are fixed. In the second, all prices, including real wages, can freely adjust. This captures the notion that prices are sticky in the short run, giving a role to monetary policy in stabilizing prices, but flexible in the medium run.

There are two financial assets: equities of firms and banks short-term deposits. Equities are long-run claims on the residual income of firms after they have paid workers. We abstract from capital accumulation and assume that there is an exogenous mass of capital K which is used for production both in period 1 and 2. In the remainder of the paper, we call this asset "capital". Capital turns into final goods and is consumed by their owners at the end of period 2. Deposits are endogenous to the financial system's and households' decisions.[5]

2.2 Households: Consumption and Portfolio Decisions

There is a continuum of identical households indexed by $j \in [0, 1]$. For clarity, we omit the j when no confusion results. They have the following preferences over the stream of consumption and labor supply:

$$\max_{c_1, c_2, \ell_2} \Big\{ \log(c_1) - v_1(\ell_1) + \beta \left(\log c_2 - v_2(\ell_2) \right) \Big\}. \tag{1}$$

[5] In a robustness section, we consider an extension with long-term government bonds, whose supply is exogenous and controlled by the government.

with $v_t(.)$ is an increasing function for $t = 1, 2$.

Households enter period 1 with a portfolio of capital k_{H1} and deposits d_1. From these investments they collect returns $\frac{Q_{K1}+r_{K1}}{Q_{K0}}$ and R_1 respectively, where r_{k1}, Q_{K1} are the endogenous dividends per unit capital and the price of capital in period 1. Returns on assets depend on the equilibrium, including on the outcome of the run.

At the beginning of the period, the run game occurs. We define the households strategies and the equilibrium of the game later in section 2.5. The outcome of the run determines the return on deposits R_1. If no run occurs or doesn't lead to the intermediaries' liquidation, returns on deposits are simply the promised rate at the end of the previous period, $\bar{R}_1$. If it is successful and banks have to be liquidated, the return on deposits R_1 is denoted R_1^*. From the perspective of households, this is a random variable, whose realized value depends on how many depositors withdraw their deposits and on the asset value of the bank relative to its liabilities in the run equilibrium. We define R_1^* formally in the above-mentioned section 2.5. Finally, households collect income from their supply of labor, $W_1\ell_1$, and receive net-of-tax lump-sum transfers from the government, T_1.

After the outcome of the run is realized and income is collected, households choose how much deposits d_2 to hold at banks, how much capital to hold k_{H2} and how much to consume c_1. Accordingly, their budget constraint in period 1 is

$$P_1 c_1 + d_2 + Q_{K1} k_{H2} = R_1 d_1 + (Q_{K1} + r_{k1}) k_{H1} + T_1 + W_1 \ell_1 \tag{2}$$

In period 2, households collect income from their portfolio of capital k_{H2} and deposits d_2, as well as labor earnings $W_2\ell_2$, and transfers T_2. Importantly, we follow GKP in assuming that households are less efficient at holding long-term assets than intermediaries:[6] the returns on their direct holdings are decreasing with the amount they hold in period 2.[7] For tractability we assume these costs are quadratic and given by $\left(\frac{\beta_K}{2}\frac{P_2 k_{H2}^2}{K}\right)$. In period 2, the budget constraint is thus given by

$$P_2 c_2 = R_2 d_2 + \left(1 + r_{k2} - \frac{\beta_K}{2}\frac{P_2 k_{H2}}{K}\right) k_{H2} + T_2 + W_2 \ell_2 \tag{3}$$

Consistent with the assumption that wages are fixed in nominal terms and prices

[6]This cost also rationalizes why intermediaries exist: they have a more efficient intermediation technology.

[7]Given that portfolio holdings from period 0 to period 1 are exogenous, we abstract from these costs in the first period.

are sticky in the first period, households supply labor perfectly elastically to firms. In the second period, we assume that $v(\ell_2) = 0$ for $\ell_2 < \bar{\ell}$ and $v(\bar{\ell}) = +\infty$, which implies that they supply inelastically $\bar{\ell}_2$. Households are price-takers in all markets: they take the path of wages W_1, W_2, final goods prices P_1, P_2, asset prices Q_{K1}, Q_{K2}, dividends per unit capital r_{k1}, r_{k2} and of the interest rate on deposits R_1, R_2 as given.

Optimality conditions. Taking the first-order conditions for c_1, c_2, d_2 and k_{H2}, the household' optimality conditions are given by

$$1 = \beta \frac{R_2}{1+\pi_2} \frac{c_1}{c_2}, \quad \text{and} \quad R_2 = \frac{1 + r_{K2} - \beta_K \frac{P_2 k_{H2}}{K}}{Q_{K1}} \tag{4}$$

where $\pi_2 \equiv P_2/P_1 - 1$. The first condition is the traditional Euler equation governing the allocation of consumption between period 1 and 2 and the second is the no-arbitrage conditions between deposits and long-term assets. There is no optimal labor supply condition in period 1, consistent with our assumption of fixed wages and elastic labor supply. In period 2, labor is simply $\ell_2 = \bar{\ell}$.

2.3 Final Good Firms

Final good firms buy intermediate goods to produce final goods which they sell to households. They are competitive and take the price of the final good P_1, P_2 and of intermediates $\{P_{1i}, P_{2i}\}_i$ as given. The technology to produce the final good has constant elasticity of substitution, ϵ. They seek to maximize profits subject to the technological constraint. Their problem is given by

$$\max_{\{Y_{ti}\}_i} P_t Y_t - \sum_i P_{ti} Y_{ti} \quad \text{subject to} \quad Y_t = \left(\int_i Y_{ti}^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Optimality conditions. The solution to their problem is given by

$$Y_{ti} = \left(\frac{P_{ti}}{P_t} \right)^{-\epsilon} Y_t. \tag{5}$$

In equilibrium, free entry ensures that final goods producers earn zero profits. Final output Y_t is determined by the goods market clearing condition.

2.4 Intermediate Good Firms

Intermediate goods are differentiated and produced by firms which are in monopolistic competition. These firms combine labor and capital and sell their variety to final goods firms. Following Rotemberg (1984), they face quadratic adjustment costs when choosing their price in the first period $\theta_1 > 0$, but not in the second $\theta_2 = 0$. To simplify the analysis, physical capital K cannot be accumulated and is firm-specific, so that it cannot be moved across firms. Taking wages in both periods (W_1, W_2) as given, their problem is given by:

$$\max_{Y_{ti}, P_{ti}, \ell_{ti}} P_{ti} Y_{ti} - W_t \ell_{ti} - \frac{\theta_t}{2} \left(\frac{P_{ti}}{P_{t-1i}} - 1 \right)^2 P_t Y_t \tag{6}$$

$$\text{subject to} \quad Y_{ti} = \ell_{ti}^{\alpha} K_{ti}^{1-\alpha} \quad \text{and} \quad Y_{ti} = \left(\frac{P_{ti}}{P_t} \right)^{-\epsilon} Y_t \tag{7}$$

Optimal pricing decisions. Their optimal pricing decisions are given by

$$(\epsilon_1 - 1) \left(\frac{\epsilon_1}{\epsilon_1 - 1} \frac{MC_{1i}}{P_{1i}} - 1 \right) = \theta_1 \pi_1 (\pi_1 + 1) \quad \text{and} \quad P_{2i} = \frac{\epsilon_2}{\epsilon_2 - 1} MC_{2i} \tag{8}$$

$$\text{with} \quad MC_{ti} = \frac{W_t}{\alpha} \left(\frac{Y_{ti}}{K_{ti}} \right)^{\frac{1-\alpha}{\alpha}} \tag{9}$$

Labor earnings and capital returns. We assume that two types of lump-sum transfers are enforced by the government: (i) the cost of price changes $\frac{\theta_t}{2} \left(\frac{P_{ti}}{P_{t-1i}} - 1 \right)^2 P_t Y_t$ are transferred to the government; (ii) each worker receive $\frac{P_{ti} - MC_{ti}}{MC_{ti}} W_t$ per unit of labor from their employer, which means that their post-transfer labor earnings are given by $W_t \ell_t = \alpha P_{ti} Y_{ti}$.

These transfers ensure that the returns on capital are a constant fraction of aggregate output. As a result, each period firms distribute to their shareholders—a mix of intermediaries and households—the dividends which are equal to $(1 - \alpha) P_{ti} Y_{ti}$. The dividends per unit equity r_{kt} and the total ex post return R_{kt} are given by

$$r_{kt} = (1 - \alpha) \frac{P_{ti} Y_{ti}}{K_{ti}} \quad \text{and} \quad R_{kt} = \frac{(1 - \alpha) \frac{P_{ti} Y_{ti}}{K_{ti}} + Q_{Kt+1}}{Q_{Kt}} \tag{10}$$

2.5 Financial Intermediaries and the Risk of Run

Financial intermediaries enter period 1 with a portfolio of claims on firms K_{F1} and they owe deposits D_1 to households. After the shocks are realized and policies are announced, each household enters the period with a portfolio of assets, (k_{1j}, d_{1j}), and decides whether to withdraw or to keep their deposits. This section briefly explains the run game. Appendix B provides more details.

Information structure and posterior beliefs. Depositors know their own individual holdings of deposits, d_{1j}, and the overall size of the banks balance sheet, but they don't perfectly observe the composition of their liability, between deposits D_1 and equity N_0.[8] They have two pieces of information to form beliefs about N_0. They know that it is drawn from a log-normal distribution around the end-of-period 0 net worth $\bar{N}_0$ with dispersion σ_N: $\log N_0 \sim \mathcal{N}(\log \bar{N}_0, \sigma_N)$. They also receive a private signal η_j, centered around N_0 with dispersion σ_η: $\log \eta \sim \mathcal{N}(\log N_0, \sigma_\eta)$. For future reference, we denote $F(\eta|N_0)$ the CDF of this distribution. The posterior belief of household j about N_0 is thus also log-normally distributed:

$$\log N_0 \sim_j \mathcal{N}\left(\mu_{N_0}(\eta_j, \bar{N}_0), \sigma^2_{NP}\right) \tag{11}$$

where μ_{N_0} is an average of η_j and $\bar{N}_0$ weighted by the signal's precision $\sigma_\eta^{-1}, \sigma_N^{-1}$. We denote the density of this posterior distribution $p(n|\eta_j, \bar{N}_0)$.

Condition for a successful run. If a sufficiently large fraction of depositors decide to withdraw, banks aren't able to repay them all and have to be liquidated. Denoting δ the share of depositors who run in equilibrium, a run is successful if:

$$R^*_{k1} Q_{K0}(K - K_{H1}) < \bar{R}_1 D_1 \delta \tag{12}$$

where the asterisk * denotes variables in the "run" equilibrium—the equilibrium in which banks have to be liquidated. In such an equilibrium, all assets have to be held by households or the government, i.e. $K^*_{F2} = B^*_{F2} = 0$, which leads to a drop in their prices

[8] While the traditional approach in the global game literature is to model uncertainty around the expected return on the bank's portfolio (Morris and Shin, 1998; Goldstein and Pauzner, 2005), uncertainty in this paper is around the initial level of bank equity. This is because returns are endogenous to the macroeconomic equilibrium. Both objects shape the depositors' payoffs if a run happens.

(which justifies the run ex post).[9] Asset prices and returns on assets are given by

$$R^*_{k1} = \frac{r^*_{K1} + Q^*_{K1}}{Q_{K0}} \qquad \text{and} \qquad Q^*_{K1} = \frac{1 + r_{K2} - \beta_K \frac{P_2(K-K_G)}{K}}{R_2} \tag{13}$$

where we used the households optimality conditions (4) to pin down asset prices. Using the previous equation (12), a run is successful if and only if the share of depositors running exceeds a threshold: $\delta > \bar{\delta}(N_0)$ with $\bar{\delta}(N_0) = \frac{R^*_{k1} Q_{K0}(K-K_{H1})}{\bar{R}_1[Q_{K0}(K-K_{H1})-N_0]}$.

Payoffs and equilibrium. If a depositor runs and the run is successful, it gets a share of the bank liquidation value proportional to its deposits d_{1j}. If they don't run, they lose all their deposits.[10] A depositor who runs always incurs a small exogenous utility cost of running ζ. Without this utility cost, running would always be a dominant strategy. Denoting the after-run indirect utility of a depositor with initial deposits d_{11}, $U(R_1 d_{1j})$ (and omitting the dependence on k_{H1j} for simplicity)[11], Table 1 summarizes the payoffs for each action and in each equilibrium outcome.

	In equilibrium the run is ...	
Individual action	Successful	Unsuccessful
Run	$U\left(\frac{R^*_{k1} Q_{K0}(K-K_{H1})}{\bar{R}_1[Q_{K0}(K-K_{H1})-N_0]\delta} d_{1j}\right) - \zeta$	$U(\bar{R}_1 d_{1j}) - \zeta$
Don't run	$U(0)$	$U(\bar{R}_1 d_{1j})$

Table 1: Payoffs in Four Cases

In equilibrium, depositors adopt a trigger strategy: they run if their private signal is below a threshold $\bar{\eta}$. The equilibrium share of depositors running is the share of those receiving a signal below this threshold, $\delta^*(N_0) = F(\bar{\eta}|N_0)$. Importantly, the threshold $\bar{\eta}$ is endogenous and should be such that a depositor with signal $\bar{\eta}$ is indifferent between running and not running:

[9] In GKP banks' equity recovers slowly as new banks enter, we make the simple assumption that no bank enters in period 2.

[10] The setup assumes that deposits are uninsured. When considering optimal policy, we allow for the possibility that governments provide deposit insurance to prevent runs.

[11] R_2 enters into depositors' payoffs only through R^*_{K1}. Note that, if the run is unsuccessful, the return on deposits is $\bar{R}1$ because these are deposits held by banks between period 0 and 1.

$$\int_0^{\max(\bar{N},0)} \left[U\left(\frac{R_{k1}^* Q_{K0}(K - K_{H1})}{\bar{R}_1 \left[Q_{K0}(K - K_{H1}) - n \right] F(\bar{\eta}|n)} D_1 \right) - U(0) \right] p(n|\bar{\eta}) dn = \zeta \tag{14}$$

where $\bar{N}$ is the level of net worth above which the run is unsuccessful. It is the solution to the following equation $F(\bar{\eta}|\bar{N}) = \frac{R_{k1}^* Q_{K0}(K - K_{H1})}{\bar{R}_1[Q_{K0}(K-K_{H1})-\bar{N}]}$. The left-hand side of this equation is decreasing in $\bar{N}$ while the right-hand-side is increasing in it.

2.6 Financial Intermediaries if No Run Happened

If a run doesn't happen or if the run isn't successful, financial intermediaries continue to operate from period 1 to period 2. Their period-1 equity depends on the returns on assets and payments on liabilities $R_1 = \bar{R}_1$ made at time 1:

$$N_1 = \bar{R}_1 N_0 + (R_{k1} - \bar{R}_1) Q_{K0} K_{F1} + N_G \tag{15}$$

where $\bar{R}_1, K_{F1}, Q_{K0}$ are all exogenous in period 1 and N_G denotes equity injection by the government. Returns on long-term assets R_{k1} are endogenous.

Unconstrained portfolio allocation and no-arbitrage. At the end of period 1, financial intermediaries collect households deposits, D_2, and invest in capital, K_{F2}. Taking all asset prices and returns as given, they seek to maximize the end of period-2 N_2. When intermediaries can freely allocate their portfolio they would arbitrage away any differences in returns:

$$R_2 = R_{K2} \tag{16}$$

Incentives-compatible balance sheet constraint. Following the financial accelerator literature, we assume that due to an agency problem the intermediaries' overall assets cannot exceed a multiplier of their equity. Denoting the maximum leverage ϕ^P, the constraint is given by

$$\phi^P N_1 \geq Q_{K1} K_{F2}. \tag{17}$$

We assume that $\phi^P > 0$ is an exogenous parameter, like for example in Di Tella and Kurlat (2021). One could endogenize ϕ^P, in which case the maximum leverage ϕ^P

would increase with the spread R_{k2}/R_2 which would partly mitigate the amplification stemming from the constraint.[12] Given that this second round mechanism doesn't qualitatively affect our results, but would substantially complicate the analysis, we keep ϕ^P exogenous in the rest of the paper.

When this balance sheet constraint binds, returns on capital are determined by the households conditions (4) and rise above the interest rate on deposits, $R_{K2} > R_2$. This is because households are the marginal buyer and they require a compensation for holdings these assets.

Macroprudential policy. Consistent with the development of macroprudential tools since the Great Financial Crisis, the government in the model can set an equity-based balance sheet constraint, ϕ^G which takes the exact same form as the incentive-based constraint (17). The effective leverage constraint faced by intermediaries is given by $\phi = \min(\phi^P, \phi^G)$. We deliberately abstract from the design of macroprudential policy *ex ante* and from its role in limiting leverage build-ups over the financial cycle. Instead, the macroprudential regulation is given at the onset of the stress episode and may become binding in equilibrium. This approach allows us to study how policymakers should adjust macroprudential requirements *ex post* once financial stress has materialized.

2.7 Government and Central Bank and the Costs of Tools

The central bank controls the rate on deposits R_2. The government can issue short-term deposits D_{G2}, purchase equities K_G, inject equity into the banking system N_G, and sets transfers T_1, T_2. These choices have to be consistent with the following budget constraints:

$$D_{G2} = T_1 + \frac{\theta_t}{2}\pi_1^2 P_t Y_t + Q_{K1} K_G + N_G \tag{18}$$

$$\left(1 + r_{k2} - \frac{\beta_G}{2}\frac{P_2 K_G}{K}\right) K_G - \frac{\beta_N P_2}{2} N_G^2 = T_2 + (1 + R_2) D_{G2} \tag{19}$$

where T_1, T_2 may be negative or positive and where we have assumed that all firms were symmetric.

Importantly, we assume that the use of tools such as equity injections, credit policy,

[12] Following Gertler and Karadi (2011), and consistent with our assumption that bankers go back to being households in period 2, ϕ^P would be given by: $\phi^P = \frac{\eta}{\lambda - \nu}$ with $\eta = 1$ and $\nu = \frac{R_{k2}}{R_2} - 1$ where λ is the fraction of funds the banker could divert (see equations 11 and 13 in their paper).

and deposit insurance, are costly. We model these costs in a reduced-form way as quadratic pecuniary losses in the government's budget constraint. These costs capture several implementation challenges and negative implications associated with the use of these tools. First, these tools can have fiscal costs. For example, purchase of risky assets exposes the central bank to financial losses. Deposit insurance by governments may trigger a large unexpected spending in case of a run (Allen et al., 2011). These fiscal costs entail real costs when taxation is distortionary. Second, generous central bank intervention, public equity injection and deposit insurance can introduce moral hazard and incentivize risk-taking (Cooper and Ross, 2002). Additional costs have been highlighted in the literature. For example, large scale credit policy could lead to mispricing of risk premia and can become "addictive" (Steeley, 2015; Karadi and Nakov, 2021). We define the joint objective of the government and the central bank in section 6.

2.8 Market Clearing

The labor and goods market clear. Given that price adjustments consume real resources in the first period, the market clearing for final goods in the first period is given by

$$Y_1 \left(1 - \frac{\theta}{2}\pi_1^2\right) = C_1 \tag{20}$$

where $C_1 = \int c_{1j} dj$ where each individual household is indexed by j. Given that real wages are flexible in the second period, the level of the price level is indeterminate. We thus normalize $P_2 = P_1$ and let W_2 adjust so that the real wage clears the labor market. This assumption allows us to abstract from inflation in goods prices from period 1 to period 2 and focus on inflation from period 0 to period 1 only, $\pi_2 = 0$.

In addition, the capital and short-term deposits markets also clear:

$$K = K_{H2} + K_{F2} + K_G \quad \text{and} \quad D_2 = D_{F2} + D_{G2} \tag{21}$$

where $D_2 = \int d_{2j} dj$ and $K_{H2} = \int k_{H2j} dj$ and j indexes an individual household.

3 Price, Output and Financial Stability Trade-offs

We characterize the equilibrium of the economy with three equations. The Phillips Curve ($\mathcal{PC}$) and the Euler Equation ($\mathcal{EE}$) always apply and the third equation depends on the financial friction: the Balance Sheet Constraint ($\mathcal{BSC}$) when banks' equity is the constraint; the Run Equation ($\mathcal{RE}$) under run risk. These frictions introduce wedges that distort the allocation. Interest rate tightening, by worsening the balance sheet constraint or by raising run risk, generates two distinct price–financial stability trade-offs in addition to the conventional output–inflation trade-off.

3.1 The Trade-off Between Output and Inflation

We derive the Phillips curve from combining the intermediary firm's optimal pricing condition (8), the optimality condition of final goods firms (5), the symmetry of intermediary firms and the production function of final goods firms $Y_{i1} = Y_1$, the definition of marginal cost (9) and the assumption that wages are fixed in period 1:

$$(\epsilon_1 - 1)\left(\frac{\epsilon_1}{\epsilon_1 - 1}\frac{\underline{W}}{(1+\pi_1)P_0\alpha}\left(\frac{Y_1}{K}\right)^{\frac{1-\alpha}{\alpha}} - 1\right) = \theta_1\pi_1(\pi_1 + 1) \quad (\mathcal{PC}) \tag{22}$$

The Phillips curve relates the level of inflation π_1 to the level of output Y_1 in period 1. The second important equation is the Euler equation, the optimality condition governing the intertemporal allocation of consumption of households which is given by (4):

$$Y_1\left(1 - \frac{\theta}{2}\pi_1^2\right) = \frac{C_2}{\beta R_2}. \quad (\mathcal{EE}) \tag{23}$$

where we used the goods market clearing condition in period 1 (20).

The Phillips curve and the Euler equation determine inflation π_1 and output Y_1 in period 1 as a function of C_2 and the policy rate R_2. The other variables related to production in period 1 are directly implied by the equilibrium level of Y_1 and π_1: consumption C_1 is closely related to output Y_1 through the market clearing condition, and labor supply adjusts to accommodate the needs of firms: $\ell_1 = \left(\frac{Y_1}{K^{1-\alpha}}\right)^{\frac{1}{\alpha}}$.

Trade-offs between price and output stabilization. The Phillips Curve $(\mathcal{PC})$, and the Euler Equation $(\mathcal{EE})$ are sufficient to illustrate the well-known static trade-off between output and inflation central banks face when setting the interest rate R_2. From these two equations, it is clear that the social planner can decrease inflation by increasing R_2, but at the cost of a lower consumption C_1 and output Y_1. We will come back to this trade-off in the calibrated model in the next subsection.

3.2 The Trade-off Between Inflation and Intermediation Capacity

We now analyze how the balance sheet constraint shapes the allocation and how policy rate increases can negatively affect the intermediation capacity of banks. We abstract from the risk of run, corresponding to an infinite cost of running $\zeta \to +\infty$. We show that this trade-off between inflation and intermediation capacity has both an extensive and an intensive margin. High interest rates can cause the balance sheet constraint to bind and a wedge to open up between the deposit rate and returns on assets. Once the economy is in this "constrained" zone additional rate hikes further decrease intermediation capacity and widen the wedge. In this zone, the model is simply described by a Phillips curve, an Euler equation and a balance sheet constraint.

Costs of Limited Intermediation and Returns Spread. The costs of limited intermediation appears in the resource constraint in the second period:

$$C_2 = \bar{\ell}^{\alpha} K^{1-\alpha} + K - \left(\frac{\beta_K}{2} \frac{K_{H2}}{K} \right) K_{H2}$$

where we used the labor supply ℓ and production technology $Y_2 = \bar{\ell}^{\alpha} K^{1-\alpha}$ and assumed no government interventions.[13] These costs are strictly increasing in K_{H2}. These costs, which lower welfare, arise when the balance sheet constraint of intermediaries binds in the first period and households hold part of the capital stock.

When the balance sheet constraint of intermediaries doesn't bind, intermediaries hold the entire stock of assets and they arbitrage away any spread between the policy rate and the rate of returns on assets $R_{K2} = R_2$. In that case financial variables are irrelevant to the real allocation and welfare. When the balance sheet constraint binds, households holds part of the capital stock and a wedge opens up between the deposit rate and the returns on asset, $R_{k2}(1-\sigma) \equiv R_2$. Using the first order condition of households (4), the wedge σ is strictly increasing in K_{H2}:

$$\sigma \equiv 1 - \frac{R_2}{R_{k2}} = \beta_K \frac{P_2 K_{H2}}{K(1 + r_{K2})}.$$

This spread is the wedge capturing the distortions implied by the balance sheet constraint which policymakers would like to close.

[13] For future reference, we can also determine W_2 from the assumption of flexible prices and wages in the second period and the normalization $P_1 = P_2$, $W_2 = \left(\frac{\epsilon_2}{\epsilon_2 - 1} \frac{1}{\alpha P_1} \left(\frac{\bar{\ell}^{\alpha} K^{1-\alpha}}{K} \right)^{\frac{1-\alpha}{\alpha}} \right)^{-1}$

Trade-off (extensive margin). A first illustration of the trade-off faced by monetary policy when hiking the interest rate is that the balance sheet constraint is more likely to bind. This is because the drop in the asset value depletes intermediaries' net worth. To see this, recall that a condition for the constraint not to bind is that intermediaries are able to hold all assets in the economy, namely

$$\bar{R}_1 N_0 + (R_{k1} - \bar{R}_1) Q_{K0}(K - K_{H1}) + N_G > \frac{Q_{K1} K}{\phi} \tag{24}$$
$$\text{with} \quad Q_{K1} = \frac{1 + r_{K2}}{R_2}, \quad R_{k1} = \frac{r_{K1} + Q_{K1}}{Q_{K0}}.$$

From equation (24), we see that the drop in asset prices caused by rate hikes leads the right-hand side to decrease faster than the left-hand side up to the point where the constraint binds with equality. The following lemma formalizes the idea that the constraint is more likely to bind whenever, for a given level of N_0, the policy rate R_2 is high enough. Figure 2 illustrates the split of the state space $(-N_0, R_2)$ between the "constrained" zone in red and the "financially stable" zone in blue.

Lemma 1. *Under regularity conditions, there exists a strictly increasing and continuous function $\bar{R}_2(N_0)$ for $N_0 \geq 0$ such that (24) holds if and only if $R_2 < \bar{R}_2(N_0)$.*

Proof. See Appendix C. □

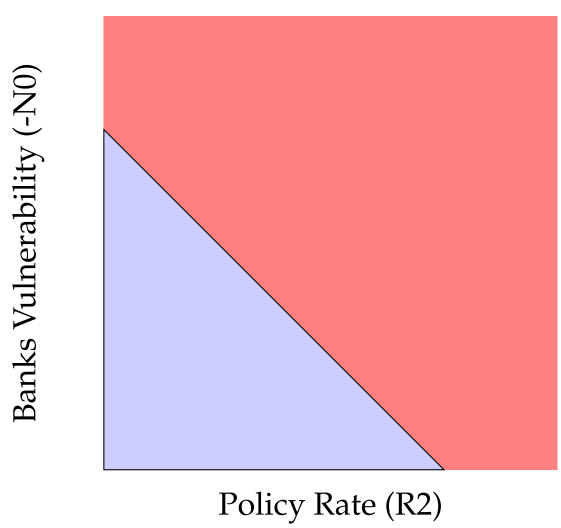

Figure 2: Constrained (red) vs. unconstrained (blue) zone

The level of interest rate $\bar{R}_2(N_0)$ is tightly connected to the concept of r^{**} in Akinci et al. (2021). In our model, it is a nominal interest rate, while it is a real interest rate in theirs, as they abstract from nominal frictions. Like r^{**}, the level of interest rate $\bar{R}_2$ is a function of the state variables and the vulnerability of the financial system, N_0.

The regularity conditions are as follows: households' holdings are not too large relative to the leverage ratio $\frac{\phi-1}{\phi} > \frac{K_{H1}}{K}$, households hold positive deposits $D_1 > 0$ and the Phillips curve is upward sloping (inflation increases with output at period 1) and not too steep. The assumptions of the lemma are in general true. For example, if households don't hold any asset from time 0 to time 1, $K_{H1} = 0$ it simply says that banks are allowed to have a positive leverage, $\phi > 1$. The second assumption simply says that banks enter period 1 with some positive leverage.

Trade-off (intensive margin). Besides this extensive margin of the trade-off implied by high policy rate, additional increases in the interest rate further lowers intermediation capacity. To illustrate this intensive margin of the trade-off between preserving intermediation capacity and inflation faced by interest rate policy R_2, we derive the balance sheet constraint, which together with the Euler equation and the Phillips curve, characterize the equilibrium path of the economy in the constrained zone. Using the optimality portfolio conditions of households (4) and the definition of returns in period 1 (10) to substitute for the equilibrium asset prices and returns in the balance sheet constraint (17), we obtain:

$$\bar{R}_1 N_0 - \bar{R}_1 Q_{K0}(K - K_{H1}) + N_G + r_{K1}(\pi_1, C_1)(K - K_{H1}) + \frac{1 + r_{K2} - \beta_K \frac{P_2 K_{H2}}{K}}{R_2}\left(K - K_{H1} - \frac{K - K_{H2} - K_G}{\phi}\right) = 0 \quad (\mathcal{BSC})$$

This $\mathcal{BSC}$ equation pins down the equilibrium portfolio holdings of households K_{H2} as a function of Y_1, π_1, and policy interventions, including the policy rate R_2. For future reference, we thus denote this function $\mathcal{BSC}(K_{H2}, K_G, R_2, N_0, N_G)$.

Figure 3 illustrates the equilibrium outcomes as a function of the interest rate policy R_2.[14] When interest rates are sufficiently low, the leverage constraint BSC is slack. In this region, raising interest rates dampens inflation π_1 but it also leads to lower output $Y1$. Higher interest rates also lead to lower price of capital Q_K. While this reduces the net worth of intermediaries, the leverage constraint remains slack and intermediaries remain the marginal buyer of capital. This keeps interest rate spreads at zero.

When interest rates are sufficiently high, the balance sheet constraint binds. In this situation, intermediaries are unable to fully intermediate all assets in the economy and

[14]Section 5 presents the calibration of the model.

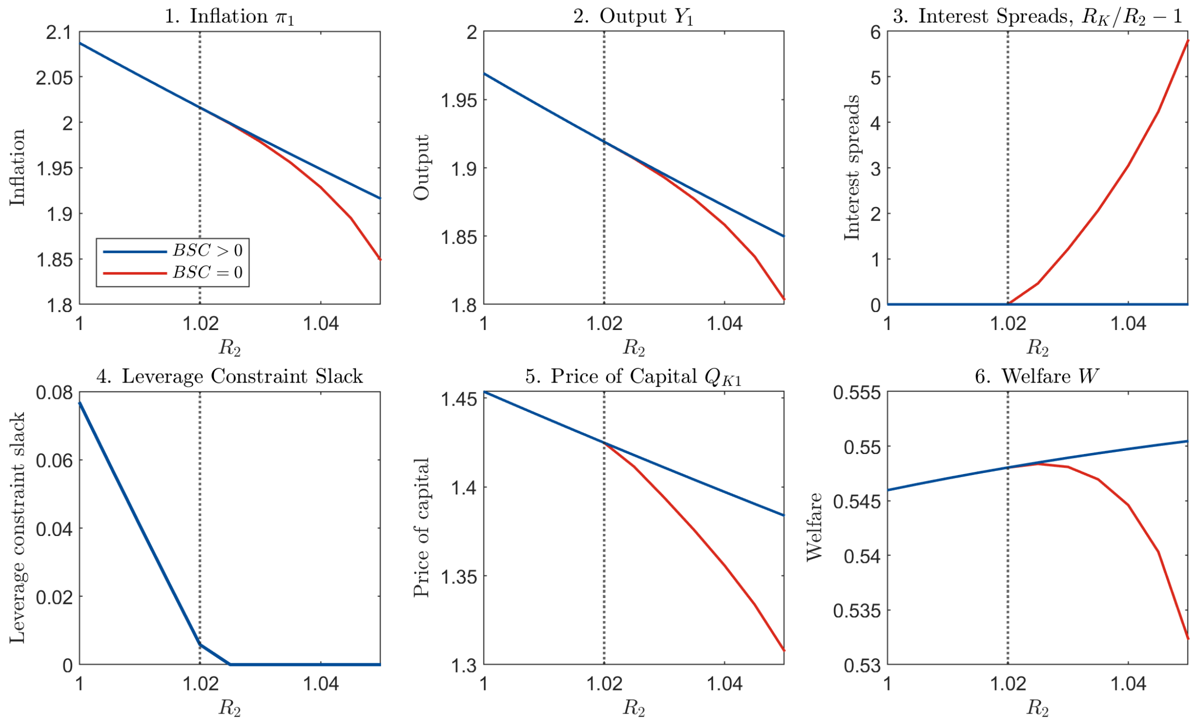

Figure 3: Equilibrium as a function of R_2. Blue line shows the equilibrium where the balance sheet constraint does not bind $BSC \equiv (\phi N_1 - Q_{B1}B_{F2} - Q_{K1}K_{F2}) > 0$ (or never binds counterfactually). The red line shows the equilibrium where the constraint binds $BSC = 0$. The vertical dashed line shows the interest rate above which the constraint binds.

households become marginal buyers. Since households need to pay an efficiency cost for holding assets, there is a positive spread between between the policy rate and the return on capital $R_K/R_2 - 1$. The positive spreads further depresses asset prices Q_{K1} and exacerbates the balance sheet constraint. Relative to a counterfactual situations where the balance sheet constraint does not bind, both inflation and output in period 1 are lower. Notably, this happens through an intertemporal effect as households save more today in anticipation of the efficiency losses associated with households' holding of capital in period 2.

3.3 The Trade-off Between Inflation and the Risk of a Run

We now analyze how higher interest rates affect run risk. We abstract from balance sheet constraints, corresponding to $\phi^P \to +\infty$. As with intermediation capacity, the trade-off operates on both an extensive and an intensive margin. On the extensive margin, sufficiently high rates push the economy into a "run zone," where the probability of a run—the wedge reflecting coordination failures—becomes positive. On the intensive margin, further rate hikes within this zone exacerbate coordination failures and raise run risk. This highlights that central banks face a risk-management problem.

The likelihood of a run. The likelihood of a successful run ξ is the second wedge related to financial frictions that policymakers would like to close. It is the probability that the share of depositors who run δ is above some threshold:

$$\xi = P\left(\delta \bar{R}_1 D_1 > R^*_{k1} Q_{K0}(K - K_{H1}) | \bar{N}_0\right) \tag{25}$$

where $D_1 = Q_{K0}(K - K_{H1}) - N_0$ is unknown since policymakers, like households, don't perfectly observe N_0. As a result, the probability is conditional on the information set policymakers have at the beginning of time 1, which is only that the true value is drawn from a log-normal distribution with mean $\bar{N}_0$.

The equilibrium share of depositors who run is equal to those who receive a signal lower than a threshold $\bar{\eta}$, namely $\delta = F(\bar{\eta}|N_0)$ (see section 2.5). This threshold is endogenous to the values of asset returns in the run equilibrium and to the interest rate, as shown in equation (14). The higher the interest rate the stronger the incentives to run, the lower the threshold, the higher δ, everything else equal.

The probability ξ, together with the equation determining $\bar{\eta}$ (14), is the third equation of the model when a run happens with positive probability. We call this (pair of) equation the Run Equation ($\mathcal{RE}$). It is easy to see that it is a function of the period-0 equity of banks N_0 and government policies, including the policy rate R_2. We denote this function $\xi(R_2, N_0, K_G, N_G)$. It is differentiable and increasing in R_2 and decreasing in N_0.

Trade-off (extensive margin). For any given level of banks equity, a run is more likely to occur with positive probability when the policy rate is high enough. To formalize this idea, recall from section 2.5 that if N_0 is above $\bar{N}$, the run is unsuccessful. An immediate corollary is that runs can occur with positive probability whenever intermediaries are less well capitalized or when the policy rate R_2 is high enough.

Lemma 2. *Under regularity conditions, there exists a strictly increasing and continuous function $\tilde{R}_2(N_0)$ for $N_0 \geq 0$ such that $\xi = 0$ if $R_2 < \tilde{R}_2(N_0)$.*

Proof. See Appendix C. □

The intuition is as follows: when interest rates increase, the asset value drops, which decreases banks equity relative to deposits. This makes it more likely that banks won't be able to repay all their depositors if it were to be liquidated. The split of the state space between "run" zone in red and "financial stability" zone in blue would be qualitatively similar to Figure 2. However, the location where the split occurs depends on model parametrization.

The regularity conditions for the lemma to hold are that households hold positive deposits $D_1 > 0$ and the Phillips curve is upward sloping (inflation increases with output in period 1) and not too steep. The proof of this result is simple. Under the regularity conditions R^*_{k1} is continuous and strictly decreasing in R_2. $\bar{N}$ is strictly decreasing and differentiable in R^*_{k1}. We can thus define a strictly increasing, differentiable function $\bar{N}(R_2)$. Given it is strictly increasing, we can invert it, and define $\tilde{R}_2(N_0)$.

Using the three equations summarizing the model—the Run Equation, the Euler Equation and the Phillips Curve—Figure 4 illustrates the equilibrium outcomes both inside and outside the run zone. When interest rates R_2 are sufficiently low, the economy lies outside the run zone—to the left of the vertical line. Here, intermediaries' net worth is sufficiently high so that there is no incentive for depositors to coordinate on a run. Inside the run zone—to the right of the vertical line—there is a positive probability that banks fail if a large enough share of depositors coordinate on withdrawing their deposits.

Trade-off (intensive margin). Once the economy enters the "run zone," interest rate policy faces a trade-off between stabilizing inflation in period 1 (conditional on no run) and raising the likelihood of a run. Figure 4 illustrates the two possible allocations: the blue line corresponds to the no-run state, and the red line to the run state.

In the no-run state (blue), higher interest rates help stabilize inflation. Yet this comes at the cost of depleting intermediaries' net worth, thereby increasing run probability. In the run state (red), a systemic run forces banks into liquidation. Households must absorb the entire capital stock, causing interest spreads $R_K/R_2 - 1$ to spike and leading to sharp declines in inflation π_1 and output Y_1. This underscores that central banks face a risk-management problem.

4 Empirical Validation

Building on recent evidence by Schularick et al. (2021), Boissay et al. (2023) and Grimm (2024), we now show that the model's core mechanisms are supported by empirical evidence based on historical global data.

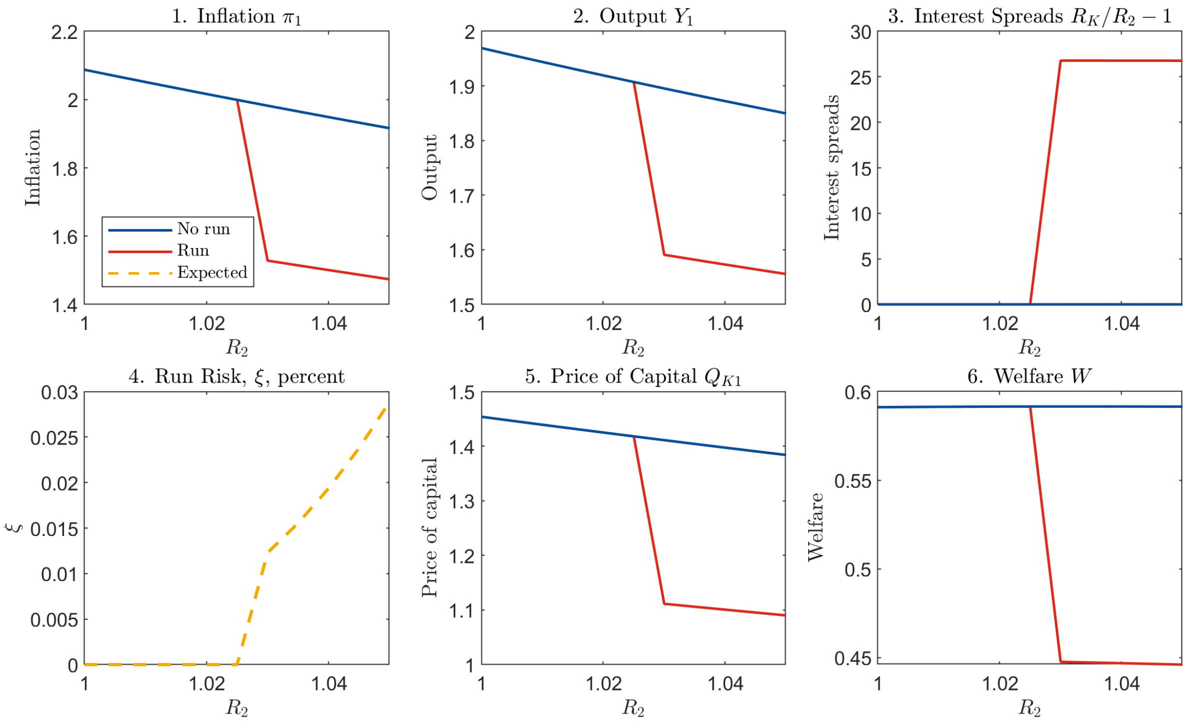

Figure 4: Equilibrium as a function of R_2. Blue line shows the equilibrium where there is no run risk, or a run does not occur. The red line shows the equilibrium where a run occurs. The dashed yellow line is the average of the two equilibria, weighted by the probability of a run. The vertical dashed line shows the interest rate above which there is positive run risk.

Data and identification. Our historical dataset uses the Jorda-Schularick-Taylor (JST) Macrohistory Database (Jordà et al., 2017) and Baron et al. (2021) which provide data on financial crises and monetary policy. The merged dataset spans over the period 1870–2016 and covers 18 advanced economies: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

Following recent approaches by Schularick et al. (2021) and Boissay et al. (2023), we provide causal estimates of the impact of interest rate tightening events on the likelihood of banking panics and bank equity crashes, asset prices and bank loans and on the differential effect of supply- and demand-driven inflation. Our identification strategy relies on a rich set of macro controls to address the issue of omitted variables and on instrumenting changes in the nominal rates with the Trilemma instrument. We provide further details in Appendix E.1.[15]

[15]We present both OLS and IV estimates in Appendix E.5. Across outcomes, the effect of rate hikes is systematically larger in magnitude under IV than under OLS, indicating attenuation bias due to the endogenous response of monetary policy to domestic economic conditions (see Jordà et al. (2020)).

Bank panics and bank equity crashes. As shown in Figure 8, we find that, consistent with the model, interest rate tightening leads to a significantly higher likelihood of bank panics and bank equity crashes. Bank equity crashes are defined as annual declines in bank equity prices exceeding 30 percent. The measure of banking panics builds on narrative sources to identify episodes of "severe and sudden withdrawal of funding by bank creditors from a significant part of the banking system," including both solvent and insolvent banks. Both measures come from Baron et al. (2021).

Asset prices and bank loans. Our second validation exercise shows that tightening events leads to lower asset prices and declines in bank intermediation activities—two key channels of transmission of monetary policy in the model. Asset prices are defined as real estate and equity prices and bank intermediation activities is defined as bank loans. Figure 9 shows that tightening leads to lower asset prices and bank assets, consistent with the model's key mechanisms.

Supply- vs demand-driven inflation. Finally, we look at whether monetary tightening has a stronger effect on financial stress in period of supply-driven inflation, as found in the U.S. for the recent decades by Boissay et al. (2023). To differentiate between inflation episodes driven by supply versus demand shocks, we follow the approach in Jump and Kohler (2022). Shocks are identified based on the signs of residuals in reduced-form regressions of real GDP growth and inflation. Further details on this methodology are provided in Appendix E.2. Consistent with the model, Figure 7 shows that the negative impact of tightening on financial stability are stronger in times of supply-driven inflation.[16]

5 Calibration

In this section, we explain how we calibrate the two-period model, extended to incorporate long-term government bonds.[17] The numerical simulations should be interpreted as a proof of concept for the framework, not as a definitive quantitative assessment.

Parameters. The subjective discount factor β, the labor share α, and the elasticity of substitution ϵ are set to the standard values in the literature. We choose the adjustment

[16] A notable difference relative to Boissay et al. (2023) is that we find that tightening worsens financial stability even in period of demand-driven inflation, while they find that it improves it.

[17] See Section 6.5 for a description of this extension.

cost parameter $\theta = 375$ following Capelle and Liu (2023). The resulting slope of the Phillips curve is 0.06, within the range of empirical estimates (Furlanetto and Lepetit, 2024). The long-term government coupon rate is set to $r_B = 0.04$. The leverage limit is $\phi = 5$, which is between the calibrated values in Gertler and Karadi (2011) and Gertler and Kiyotaki (2015) .

We set the managerial cost parameters β_K, β_L to 0.2. The managerial costs are low enough to ensure that households find it profitable to hold long-term assets when the leverage constraint binds or a bank run occurs, but high enough to induce an increase in interest spreads in those events, following Gertler and Kiyotaki (2015). We set the government's managerial costs β_G, β_N for credit policy and for equity injections smaller than those of households', but the results are qualitatively similar if they are set similar to those of households'. We investigate the role of government's managerial costs for optimal policy in the following section.

Parameters that affect the probability of a run, σ_n, σ_η, and ζ are set to target the elasticity of run probability to changes in interest rates (as documented empirically in Appendix E.5). In our current calibration, the model yields an elasticity of run probability that is smaller than those of the empirical estimates (0.02 percent versus empirical estimates ranging from 1.3 to 1.7 percent).

Table 2: Parameters

Variable	Description	Value
β	Household time preference	0.96
α	Labor share	0.6
ϵ	Markup	10
θ	Coefficient on price adjustment cost	375
r_B	Long-term government bond coupon	0.04
ϕ	Maximum leverage	5
β_K, β_L	Household managerial cost	0.2
β_G, β_N	Government managerial cost	0.05
χ	Disutility of labor	0.51
σ_n	Dispersion in prior beliefs about banks' net worth	0.75
σ_η	Dispersion in private signal about banks' net worth	0.75
ζ	Utility cost of running	2.15

Initial conditions. Labor supply in period 2, $\bar{l}$, is chosen so that the equilibrium real interest rate is 2%. The gross nominal interest rate in the first period, R_1, is set to 1.04 to target an initial inflation rate of 2%.

We normalize the capital stock K, the long-term government bond stock B, and the level of wages in period 1, $\overline{W}$, to 1. We assume banks hold all capital and long-term bond holdings in period 0, so $K_{F1} = K$ and $B_{F1} = B$. The remaining initial state variables (Q_{K0}, Q_{L0}, N_0) are chosen so that the leverage constraint does not bind in the baseline equilibrium.

Table 3: Initial Values

Variable	Description	Value
$\bar{l}$	Labor supply in period 2	0.6
R_1	Interest rate in period 0	1.04
K, B	Capital and long-term government bond stock	1
$\overline{W}$	Wage level in period 1	1
K_{F1}, B_{F1}	Bank capital and bond holdings (period 0)	1
P_0	Price of final output (period 0)	1.5096
Q_{K0}, Q_{L0}	Price of capital and bond in period 0	2.7
N_0	Net worth in period 0	2.3

6 Optimal Policies with Financial Fragility

Section 3 described the financial-frictions-implied wedges (σ, ξ) and the trade-offs faced by central banks. We now characterize the optimal combination of interest rate policy and other tools in times of financial fragility, and analyze how it depends on the costs of the tools (β_G, β_N) and the degree of financial vulnerability. We start with the case in which the leverage constraint binds and then turn to the case of the systemic run. We close the section with a set of robustness checks.

6.1 Policymakers' Objectives and Instruments

The baseline objective of policymakers is to maximize the expected households' welfare, which is a weighted sum of (log) consumption in the current and future period and inflation subject to the restriction that the allocation is a competitive equilibrium. Policymakers have three instruments $\{K_G, N_G, R_2\}$ which are chosen after the shock is realized, but before the run game happens.

Definition 1 (Optimal Policies). *A Ramsey-optimal allocation is a set of quantities* $\{Y_1, C_1, Y_2, C_2, \ell_1, \ell_2, K_{F2}, K_{H2}\}$, *policies* $\{K_G, N_G, R_2\}$ *and prices and returns*

$\{W_1, W_2, \pi_1, \pi_2, Q_{k1}, r_{K1}, r_{K2}\}$ *which solve*

$$W = \max_{C_1,C_2,Y_1,Y_2,\ell_1,\ell_2,K_{F2},K_{H2},R_2,N_G,K_G} \mathbb{E}\left[(\log C_1 - v(\ell_1)) + \beta(\log C_2 - v(\ell_2))|\bar{N}_0\right],$$

subject to the constraints that the allocation is a competitive equilibrium of the economy.

The expectation $\mathbb{E}(.)$ captures only the uncertainty about the risk of a run. It is taken under the information set available to policymakers at the beginning of time 1. The only variable they can't perfectly predict is N_0. As discussed above, we assume they know the mean $\bar{N}_0$ of the distribution from which it is drawn.

We also consider an alternative objective whereby central banks seek to minimize deviation of inflation from a target, which we denote $\bar{\pi}$ and the rest of the government chooses other tools to maximize welfare. This alternative approach is more in line with a strict inflation targeting mandate that some central banks in the world abide by. We analyze this case in section 6.5 and show that the qualitative results are robust.

6.2 Outside of the Run or the Constrained Zones

We start by considering the benchmark situation where the economy is in the part of the state space where there is no risk of run, $\xi = 0$, and the balance sheet constraint doesn't bind, $\sigma = 0$. This occurs when inflation is small or when banks are well-capitalized, as shown in section 3.2. In that case, the central bank balances the maximization of output with inflation stabilization. The key result is that there is no basis for the government and the central bank to take into account financial frictions in setting interest rates and there is no role for additional tools.

Outside of the run and the constrained zones, we can re-express the social planner's problem described by Definition 1 as a simpler maximization problem subject to the Phillips curve. Recall from section 3.1 that in this part of the state space, the economy is fully characterized by two equations, the Phillips curve and the Euler equation. Given that R_2 and C_1 are related one-to-one through the Euler equation, the social planner can simply choose C_1 and back out the level of R_2 that implements it. Using the market clearing condition to substitute for C_1, assuming that the disutility of labor in period 1 is given by

$$v_1(\ell_1) = \chi \log \ell_1, \tag{26}$$

with $\chi < \alpha$ and using the firm's production technology $Y = \ell^\alpha K^{1-\alpha}$ to substitute for

ℓ_1 as a function of Y_1, the problem of the social planner becomes

$$W = \max_{Y_1,\pi_1,K_G,N_G} \left\{ \left(1-\frac{\chi}{\alpha}\right)\log Y_1 + \log\left(1-\frac{\theta}{2}\pi_1^2\right) + \beta \log C_2 \right\} \quad \text{s.t.} \quad 0 = \mathcal{PC}(Y_1,\pi_1)$$

The optimality condition is given by

$$MRS = \frac{\left(1-\frac{\chi}{\alpha}\right)\left(1-\frac{\theta}{2}\pi_1^2\right)}{\theta\pi_1 Y_1} = -\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi} = MRT.$$

Intuitively, the marginal rate of substitution (MRS) between output and inflation $\frac{\left(1-\frac{\chi}{\alpha}\right)\left(1-\frac{\theta}{2}\pi_1^2\right)}{\theta\pi_1 Y_1}$ should be equal to the marginal rate of transformation (MRT) which is also equal to the slope of the Phillips curve $-\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}$. If there was a wage subsidy to offset the markup distortion, the implied level of output would coincide with the flexible price allocation. We can then use the Euler equation and the goods market clearing condition to express the implied interest rate:

Lemma 3 (Baseline)**.** *The optimal interest rate is given by*

$$R_2 = \underbrace{-\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}}_{\text{Slope of Phillips curve (MRT)}} \underbrace{\frac{\theta\pi_1}{\left(1-\frac{\chi}{\alpha}\right)\left(1-\frac{\theta}{2}\pi_1^2\right)^2}}_{\text{Welfare cost of inflation}} \frac{C_2}{\beta} \tag{27}$$

This formula provides intuition on the determinants of the optimal trade-off between output maximization and inflation stabilization. More specifically, the higher the level of inflation π_1, the steeper the Phillips curve $-\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}$ and the higher future consumption C_2, the higher the optimal interest rate. Finally, given that credit policy and equity injection play no role in this part of the state space, we have $K_G = N_G = 0$.

6.3 Inside the Constrained Zone

We now turn to the case where the balance sheet constraint binds and there is no risk of a run ($\zeta \to +\infty$). As shown in section 3.2, this happens when the level of inflation requires a large rate hike or when banks are less well-capitalized. Inside the constrained zone, the economy is characterized by three equations: the Phillips curve, the Euler equation and the balance sheet constraint. Like in the benchmark problem analyzed before, the Euler equation is omitted because the social planner can simply choose C_1

and back out the level of R_2. As a result, the social planner's problem is given by:

$$W = \max_{Y_1,\pi_1,K_G,N_G,K_{H2}} \left\{ \left(1-\frac{\chi}{\alpha}\right)\log Y_1 + \log\left(1-\frac{\theta}{2}\pi_1^2\right) + \beta C_2(K_{H2},K_G,N_G) \right\}$$
$$\text{s.t.} \quad 0 = \mathcal{PC}(Y_1,\pi_1) \quad \text{and} \quad 0 = \mathcal{BSC}(K_{H2},K_G,Y_1,N_0,N_G)$$

Note that because we introduce one more constraint ($\mathcal{BSC}$), we will take an additional first order condition with respect to K_{H2}.

When the balance sheet constraint binds, a wedge (σ) opens up between the efficient and the actual allocation. From section 3.2, we know that the spread between the return on capital and on deposits σ is tightly related to $\beta_K \frac{P_2 K_{H2}}{K}$ and is exacerbated by further increases in the policy rate R_2. In appendix C.2 and in the following lemma, we show that it is also closely linked to the shadow cost of the $\mathcal{BSC}$ constraint (i.e. the Lagrange multiplier). Policymakers should deploy other tools such as credit policy or equity injection to close the wedge and should moderate their policy rate hikes when these tools are costly. The following lemma gives an analytical expression for the optimal interest rate when tools are costly.

Lemma 4 (Constrained - optimal interest rate)**.** *When the other tools are costly $\beta_G, \beta_N > 0$, the optimal interest rate is given by*

$$R_2 = \Omega \bar{R}_2 \tag{28}$$

$$\textit{with} \quad \Omega = \frac{1}{1+\Omega_0 \sigma \frac{dK_{H2}}{dR_2}} < 1 \quad \textit{and} \quad \Omega_0 = \frac{1+r_{K2}}{P_2} \frac{1}{\left(1-\frac{\theta}{2}\pi_1^2\right)\left(1-\frac{\chi}{\alpha}\right)} \frac{1}{Y_1} \tag{29}$$

The formula reveals an additional term $\Omega_0 \sigma \frac{dK_{H2}}{dR_2}$ that calls for a lowering of the optimal interest rate relative to the policy rate in the absence of financial frictions $\bar{R}_2$. A less aggressive tightening avoids the drop in physical returns and asset prices today, which hurts the intermediaries' balance sheets. This deviation is proportional to the shadow utility cost of the balance sheet constraint, which is directly related to the spread between the return on capital and deposits, $\sigma = \frac{\beta_K P_2 K_{H2}}{K(1+r_{K2})}$. It is larger when households hold more of the capital stock (K_{H2}/K) and when these holdings are costly (β_K).

The extent to which this welfare cost should translate into a lower interest rate depends on the sensitivity of the banking sector's intermediation capacity, which is the complement of the households' holdings of capital K_{H2}, to the interest rate R_2. This is captured by the term the term $\frac{dK_{H2}}{dR_2}$. Mathematically, this term is related to the sensitivity of the balance sheet constraint :

$$-\frac{dK_{H2}}{dR_2} = \underbrace{\frac{\mathcal{BSC}_Y}{\mathcal{BSC}_{K_{H2}}}\left(1 + \frac{\mathcal{BSC}_\pi}{\mathcal{BSC}_Y}\frac{\mathcal{PC}_Y}{-\mathcal{PC}_\pi}\right)}_{-dK_{H2}/dY_1}\frac{dY_1}{dR_2}.$$

The bracket includes two additive terms: the direct effect of the interest rate on output and asset prices, and an indirect effect through inflation.

When credit policy are available, they should be used to address the source of financial stress, alleviate the trade-off for interest rate policy and allow monetary policy to focus on price and output stabilization.

Lemma 5 (Constrained - Credit Policies)**.** *If $\beta_G > 0$, the optimal level of credit policy is given by*

$$K_G = \underbrace{\frac{\beta_K}{\beta_G}}_{\text{Relative cost of credit policy}} \underbrace{K_{H2}}_{\text{HH holding}} \underbrace{\frac{\mathcal{BSC}_{K_G}}{\mathcal{BSC}_{K_H}}}_{\text{Relative efficacy of credit policy}}$$

If $\beta_G = 0$, the optimal interest rate is the same as outside the constrained zone (equation 27) and credit policy are given by

$$K_G > \underline{K}_{G2}$$

$$\underline{K}_{G2} = \frac{\phi R_2}{1+r_{K2}}\left[\bar{R}_1 N_0 - \bar{R}_1 Q_{K0}(K - K_{H1}) + r_{K1}(R_2)(K - K_{H1}) + \frac{1+r_{K2}}{R_2}\left(K - K_{H1} - \frac{K}{\phi}\right)\right]$$

The first formula shows that these other tools should be used in proportion to their costs and benefits. Optimal credit policy are higher when the government is efficient at intermediating $1/\beta_G$, when households hold more assets K_{H2}, when households are less efficient at holdings capital β_K and when the government's holdings relax the balance sheet constraint of intermediaries relative to households $\frac{\mathcal{BSC}_{K_G}}{\mathcal{BSC}_{K_H}}$.

More broadly, the optimal mix of policy rate moderation and credit policy and the degree of separation of financial stability objectives depend on the cost of other tools, β_G. The more costly credit policies, the less they should be used and the larger the deviation of interest rate policy from its level outside of the constrained zone. When these other tools are prohibitively costly, or not available, $\beta_K \to +\infty$, policymakers should implement $K_{H2} = 0$ and the deviations of the interest rate policy are largest to preserve the intermediation capacity of banks. This is a case where separation of financial stability and price stability is impossible.

At the other extreme, when credit policy have no cost $\beta_G = 0$, policymakers can achieve perfect separation of financial stability objectives. By intermediating at no cost the assets that the private intermediate cannot hold, public credit policy can restore the first best allocation. Formally, when credit policy are not costly, we can drop the $\mathcal{BSC}$ constraint. The interest rate is then chosen only to trade-off price stabilization and output maximization while credit policy should address balance sheet constraint. The following lemma formalizes this separation result.

Intuitively, the optimal level of government's asset purchases K_G should be at least $\underline{K}_{G2}$ which is the amount necessary so that private intermediaries can hold all the remaining capital stock $K - K_G$.

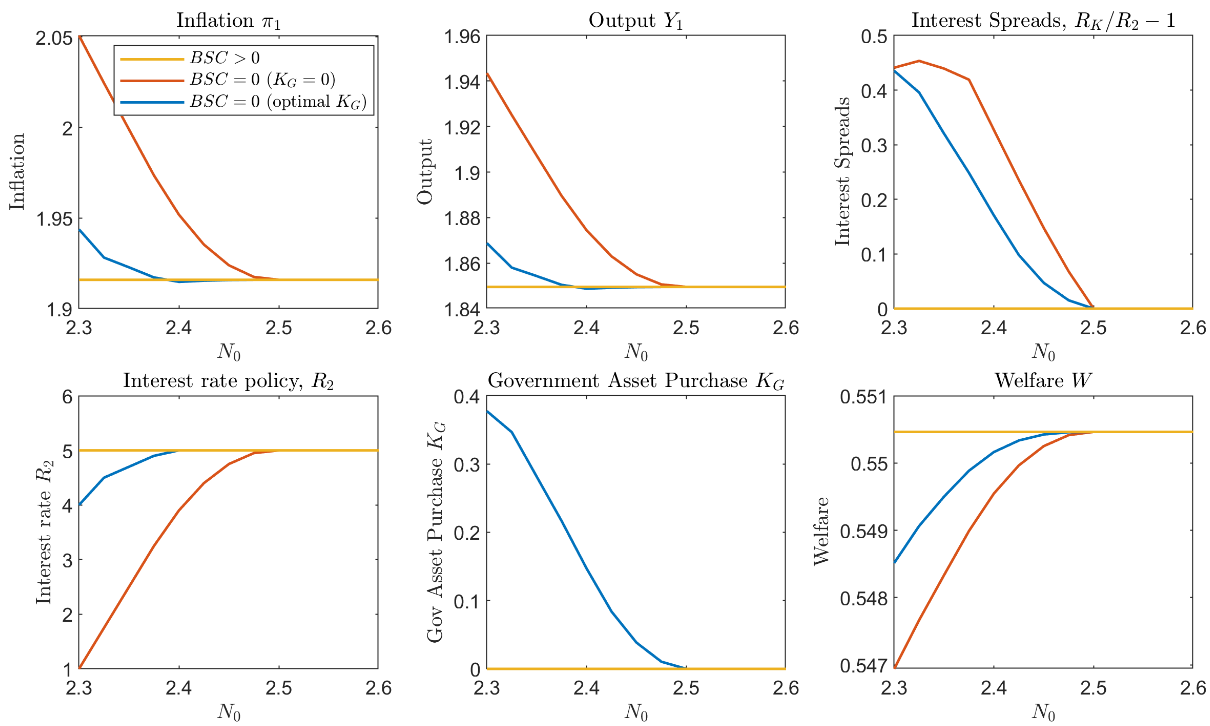

Figure 5: Equilibrium as a function of N_0. Yellow line shows outcomes when there is the leverage constraint does not bind. Orange line shows outcomes given the optimal policy rate R_2 and no alternative policies, when the leverage constraint binds. Blue line shows expected outcomes given the optimal policy rate R_2 and use of alternative policies, when the leverage constraint binds.

Equity injection. We now consider equity injections as an alternative tool policymakers can use to address financial frictions. When they are not costly to use $\beta_N = 0$, equity injections should be used to fully address the source of financial distortions and allow interest rate policy to focus on the trade-off between output and inflation. Using the same reasoning as for credit policy, we use the constraint $\mathcal{BSC}$ to solve for the minimum level of N_G such that households hold no long-term asset in equilibrium $K_{H2} = 0$. This minimum level is given in the following lemma. When equity injection is costly $\beta_G > 0$, the formula for the optimal rate is the same as for the case where other tools are not available shown before and the optimal choice of other tools is given by the following lemma.

Lemma 6 (Constrained - Equity injection)**.** *If $\beta_N > 0$, the optimal interest rate is the same as in the baseline (equation 27) and the optimal level of equity injection should be*

$$N_G = \underbrace{\frac{\beta_N}{\beta_G}}_{\text{Relative costs of equity injection}} \underbrace{\frac{K_{H2}}{K}}_{\text{HH holding}} \underbrace{\frac{\mathcal{BSC}_{N_G}}{\mathcal{BSC}_{K_H}}}_{\text{Relative efficacy of equity injection}}$$

If $\beta_N = 0$, the optimal interest rate is the same as in the baseline (equation 27) and the minimum level of bank equity should be

$$\underline{N}_G = \left[-\bar{R}_1 N_0 + \bar{R}_1 Q_{K0}(K - K_{H1}) - r_{K1}(R_2)(K - K_{H1}) - \frac{1 + r_{K2}}{R_2} \left(K - K_{H1} - \frac{K}{\phi} \right) \right]$$

Macroprudential policies and other tools. Macroprudential policies such as capital requirements can be relaxed *ex post* by increasing ϕ^G to raise the intermediaries' overall capacity to invest and close the spreads.[18] However, this would be effective if and only if the macroprudential regulation is binding, i.e. $\phi^G < \phi^P$. Like for equity injections and credit policy, the extent to which it should be used to address the source of the stress depends on its relative costs and benefits. If the binding constraint is the incentives-based one, relaxing macroprudential policies wouldn't have any effect. Finally other tools such as deposit insurance or lender of last resort wouldn't be effective when the distortion comes from a binding collateral constraint.

Example: the U.S. Savings and Loan crisis. During the early stages of the 1980s U.S. Savings and Loan crisis, the Federal Reserve increased its lending support amid a surge in bank failures. This intervention played a key role in helping solvent banks maintain

[18] While macroprudential policies aim to limit the *ex ante* buildup of vulnerabilities, financial stress may still materialize, raising the question of how macroprudential requirements should be adjusted *ex post*.

operations despite facing temporary liquidity shortages and helped the Fed keep its focus on a tight monetary stance.

6.4 Inside the Run Zone

We now consider the case where the economy is in the "run" zone, i.e. where there is a positive probability of a systemic run $\xi > 0$. As shown in section 2.5, this happens when the level of inflation requires a strong rate hikes or when banks are less well-capitalized. Given that policymakers need to decide on the policy rate before knowing the outcome of the run, they maximize expected welfare conditional on the public signal $\bar{N}_0$. Abstracting from the balance sheet constraint, the social planner faces two constraints ($\phi^P \to +\infty$): the Phillips curve in the good and in the run equilibrium. Like before, we omit the Euler equation, choose C_1 and then back out the level of R_2. The problem of policy-makers is given by

$$W = \max_{Y_1, Y_1^*, \pi_1, \pi_1^*, K_{H2}, K_{H2}^*, N_G, K_G} \Bigg\{ (1-\xi)\left(\left(1-\frac{\chi}{\alpha}\right)\log Y_1 + \log\left(1-\frac{\theta}{2}\pi_1^2\right) + \beta \log C_2\right)$$
$$+\xi\left(\left(1-\frac{\chi}{\alpha}\right)\log Y_1^* + \log\left(1-\frac{\theta}{2}(\pi_1^*)^2\right) + \beta\log C_2^*\right)\Bigg\}$$
$$\text{s.t.} \quad 0 = \mathcal{PC}(Y_1, \pi_1) = \mathcal{PC}\left(Y_1^*, \pi_1^*\right)$$

where $\xi\left(Y_1^*, \pi_1^*, \frac{C_2^*}{\beta C_1^*}, K_G\right)$ is the probability of a run.[19]

Due to the risk of coordination failures among depositors, an additional wedge—the risk of a run given by ξ—opens up between the efficient and the actual allocation which policies should try to address. When other tools are available, including asset purchases, equity injections, lender of last resort facilities and deposit insurance, they should be used, in proportion to their costs, to decrease the distortions implied by the coordination failure. When there are costly, the central bank should internalize the impact of interest rate hikes on the risk of run. Denoting the optimal rate in the no-run state of the world $\bar{R}_2 = \frac{\theta \pi_1 C_2}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi_1)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}$ and the optimal rate in the run state of the world $\underline{R}_2 = \frac{\theta \pi_1^* C_2^*}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi_1^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}$, the following lemma gives the optimal rate *ex*

[19] For simplicity, the social planner's welfare function abstracts from the utility costs ζ incurred by households who decide to withdraw their deposits. These shoe-leather costs are empirically small relative to the macroeconomic costs emphasized in this paper. Incorporating them would only strengthen our result.

ante.

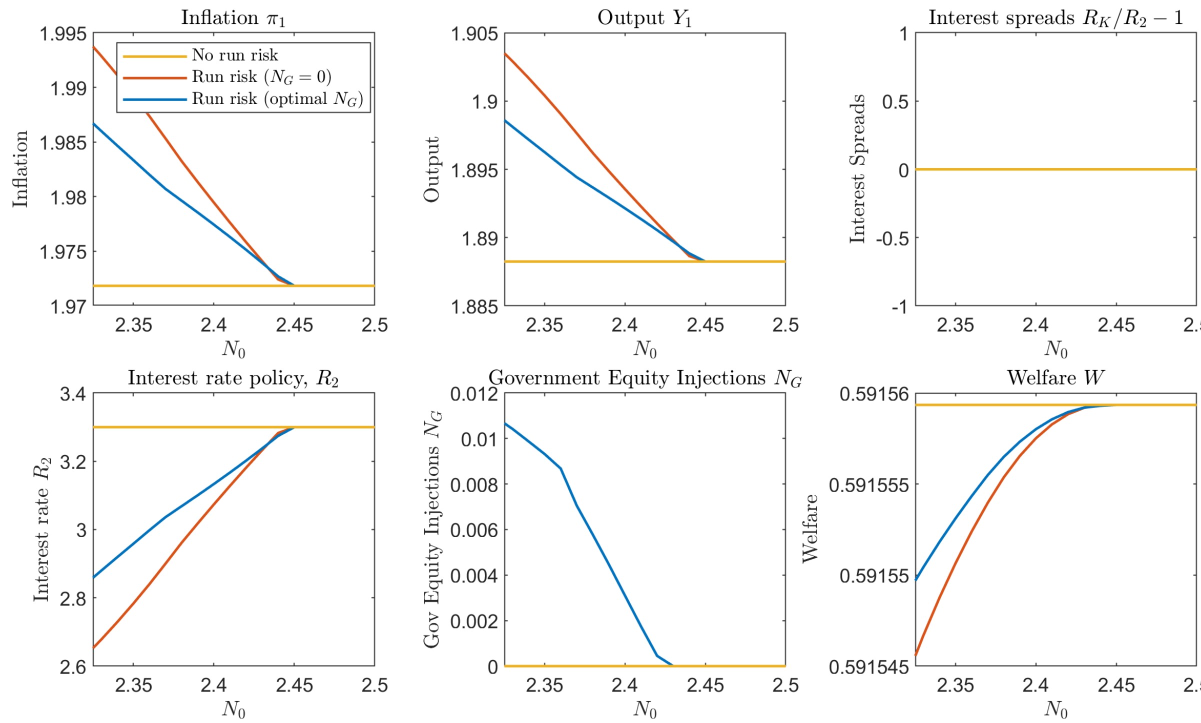

Figure 6: Equilibrium as a function of N_0. Yellow line shows outcomes when there is no run risk. Orange line shows expected outcomes with run risk, at the optimal policy rate R_2 and no alternative policies. Blue line shows expected outcomes when alternative policies are used.

Lemma 7 (Run - optimal interest rate policy)**.** *When the other tools are costly $\beta_G, \beta_N > 0$, the optimal rate is given by*

$$R_2 = (1 - \xi\Omega_1)\bar{R}_2 + \xi\Omega_1\underline{R}_2 - \xi'\Omega_2 \log\frac{C_2^*}{C_2} \tag{30}$$

$$\text{with} \quad \Omega_1 = \frac{\frac{\chi}{\alpha}}{1 - (1 - \frac{\chi}{\alpha})\frac{\underline{R}_2}{\bar{R}_2}}, \quad \Omega_2 = \frac{\alpha}{\chi}\frac{(1+\beta)}{(1-\frac{\chi}{\alpha})}Y_1^*\Omega_1 \quad \text{and} \quad \xi' = \xi_Y + \xi_\pi\frac{-\mathcal{P}\mathcal{C}_Y}{\mathcal{P}\mathcal{C}_\pi}$$

where ξ' is the total derivative of the probability of a run with respect to Y_1 (through R_2), taking into the account the effect through inflation, $\bar{R}_2, \underline{R}_2$ are the "shadow optimal interest rates" if the central bank could implement a state-contingent interest rate policy.

The formula above makes clear that the optimal interest rate should be below its optimal level without a risk of run $\bar{R}_2$ to take into account the risk of a run. This happens through two separate mechanisms.

The first effect, $\xi\Omega_1 (\bar{R}_2 - \underline{R}_2)$, is proportional to the probability of the run and to the gap between the shadow optimal rates in the no run and in the run equilibria $\bar{R}_2 - \underline{R}_2 > 0$. The intuition is as follows: since aggregate demand and inflation drop in case of a run, the central bank is willing to tolerate higher inflation in the good state of the world to avoid a deeper recession in the case of a run. It is worth noting that it is as if the central bank was targeting a weighted average of the two "shadow optimal interest rates" where the weights are $(1 - \xi\Omega_1)$ on the good state and $\xi\Omega_1$.[20]

This first effect implies that monetary policy should adopt a risk-management approach when there is a risk of bank runs. It is worth noting that this remains true even if the probability was insensitive to marginal changes in the interest rate, $\xi' = 0$. This is in sharp contrast with the case of binding collateral constraints analyzed before. In this case interest rate policy should take the level of the distortions—the spreads—into account only if these distortions are sensitive to marginal changes in the interest rate.

The second effect, $\xi'\Omega_2 \log \frac{C_2^*}{C_2}$, calls for a lowering of the interest rate since $\xi' < 0$ and $\log \frac{C_2^*}{C_2} < 0$. It captures how interest rate tightening affects the run probability, which is given by the sensitivity of the probability of a run with respect to the interest rate ξ'. The extent to which that sensitivity translates into a lower interest rate depends on the welfare losses in case of a run, $(1 + \beta) \log \frac{C_2^*}{C_2}$. The larger the loss given a run, the lower the optimal interest rate.

When other instruments are available, they should be used to address the source of financial stress, alleviate the trade-off for interest rate policy and allow monetary policy to focus on price and output stabilization. Equity injection directly helps strengthening banks' balance sheets. Indirectly, they also help improve the allocation by boosting asset prices. Both the direct and indirect channels contribute to lowering the risk of a run.

Lemma 8 (Run - other tools)**.** *If $\beta_G, \beta_N > 0$, the optimal level of equity injection is given by*

$$N_G = \Omega_3 \frac{\xi_{N_G}}{\beta_N} \log \frac{C_2^*}{C_2}$$

Similarly the optimal level of asset purchases is given by:

$$K_G = \Omega_3 \frac{\xi_{K_G}}{\beta_G} K \log \frac{C_2^*}{C_2}$$

[20] Maybe surprisingly, the weight on the run state of the world is not the probability that this state occurs (ξ) but is smaller by a factor $\Omega_1 < 1$ which is increasing in the dis-utility of labor χ. The bigger the dis-utility from labor, the more weight the central bank puts on the run state of the world because agents work less in that state.

Equity injections, and maybe more surprisingly asset purchases, can help improve the allocation and should be used in proportion to their costs and benefits in decreasing the risk of run. The less costly equity injection and asset purchases $1/\beta_N, 1/\beta_G$. the higher the efficacy of these policies at bringing down the risk of a run ξ'_{N_G}, ξ'_{K_G}, and the more costly the run in terms of consumption $\frac{C_2^*}{C_2}$, the more the government should use them.

The fact that asset purchases can also help improve welfare where there is a risk of runs is a more unexpected result. It directly relates to the fact that the run comes from an inefficient fire sale which depresses prices. By scaling up its balance sheet and intermediating long-term assets, the government and the central bank can mitigate the drop in asset prices and hence the risk of a run.

The optimal mix of policy rate moderation, equity injections and asset purchases and the degree of separation of financial stability objectives depend on their cost, β_N, β_K. The less costly equity injections, the more they should be used and the more the policy rate R_2 can focus on inflation (still trading-off output losses) and tighten more aggressively than in the case without these other tool. This is because the equilibrium level of inflation and period-2 consumption in the run equilibrium C_2^*, π_1^* are higher and the run-implied loss C_2^*/C_2 is smaller. The effect through the change in the probability of a run ξ' is ambiguous as it depends on the convexity of the $\xi(.)$ function. In the extreme case where other tools are prohibitively costly, or not available, $\beta_N \to +\infty$, then $N_G = 0$ and separation of financial stability objectives is impossible: the central bank should tighten significantly less to account for its effect on the probability of a run.

By contrast, when equity injections or asset purchases have no cost $\beta_N = \beta_G = 0$, policymakers can fully achieve their financial stability goal and separate them from the price/output stability trade-off. The government should inject equity or purchase assets up to the point where intermediaries' balance sheets are repaired and the likelihood of a run is eliminated, $\xi = 0$. Interest rate policy should then focus only on the price-output trade-off.

Lemma 9 (Run - complete separation)**.** *If $\beta_N = 0$, the optimal interest rate is the same as in the baseline (equation 27) and equity injection is given by*

$$N_G > \bar{N} - N_0$$

If $\beta_G = 0$, the optimal interest rate is the same as in the baseline (equation 27) and asset *purchases K_G are such that*

$$\frac{(R^*_{k1}(K_G) - \bar{R}_1)Q_{K0}(K - K_{H1})}{\bar{R}_1} + N_0 > 0$$

Intuitively, the optimal level of government's equity injection N_G should be sufficient to bring the level of intermediaries' equity above $\bar{N}$, the level of net worth such that even if all depositors run, the run is unsuccessful. Similarly asset purchases should boost prices and thereby recapitalize banks up to the point where the level of banks net worth is such that there is no risk of run.

Deposit insurance, lender of last resort and other policies. The analysis above has focused on equity injection and asset purchases. Two more traditional tools can more directly address coordination failures among depositors: deposit insurance and lender of last resort facilities. Through the lens of the model, deposit insurance and lender of last resort facilities are state-contingent versions of asset purchases. They are contingent credit lines to banks only if the run happens. Our previous results would thus hold for these two policies too. Arguably they are less costly—since they are activated only in the state of the world where a run happens—so they would correspond to a lower β_N. This would in turn imply a higher degree of separation, more aggressive use of the tools and more aggressive interest rate tightening.

Example: the 2023 U.S. regional banking turmoil. In the wake of the post-pandemic tightening in 2022, many regional banks experienced run on their deposits. To face this turmoil, the FDIC invoked the systemic risk exception under the Federal Deposit Insurance Act to override the least-cost resolution requirement and guarantee both insured and uninsured deposits at Silicon Valley Bank and Signature Bank. Furthermore, fiscal support mechanisms play a critical role in reinforcing the credibility of deposit insurance and shielding the central bank's balance sheet from potential losses tied to asset purchases and emergency lending. During the same crisis, the U.S. Treasury allocated $25 billion to support the Federal Reserve's newly established lending facility.

6.5 Robustness

Long-term government bonds. The previous analysis has abstracted from long-term bonds and focused on firms equity. In practice, valuation effects in intermediaries' balance sheet also work through the decline in the price of long-term bonds. In Appendix A, we show that all our results go through when including long-term bonds.

Strict inflation targeting. All the previous results still hold, at least qualitatively, when the central bank follows a strict inflation targeting mandate. In particular, as long as other tools are costly to use the central bank should adopt a less aggressive policy stance. We provide a formal proof in Appendix D.

When tools are costly, the optimal interest rate is lower in the "constrained" zone than outside and strictly decreasing in K_{H2} and L_{H2}. The reason is that a binding balance sheet constraint depresses future consumption ($C_2 < Y_2 + K$), which depresses current consumption ($C_1 = C_2/(\beta R_2)$). This in turn means the interest rate doesn't need to be as high to control inflation. In that case, credit policy and equity injection can help offset the loss of consumption in period 2. If other tools are not costly, governments should use them to address the financial distortions stemming from the financial constraint of intermediaries. In equilibrium, households shouldn't hold any assets, there shouldn't be any spread between the policy rate and asset returns, and $C_2 = Y_2 + K$. This implies that the policy rate should follow the same path as the one outside the "constrained" zone.

In the case of a run, the inflation rate depends on the materialization of a run. Given that policies are set before the realization of a run, policy-makers can't achieve perfect price stabilization ex post. The best they can do is to achieve price stability on average across states of the world. Accordingly, we assume that the objective of the central bank is to minimize the expected squared deviations from target. In Appendix D, we show that the optimal policy rate is strictly lower than the one outside of the run zone. This is because the central bank reduces the average deviation by tolerating some inflation in the no-run path, to avoid a drop in inflation in a run period. This again illustrates that monetary policy should adopt a risk-management approach when there is a risk of a run.

When credit policy, equity injection or deposit insurance are available, they should be used to decrease the risk of runs by boosting asset prices, strengthening intermediaries' balance sheets or directly reassuring depositors, which in turns improve intermediation and raise consumption in period 2. This in turn boosts expected inflation and leads the central bank to raise its policy rate.

Large financial crisis. Finally, we consider a large financial crisis which we model as a case in which the run happens with probability one, $\xi = 1$. In equilibrium, banks lose all their net worth $N_1 = 0$, and absent government's interventions all the capital is intermediated by households which results in a drop in consumption in period 2, and hence in period 1.

Given that the run, the implied disruption of financial intermediation and the large drop in output occur independently of the stance of monetary policy, the only trade-off the central bank faces is between inflation stabilization and preserving output - the same one it faced outside of the constrained and of the run zones. As a result the optimal interest rate policy is simply given by $R_2 = -\frac{\theta \pi_1^*}{\left(1-\frac{\theta}{2}(\pi_1^*)^2\right)\left(1-\frac{\chi}{\alpha}\right)} \frac{\mathcal{PC}_Y^*}{\mathcal{PC}_\pi^*} C_2^*$. When credit policy are available, they should be used to intermediate part of the private assets in the economy and minimize the drop in C_2^*. We provide further details in Appendix C.4.

7 Conclusion

This paper presents a two-period NK framework featuring a financial sector that finances itself with short-term deposits and invests in long-term assets, while facing a leverage constraint and the risk of depositor runs. The model explains how monetary tightening, particularly during periods of high inflation and financial vulnerability, can increase financial instability, erode bank equity, and trigger banking panics—patterns consistent with empirical findings from global historical data. We then use the model to characterize the optimal combination of interest rate policy, credit policy, equity injections, deposit insurance, and macroprudential measures. The model's tractability enables us to derive intuitive expressions for optimal policy design, shedding light on the key factors that shape the ideal policy mix.

Our findings show that policymakers can theoretically achieve a full separation of price and financial stability goals through alternative policy tools such as credit policy, equity injections, deposit insurance, and macroprudential measures. However, this separation is feasible only if these tools can be deployed without costs. In the more realistic scenario where alternative policy tools face implementation challenges but remain viable options, policymakers should adopt a mixed approach, combining a less aggressive interest rate policy with a sound use of these tools. The degree to which interest rate policy must accommodate financial stability and the extent to which alternative tools should be expanded depend on the severity of financial vulnerabilities and the costs of tools.

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Appendix

A General Model Formulation

In this Appendix, we briefly summarize the general model with long-term government bonds.

Households. Households enter period 1 with a portfolio of long-term bonds b_{h1}, capital k_{H1} and deposits d_1. From these investments they collect returns $\frac{Q_{B1}+r_B}{Q_{L0}}$, $\frac{Q_{K1}+r_{K1}}{Q_{K0}}$ and R_1 respectively, where r_B, Q_{B1} are the exogenous interest rate and the endogenous price of long-term bonds, and r_{k1}, Q_{K1} are the endogenous dividends per unit capital and the price of capital in period 1.

Their budget constraint in period 1 is given by

$$P_1 c_1 + d_2 + Q_{B1} b_{H2} + Q_{K1} k_{H2} = R_1 d_1 + (Q_{b1} + r_B) b_{H1} + (Q_{K1} + r_{k1}) k_{H1} + T_1 + W_1 \ell_1 \tag{31}$$

In period 2, the budget constraint is thus given by

$$P_2 c_2 = R_2 d_2 + \left(1 + r_B - \frac{\beta_B}{2} \frac{P_2 b_{h2}}{B}\right) b_{h2} + \left(1 + r_{k2} - \frac{\beta_K}{2} \frac{P_2 k_{H2}}{K}\right) k_{H2} + T_2 + W_2 \ell_2 \tag{32}$$

The household' optimality conditions are given by

$$1 = \beta \frac{R_2}{1+\pi_2} \frac{c_1}{c_2}, \qquad R_2 = \frac{1 + r_B - \beta_B \frac{P_2 b_{h2}}{B}}{Q_{B1}} \quad \text{and} \quad R_2 = \frac{1 + r_{K2} - \beta_K \frac{P_2 k_{H2}}{K}}{Q_{K1}} \tag{33}$$

Financial Intermediaries if No Run Happened If a run doesn't happen or if the run isn't successful, financial intermediaries continue to operate from period 1 to period 2. Their period-1 equity depends on the returns on assets and payments on liabilities

$R_1 = \bar{R}_1$ made at time 1:

$$N_1 = \bar{R}_1 N_0 + (R_{k1} - \bar{R}_1) Q_{K0} K_{F1} + (R_{B1} - \bar{R}_1) Q_{L0} B_{F1} + N_G \tag{34}$$

where $\bar{R}_1, B_{F1}, K_{F1}, Q_{K0}$ are all exogenous in period 1 and N_G denotes equity injection by the government. Returns on both types of assets R_{k1} and R_{B1} are endogenous.

At the end of period 1, financial intermediaries collect households deposits, D_2, and invest in capital, K_{F2} and in long-term bonds B_{F2}. Taking all asset prices and returns as given, they seek to maximize the end of period-2 N_2. When intermediaries can freely allocate their portfolio they would arbitrage away any differences in returns:

$$R_2 = R_{K2} = R_{B2} \tag{35}$$

Their balance sheet constraint is given by

$$\phi^P N_1 \geq Q_{B1} B_{F2} + Q_{K1} K_{F2}. \tag{36}$$

Government The government can choose the stock of debt B at interest rate r_B. The government has to pay interest on long-term bonds B in period 1, and repay the principal in period 2. These choices have to be consistent with the following budget constraints:

$$D_{G2} = T_1 + \frac{\theta_t}{2} \pi_1^2 P_t Y_t + r_B B + Q_{K1} K_G + N_G \tag{37}$$

$$\left(1 + r_{k2} - \frac{\beta_G}{2} \frac{P_2 K_G}{K}\right) K_G - \frac{\beta_N P_2}{2} N_G^2 = T_2 + (1 + r_B) B + (1 + R_2) D_{G2} \tag{38}$$

Market clearing. The long-term bonds market clear:

$$B = B_{H2} + B_{F2} \tag{39}$$

with $B_{H2} = \int l_{H2j} dj$.

B Global Game Microfoundation

The run game happens at the beginning of the period. Consumers enter period 1 with their holdings of capital, long-term bonds and deposits, K_{H1}, B_{H1}, D_1.

Information structure and posterior beliefs. Depositors are uncertain about the structure of the banks' liabilities, how much it owes to depositors and how much it owns. The true equity of the bank N_0 is imperfectly known before the run happens. Agents know that it is drawn from a distribution centered around the end-of-period 0 level of net worth $\bar{N}_0$ and with some dispersion given by σ_N. For simplicity, we will consider the case where net worth is log-normally distributed around $\bar{N}_0$,

$$\log N_0 \sim \mathcal{N}\left(\log \bar{N}_0, \sigma_N\right). \tag{40}$$

Agents also receive an idiosyncratic signal about the equity of the bank, η. It is common knowledge that it is drawn from a distribution that is centered around the true level of equity N_0 but with some noise, σ_η. We assume that the signals are also log-normally distributed around N_0,

$$\log \eta \sim \mathcal{N}\left(\log N_0, \sigma_\eta\right). \tag{41}$$

For future reference, we denote $F(\eta|N_0)$ the CDF of this distribution.

When deciding whether to run or not, depositors need to form expectations about the likely payoffs which depend on the behaviors (and the signals) of others, given their idiosyncratic signal. When priors and signals are log-normal, the posterior is also log-normal and given by

$$\log N_0 \sim \mathcal{N}\left(\frac{(\sigma_N^2)^{-1}}{(\sigma_N^2)^{-1}+(\sigma_\eta^2)^{-1}}\log \bar{N}_0 + \frac{(\sigma_\eta^2)^{-1}}{(\sigma_N^2)^{-1}+(\sigma_\eta^2)^{-1}}\log \eta, \frac{1}{(\sigma_N^2)^{-1}+(\sigma_\eta^2)^{-1}}\right) \tag{42}$$

Denote $p(N_0|\eta_i)$ the pdf of the posterior belief of agent i about the distribution of N_0 given its signal η_i, $\mu_{N_0}(\eta)$ the mean and σ_{NP}^2 the variance of this distribution given in (42), the density of the posterior is given by

$$p(n|\eta) = \frac{1}{n\sigma_{NP}\sqrt{2\pi}} \exp\left(-\frac{(\ln n - \mu_{N_0}(\eta))^2}{2\sigma_{NP}^2}\right).$$

Conditional on a given level of net worth N_0, the posterior beliefs on the signals received by other depositors is given by (41). And the share of people who receives a signal below η' is thus believed to be, $F(\eta'|N_0, \sigma_\eta)$.

Condition for successful run. There are two outcomes to the run game. Either the run is "successful" in the sense that the banks have to liquidate, or it is not. Denoting $\delta \in [0, 1]$ the share of individuals who decide to run, a necessary and sufficient condition for a run to be successful is that the bank doesn't have enough to repay the depositors who run even if it liquidate all its assets:

$$R^*_{k1} Q_{K0}(K - K_{H1}) + R^*_{B1} Q_{L0}(L - L_{H1}) < \bar{R}_1 D_1 \delta \tag{43}$$

$$\text{with} \quad Q^*_{K1} = \frac{1 + r^*_{K2} - \beta_K + (\beta_K - \beta_G)\frac{K_G}{K}}{R_2} \quad \text{and} \quad Q^*_{B1} = \frac{1 + r_B}{R_2} \tag{44}$$

$$R^*_{k1} = \frac{r_{K1} + Q^*_{K1}}{Q_{K0}} \quad \text{and} \quad R^*_{B1} = \frac{r_B + Q^*_{B1}}{Q_{L0}} \tag{45}$$

where we use the run price to evaluate both the capital stock and the long-term bonds since this is a case where the bank liquidates.

The condition for a successful run is thus that the share of depositors who run is large enough:

$$\delta > \bar{\delta}(N_0) \quad \text{with} \quad \bar{\delta}(N_0) = \frac{R^*_{k1} Q_{K0}(K - K_{H1}) + R^*_{B1} Q_{L0}(L - L_{H1})}{\bar{R}_1 \left[Q_{K0}(K - K_{H1}) + Q_{L0}(L - L_{H1}) - N_0\right]} \tag{46}$$

where N_0 is the true level of net worth which is not perfectly known by agents.

For future reference, we define $\bar{N}$ the level of net worth such that a run is not successful, it is solution to

$$F(\bar{\eta}|\bar{N}) = \frac{R^*_{k1} Q_{K0}(K - K_{H1}) + R^*_{B1} Q_{L0}(L - L_{H1})}{\bar{R}_1 \left[Q_{K0}(K - K_{H1}) + Q_{L0}(L - L_{H1}) - \bar{N}\right]}. \tag{47}$$

The left-hand side is decreasing in $\bar{N}$ while the right-hand-side is increasing in it. They cross at most once. A run is successful for lower levels of net worth and not successful otherwise.

Trigger strategy. Each depositor has two strategies: to run or not to run. Following the literature, we guess that the equilibrium strategy is a trigger strategy, where depositors run if and only if the signal they receive is lower than a threshold $\bar{\eta}$, which is common across all depositors and common knowledge. Denoting $\delta^*(N_0)$ the equilibrium mass of depositors who run when the level of equity is n, a direct implication is that the mass of depositors who decide to run is simply given by $\delta^*(N_0) = F(\bar{\eta}|N_0)$, i.e. the depositors who have a signal below the threshold $\bar{\eta}$.

Payoffs of Depositors. Given that there are two aggregate outcomes (successful and not successful) and two strategies, we can consider four different cases, which are shown in Table B. A depositor would prefer to run in the case where the run is successful because in that case it gets a share of the bank liquidation value proportional to its deposits d_1 (which is equal to D_1 in aggregate), $\frac{R^*_{k1}Q_{K0}(K-K_{H1})+R^*_{B1}Q_{L0}(L-L_{H1})}{\bar{R}_1[Q_{K0}(K-K_{H1})+Q_{L0}(L-L_{H1})-N_0]\delta}d_1$ where δ is the share of depositors who run, while it loses its deposits if it doesn't run. A depositor would prefer not to run if the run is not successful because it incurs a small utility cost of running ζ.

To formalize the depositor's problem, we define $U(I)$ the indirect utility of a depositor which receives I from the bank at the end of the run game. I is equal to $R_1 d_1$ in case the run is unsuccessful. If the depositor runs, its utility is thus given by $U(R_1 d_1) - \zeta$. We omit the other holdings (of long term bonds and capital) for clarity. In case the run is successful, and the depositors doesn't run, its indirect utility is denoted $U^*(0)$ where as before $*$ denotes "run equilibrium." If it runs, its gets $U\left(\frac{R^*_{k1}Q_{K0}(K-K_{H1})+R^*_{B1}Q_{L0}(L-L_{H1})}{\bar{R}_1[Q_{K0}(K-K_{H1})+Q_{L0}(L-L_{H1})-N_0]\delta}d_1\right) - \zeta$.

	Successful	Unsuccessful
Run	$U\left(\frac{R^*_{k1}Q_{K0}(K-K_{H1})+R^*_{B1}Q_{L0}(L-L_{H1})}{\bar{R}_1[Q_{K0}(K-K_{H1})+Q_{L0}(L-L_{H1})-N_0]\delta}d_1\right) - \zeta$	$U(R_1 d_1) - \zeta$
Don't run	$U(0)$	$U(R_1 d_1)$

Given the guess of a trigger strategy and the posterior beliefs, we can define the expected payoffs in case the depositors decide to run:

$$\int_0^{\max(\bar{N},0)} U\left(\frac{R^*_{k1}Q_{K0}(K-K_{H1})+R^*_{B1}Q_{L0}(L-L_{H1})}{\bar{R}_1\left[Q_{K0}(K-K_{H1})+Q_{L0}(L-L_{H1})-n\right]F(\bar{\eta}|n)}d_1\right)p(n|\eta_i)dn \\ +U(R_1 d_1)\int_{\max(\bar{N},0)}^{\infty} p(n|\eta_i)dn - \zeta$$

where $\bar{N}$ is the maximum level of net worth above which a run cannot be successful defined above. The expected payoffs in case the depositors decide not to run:

$$\int_0^{\max(\bar{N},0)} U(0)\,p(n|\eta_i)dn + U(R_1 d_1)\int_{\bar{N}}^{\infty} p(n|\eta_i)dn$$

Equilibrium. Assuming that in period 0 all consumers were identical, we have $d_1 = D_1 = Q_{K0}(K-K_{H1}) + Q_{L0}(L-L_{H1}) - \bar{N}_0$. A necessary condition for $\bar{\eta}$ is that a depositor with this signal is indifferent between running and not running:

$$\int_0^{\max(\bar{N},0)} \left[U\left(\frac{R^*_{k1} Q_{K0}(K-K_{H1}) + R^*_{B1} Q_{L0}(L-L_{H1})}{\bar{R}_1 \left[Q_{K0}(K-K_{H1}) + Q_{L0}(L-L_{H1}) - n \right] F(\bar{\eta}|n)} D_1 \right) - U(0) \right] p(n|\bar{\eta}) dn = \zeta \tag{48}$$

The ex ante probability of a run ξ is given by:

$$\xi = P\left(\delta^*(N_0)\bar{R}_1 D_1 > R^*_{k1} Q_{K0}(K-K_{H1}) + R^*_{B1} Q_{L0}(L-L_{H1}) | \bar{N}_0 \right) \tag{49}$$

$$\text{with} \quad D_1 = Q_{K0}(K-K_{H1}) + Q_{L0}(L-L_{H1}) - N_0 \tag{50}$$

where the equilibrium share of depositors running is given by $\delta^*(N_0) = F(\bar{\eta}|N_0)$.

This probability is a function of $\bar{N}_0$ since this is the only signal policymakers have about the true level of banks equity, of Y_1^* and π^* through r^*_{k1}, and of K_G and R_2 through Q^*_{K1} hence through R^*_{k1}.

C Proof Model

C.1 Outside of the "Constrained" and "Run" Zones

Lemma

Proof. We can rewrite the condition (24) as

$$\begin{aligned}
&\mathcal{LHS}(N_0) > \mathcal{RHS}(R_2) \\
\text{with} \quad &\mathcal{LHS}(N_0) = \bar{R}_1 N_0 - \bar{R}_1 \left(Q_{K0}(K-K_{H1}) + Q_{L0}(L-L_{H1}) \right) \\
&\mathcal{RHS}(R_2) = -r_{K1}(R_2)(K-K_{H1}) - r_B(L-L_{H1}) \\
&\qquad - \frac{1+r_B}{R_2}\left(L - L_{H1} - \frac{B}{\phi} \right) - \frac{1+r_{K2}(R_2)}{R_2}\left(K - K_{H1} - \frac{K}{\phi} \right)
\end{aligned}$$

where $r_{K1}(R_2), r_{K2}(R_2)$ are general equilibrium functions.

Assumption 1. *We define the following three regularity conditions:*

- *Households hold positive deposits* $D_1 > 0$.
- *The Phillips curve is upward-sloping and not too steep, i.e. the implicit function* $\pi_1(Y_1)$ *defined by equation (22) is continuous and strictly increasing and* $\theta\pi_1^2\left(\frac{1}{2} + \epsilon_{\pi_1/Y_1}\right)$ *where* $\epsilon_{\pi_1/Y_1} = \frac{\partial \pi_1}{\partial Y_1}\frac{Y_1}{\pi_1}$ *is the elasticity of inflation to output implied by the Phillips curve.*

- *Households' holdings are not too large relative to the leverage ratio:* $\frac{\phi-1}{\phi} > \max\left(\frac{K_{H1}}{K}, \frac{B_{H1}}{B}\right)$

We start with showing that the functions $r_{K1}(.), r_{K2}(.)$ is decreasing in R_2 through a decrease in current consumption and the price level in period 1 through the firms' optimal pricing decision. The definition of r_{K1} is given by

$$\begin{aligned} r_{K1} &= (1-\alpha)\frac{P_1 Y_1}{K} \\ &= (1-\alpha)\frac{P_0(1+\pi_1(Y_1))Y_1}{K} \end{aligned}$$

We first use the Phillips curve to define π_1 implicitly as a function of Y_1, with $\pi_Y > 0$. We then use the market clearing condition for final goods:

$$Y_1\left(1 - \frac{\theta}{2}\pi_1(Y_1)^2\right) = C_1 = \frac{C_2}{\beta R_2}$$

to show that Y_1 decreases in R_2 under one regularity condition. First note that C_2 is exogenous in the case where the financial constraint of intermediaries doesn't bind. Then the right-hand side of the equation above is decreasing in R_2. The left hand side increases in Y_1 if and only if $\theta\pi_1^2\left(\frac{1}{2} + \epsilon_{\pi/Y}\right)$ where $\epsilon_{\pi/Y}$ is the elasticity of inflation to output implied by the Phillips curve (22). since $\pi_1(Y_1)$ increases in Y_1. We can thus define a continuous and decreasing function $Y_1(R_2)$.

Under these regularity conditions on the Phillips curve, we have that an increase in R_2 leads to a decrease in $P_1 Y_1$. This in turn allows us to define a decreasing function $r_{K1}(R_2)$. Given that r_{K2} in increasing in P_2 and Y_2 is exogenous in the case where the financial constraint of intermediaries doesn't bind, and given the assumption that $P_2 = P_1$ and the result that P_1 is continuous and decreasing in R_2 we have that r_{K2} is continuous and decreasing in R_2.

$\mathcal{LHS}(N_0)$ is increasing and continuous in N_0. In addition, it is strictly negative under the assumption that $D_1 > 0$. Under the assumptions that $\frac{\phi-1}{\phi} > \max\left(\frac{K_{H1}}{K}, \frac{B_{H1}}{B}\right)$, $\mathcal{RHS}(R_2)$ is increasing and continuous in R_2. In addition, it converges to 0 for $R_2 \to +\infty$. By continuity, for all $N_0 > 0$ such that $D_1 > 0$, there exists a unique $\bar{R}_2$ such that $\mathcal{LHS}(N_0) = \mathcal{RHS}(\bar{R}_2)$. We can thus define a new function $\bar{R}_2(N_0)$. This function is strictly increasing in N_0. □

Optimal policy

Proof. We start from

$$\begin{aligned} W &= \max_{R_2} \left(1 - \frac{\chi}{\alpha}\right) \log Y_1 + \log\left(1 - \frac{\theta}{2}\pi_1^2\right) + \beta \log C_2 \\ \text{s.t.} \quad \theta\pi_1(\pi_1 + 1) &= (\epsilon_1 - 1)\left(\frac{\epsilon_1}{\epsilon_1 - 1}\frac{\underline{W}}{(1+\pi_1)P_0\alpha}\left(\frac{Y_1}{K}\right)^{\frac{1-\alpha}{\alpha}} - 1\right) \\ C_1 &= \frac{C_2}{\beta R_2} \end{aligned}$$

After substituting C_1 using the second constraint, and given that C_2 is unaffected by the policy rate since Y_2 is exogenous and the households doesn't hold any asset, we can drop C_2 from the definition of welfare. The problem simplifies to what is in the main text. The problem can thus be rewritten as

$$W = \max_{Y_1,\pi_1} \left(1 - \frac{\chi}{\alpha}\right) \log Y_1 + \log\left(1 - \frac{\theta}{2}\pi_1^2\right) + \beta\mathbb{E}C_2 \quad \text{s.t.} \quad 0 = \mathcal{PC}(Y_1, \pi_1)$$

Denoting λ the lagrange multipler, the associated FOCs are

$$\begin{aligned} \frac{\left(1 - \frac{\chi}{\alpha}\right)}{Y_1} + \lambda\mathcal{PC}_Y &= 0 \\ -\frac{\theta\pi_1}{\left(1 - \frac{\theta}{2}\pi_1^2\right)} + \lambda\mathcal{PC}_\pi &= 0 \Rightarrow \ \lambda = \frac{\theta\pi_1}{\left(1 - \frac{\theta}{2}\pi_1^2\right)\mathcal{PC}_\pi} \end{aligned}$$

Combining both equations gives

$$\frac{\left(1 - \frac{\chi}{\alpha}\right)}{Y_1} = -\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}\frac{\theta\pi_1}{\left(1 - \frac{\theta}{2}\pi_1^2\right)}$$

Using $C_2 = \beta R_2 C_1$ and the goods market condition we get

$$R_2 = -\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}\frac{\theta\pi_1}{\left(1 - \frac{\chi}{\alpha}\right)\left(1 - \frac{\theta}{2}\pi_1^2\right)^2}\frac{C_2}{\beta}.$$

□

C.2 Inside the "Constrained" Zone

We start by defining the constraint $\mathcal{BSC}$.

$$\begin{aligned}
\mathcal{BSC}(K_{H2}, K_G, \pi_1, Y_1, R_2, N_0, N_G) &= \mathcal{LHS}(N_0, N_G) - \mathcal{RHS}(K_{H2}, K_G, \pi_1, Y_1, R_2) \\
\text{with} \quad \mathcal{LHS}(N_0, N_G) &= \bar{R}_1 N_0 - \bar{R}_1 \left(Q_{K0}(K - K_{H1}) + Q_{L0}(L - L_{H1})\right) + N_G \\
\mathcal{RHS}(K_{H2}, K_G, \pi_1, Y_1, R_2) &= -r_{K1}(\pi_1, Y_1)(K - K_{H1}) - r_B(L - L_{H1}) \\
&\quad - \frac{1 + r_B - \beta_B \frac{P_2 L_{H2}}{L}}{R_2} \left(L - L_{H1} - \frac{L - L_{H2}}{\phi}\right) \\
&\quad - \frac{1 + r_{K2} - \beta_K \frac{P_2 K_{H2}}{K} - \beta_G \frac{K_G}{K}}{R_2} \left(K - K_{H1} - \frac{K - K_{H2} - K_G}{\phi}\right)
\end{aligned}$$

where $r_{K1}(\pi_1, Y_1) = (1 - \alpha)\frac{P_0(1+\pi_1)Y_1}{K}$. Replacing R_2 by its expression from the Euler equation as a function of C_1 and C_2, the constraint $\mathcal{BSC}$ is no longer a function of R_2:

$$\begin{aligned}
\mathcal{BSC}(K_{H2}, K_G, \pi_1, Y_1, N_0, N_G) &= \mathcal{LHS}(N_0, N_G) - \mathcal{RHS}(K_{H2}, K_G, \pi_1, Y_1) \\
\text{with} \quad \mathcal{LHS}(N_0, N_G) &= \bar{R}_1 N_0 - \bar{R}_1 \left(Q_{K0}(K - K_{H1}) + Q_{L0}(L - L_{H1})\right) + N_G \\
\mathcal{RHS}(K_{H2}, K_G, \pi_1, Y_1) &= -r_{K1}(\pi_1, Y_1)(K - K_{H1}) - r_B(L - L_{H1}) \\
-\frac{1 + r_B - \beta_B \frac{P_2 B_{H2}}{B}}{C_2(K_{H2}, K_G, N_G)} &\left(L - L_{H1} - \frac{B - B_{H2}}{\phi}\right) \beta Y_1 \left(1 - \frac{\theta}{2}\pi_1^2\right) \\
-\frac{1 + r_{K2} - \beta_K \frac{P_2 K_{H2}}{K}}{C_2(K_{H2}, K_G, N_G)} &\left(K - K_{H1} - \frac{K - K_{H2} - K_G}{\phi}\right) \beta Y_1 \left(1 - \frac{\theta}{2}\pi_1^2\right)
\end{aligned}$$

Note that $\mathcal{RHS}(K_{H2}, K_G, \pi_1, Y_1)$ is linear and decreasing in Y_1. It is also decreasing in π_1. Hence $\mathcal{BSC}(K_{H2}, K_G, \pi_1, Y_1, N_0, N_G)$ is increasing in Y_1 and in π_1.

Other tools not available.

Proof. The problem of policy-makers is given by

$$\begin{aligned}
W = \max_{Y_1, \pi_1, K_{H2}} &\left(1 - \frac{\chi}{\alpha}\right) \log Y_1 + \log\left(1 - \frac{\theta}{2}\pi_1^2\right) + \beta \ln C_2\left(K_{H2}, K_G, N_G\right) \\
\text{s.t.} \quad & 0 = \mathcal{PC}\left(Y_1, \pi_1\right) \\
& 0 = \mathcal{BSC}(K_{H2}, K_G, \pi_1, Y_1, N_0, N_G)
\end{aligned}$$

Denoting λ and μ the lagrange multipliers, the associated FOCs are

$$\frac{\left(1-\frac{\chi}{\alpha}\right)}{Y_1}+\lambda\mathcal{PC}_Y+\mu\mathcal{BSC}_Y=0$$

$$-\frac{\theta\pi_1}{\left(1-\frac{\theta}{2}\pi_1^2\right)}+\lambda\mathcal{PC}_\pi+\mu\mathcal{BSC}_\pi=0\Rightarrow\ \lambda=\frac{\frac{\theta\pi_1}{\left(1-\frac{\theta}{2}\pi_1^2\right)}-\mu\mathcal{BSC}_\pi}{\mathcal{PC}_\pi}$$

$$-\beta\frac{\beta_K K_{H2}}{C_2K}+\mu\mathcal{BSC}_{KH2}=0\Rightarrow\ \mu=\beta\frac{\beta_K K_{H2}}{C_2K\mathcal{BSC}_{KH2}}$$

Substituting for λ and μ in the first equation gives

$$\frac{\left(1-\frac{\chi}{\alpha}\right)}{Y_1}=-\frac{\mathcal{PC}_Y\theta\pi_1}{\left(1-\frac{\theta}{2}\pi_1^2\right)\mathcal{PC}_\pi}-\beta\frac{\beta_K K_{H2}\mathcal{BSC}_Y}{C_2K\mathcal{BSC}_{KH2}}\left(1+\frac{\mathcal{BSC}_\pi}{\mathcal{BSC}_Y}\frac{\mathcal{PC}_Y}{-\mathcal{PC}_\pi}\right)$$

Using $C_1=C_2/(\beta R_2)$ and the goods market condition to substitute for Y_1 gives the result:

$$R_2=\left[\underbrace{-\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}\frac{\theta\pi_1}{\left(1-\frac{\theta}{2}\pi_1^2\right)}\frac{C_2}{\beta}}_{\text{Baseline term}}-\underbrace{\beta_K}_{\text{Cost}}\underbrace{\frac{K_{H2}}{K}}_{\text{HH's holdings}}\underbrace{\frac{\mathcal{BSC}_Y}{\mathcal{BSC}_{K_{H2}}}\left(1+\frac{\mathcal{BSC}_\pi}{\mathcal{BSC}_Y}\frac{\mathcal{PC}_Y}{-\mathcal{PC}_\pi}\right)}_{\text{Sensitivity of balance sheet to }R_2}\right]\frac{1}{\left(1-\frac{\theta}{2}\pi_1^2\right)\left(1-\frac{\chi}{\alpha}\right)}$$

We then show how $\beta_K\frac{K_{H2}}{K}$ related to the wedge σ. From the FOC of households, we directly obtain

$$\sigma=1-\frac{R_2}{R_{k2}}=\frac{\beta_K P_2 K_{H2}}{K(1+r_{K2})}$$

Note that $\frac{\mathcal{BSC}_Y}{\mathcal{BSC}_{K_{H2}}}\left(1+\frac{\mathcal{BSC}_\pi}{\mathcal{BSC}_Y}\frac{\mathcal{PC}_Y}{-\mathcal{PC}_\pi}\right)$ is equal to the total derivative of K_{H2} to Y (total because it incorporates its effect through inflation) implied by the balance sheet constraint, $\frac{dK_{H2}}{dY}$. We can thus express the optimal interest rate as:

We can express $\frac{dK_{H2}}{dY_1}$ in terms of the sensitivity of the bank's asset to the interest rate:

$\frac{dK_{H2}}{dR_2}$ as follows

$$\frac{dK_{H2}}{dR_2} = \frac{dK_{H2}}{dY_1}\frac{dY_1}{dR_2}$$
$$\frac{dK_{H2}}{dR_2} = \frac{dK_{H2}}{dY_1}\frac{dY_1}{dC_1}\frac{dC_1}{dR_2}$$
$$\frac{dK_{H2}}{dR_2} = \frac{dK_{H2}}{dY_1}\frac{1}{\left(1-\frac{\theta}{2}\pi_1^2\right)}\left(\frac{-C_1}{R_2}\right)$$
$$= \frac{dK_{H2}}{dY_1}\left(\frac{-Y_1}{R_2}\right)$$

We end up with

$$\frac{dK_{H2}}{dY_1} = \frac{dK_{H2}}{dR_2}\left(\frac{-R_2}{Y_1}\right) \tag{51}$$

(52)

Substituting into the expression for R_2 we get:

$$R_2 = \underbrace{\bar{R}_2}_{\text{Baseline term}} - \frac{1+r_{K2}}{P_2}\frac{1}{\left(1-\frac{\theta}{2}\pi_1^2\right)\left(1-\frac{\chi}{\alpha}\right)}\frac{1}{Y_1}\underbrace{\sigma}_{\text{Wedge}}\underbrace{\frac{dK_{H2}}{dR_2}}_{\text{Sensitivity of bank intermediation to } R_2}\left(\frac{-R_2}{Y_1}\right)$$

which then gives

$$R_2 = \Omega\bar{R}_2$$
$$\text{with} \quad \Omega = \frac{1}{1+\Omega_0\sigma\left(-\frac{dK_{H2}}{dR_2}\right)} < 1$$
$$\Omega_0 = \frac{1+r_{K2}}{P_2}\frac{1}{\left(1-\frac{\theta}{2}\pi_1^2\right)\left(1-\frac{\chi}{\alpha}\right)}\frac{1}{Y_1}$$

We can also re-express the derivative $\frac{dK_{H2}}{dR_2}$ terms of the derivative of the wedge to the interest rate $\frac{d\sigma}{dR_2}$:

$$\frac{d\sigma}{dR_2} = \frac{d\sigma}{dK_{H2}} \frac{dK_{H2}}{dR_2}$$
$$\Rightarrow \frac{dK_{H2}}{dR_2} = \frac{\frac{d\sigma}{dR_2}}{\frac{d\sigma}{dK_{H2}}} = \frac{d\sigma}{dR_2} \frac{K(1 + r_{K2})}{\beta_K P_2}$$

□

Credit policy.

Proof. We now consider the choice of credit policy. The FOCs w.r.t to K_G and K_{H2} are given by:

$$-\beta \frac{\beta_K K_{H2}}{C_2 K} + \mu \mathcal{BSC}_{KH2} = 0 \Rightarrow \mu = \beta \frac{\beta_K K_{H2}}{C_2 K \mathcal{BSC}_{KH2}}$$
$$-\beta \frac{\beta_G K_G}{C_2 K} + \mu \mathcal{BSC}_{K_G} = 0$$

When $\beta_G > 0$, the optimal choice of other tools is given by combining these two FOCs, which gives

$$K_G = \underbrace{\frac{\beta_K}{\beta_G}}_{\text{Efficiency of CB intermediation}} \underbrace{K_{H2}}_{\text{HH holding}} \frac{\mathcal{BSC}_{K_G}}{\mathcal{BSC}_{K_H}}$$

When $\beta_G = 0$, the FOC with respect to K_G implies $\mu = 0$, which means that the balance sheet constraint of banks has no welfare cost, or in another words credit policy should intermediate capital up to the point where the banks balance sheet are no longer a constraint on intermediation. This in turn implies $K_{H2} = 0$ through the FOC for K_{H2}. From the FOC for the interest rate analyzed above we obtain the same optimal rate as the one outside the "constrained" zone. Finally, we use the constraint $\mathcal{BSC}$ to solve for the minimum level of K_G such that households hold no asset in equilibrium

$K_{H2} = B_{H2} = 0$:

$$\underline{K}_{G2} = \frac{\phi R_2}{1 + r_{K2}} \Bigg[-\bar{R}_1 N_0 + \bar{R}_1 \left(Q_{K0}(K - K_{H1}) + Q_{L0}(L - L_{H1}) \right) - r_{K1}(R_2)(K - K_{H1}) - r_B(L - L_{H1}) - \frac{1 + r_B}{R_2} \left(L - L_{H1} - \frac{L}{\phi} \right) - \frac{1 + r_{K2}}{R_2} \left(K - K_{H1} - \frac{K}{\phi} \right) \Bigg]$$

□

Equity injection When $\beta_G > 0$, the formula for the optimal rate is the same as for the case where other tools are not available shown before and the optimal choice of other tools is given by combining the FOC for K_{H2} and N_G, which gives

$$N_G = \underbrace{\frac{\beta_N}{\beta_G}}_{\text{Efficiency of CB intermediation}} \underbrace{\frac{K_{H2}}{K}}_{\text{HH holding}} \frac{\mathcal{BSC}_{N_G}}{\mathcal{BSC}_{K_H}}$$

Using the same reasoning as for credit policy, we use the constraint $\mathcal{BSC}$ to solve for the minimum level of N_G such that households hold no asset in equilibrium $K_{H2} = B_{H2} = 0$:

$$\underline{N}_G = \Bigg[-\bar{R}_1 N_0 + \bar{R}_1 \left(Q_{K0}(K - K_{H1}) + Q_{L0}(L - L_{H1}) \right) - r_{K1}(R_2)(K - K_{H1}) - r_B(L - L_{H1}) - \frac{1 + r_B}{R_2} \left(L - L_{H1} - \frac{L}{\phi} \right) - \frac{1 + r_{K2}}{R_2} \left(K - K_{H1} - \frac{K}{\phi} \right) \Bigg]$$

C.3 Inside the Run Zone

Using the Euler equation and the equality of R_2 in both states of the world, we obtain an additional constraint: $C_1/C_2 = C_1^*/C_2^*$. The problem of the social planner can be rewritten as

$$\begin{aligned} W = \max_{C_1, C_1^*, \pi_1, \pi_1^*, N_G, K_G, K_{H2}} & (1 - \xi) \left(\log C_1 - \chi \log \ell_1 + \beta \log C_2(K_G, N_G, K_{H2}) \right) \\ & + \xi \left(\log C_1^* - \chi \log \ell_1^* + \beta \log C_2^*(K_G, N_G, K_{H2}) \right) \\ \text{s.t.} \quad & 0 = \mathcal{PC}(Y_1, \pi_1) = \mathcal{PC}\left(Y_1^*, \pi_1^*\right) \quad \text{and} \quad C_1/C_2 = C_1^*/C_2^* \end{aligned}$$

with $\xi = \xi\left(Y_1^*, \pi_1^*, \frac{C_2^*}{\beta C_1^*}, K_G, N_G, K_{H2}\right)$

Denoting μ the lagrange multiplier associated with the third constraint, the problem can be rewritten as

$$
\begin{aligned}
W = \max_{C_1, C_1^*, \pi_1, \pi_1^*, N_G, K_G, K_{H2}} & (1-\xi+\mu)\log C_1 - (1-\xi)\log \ell_1 + [(1-\xi)\beta - \mu]\log C_2(K_G, N_G, K_{H2}) \\
& + (\xi-\mu)\log C_1^* - \xi \log \ell_1^* + (\xi\beta+\mu)\log C_2^*(K_G, N_G, K_{H2}) \\
\text{s.t.} \quad 0 = \mathcal{PC}(Y_1, \pi_1) & = \mathcal{PC}\left(Y_1^*, \pi_1^*\right)
\end{aligned}
$$

Denoting λ, λ^* the lagrange multipliers associated with the two Phillips curve constraints, the FOCs are

$$
\begin{aligned}
\frac{(1-\xi+\mu)}{Y_1} - \frac{\chi}{\alpha}\frac{(1-\xi)}{Y_1} + \lambda \mathcal{PC}_Y(Y_1, \pi_1) &= 0 \\
(1-\xi+\mu)\frac{-\theta\pi_1}{\left(1-\frac{\theta}{2}(\pi)^2\right)} + \lambda \mathcal{PC}_\pi(Y_1, \pi_1) &= 0 \\
\frac{(\xi-\mu)}{Y_1^*} - \frac{\chi}{\alpha}\frac{\xi}{Y_1^*} + \lambda^* \mathcal{PC}_{Y^*}(Y_1^*, \pi_1^*) + \xi_{Y^*}\left[\log\frac{C_1^*}{C_1} + \beta\log\frac{C_2^*}{C_2}\right] &= 0 \\
(\xi-\mu)\frac{-\theta\pi_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)} + \lambda^* \mathcal{PC}_{\pi^*}(Y_1^*, \pi_1^*) + \xi_{\pi^*}\left[\log\frac{C_1^*}{C_1} + \beta\log\frac{C_2^*}{C_2}\right] &= 0 \\
\xi_{N_G}\beta\log C_2^*/C_2 &= \beta_N N_G\left(\frac{\beta(1-\xi)-\mu}{C_2} + \frac{\xi\beta+\mu}{C_2^*}\right) \\
\xi_{K_G}\beta\log C_2^*/C_2 &= \frac{\beta_G K_G}{K}\left(\frac{\beta(1-\xi)-\mu}{C_2} + \frac{\xi\beta+\mu}{C_2^*}\right) \\
\xi_{K_{H2}}\beta\log C_2^*/C_2 &= \frac{\beta_K K_{H2}}{K}\left(\frac{\beta(1-\xi)-\mu}{C_2} + \frac{\xi\beta+\mu}{C_2^*}\right) \\
0 = \mathcal{PC}(Y_1, \pi_1) &= \mathcal{PC}\left(Y_1^*, \pi_1^*\right) \\
\log C_1/C_1^* &= \log C_2/C_2^*
\end{aligned}
$$

We use the second and fourth equations we can solve for λ and λ^* to substitute back into the first and third equations. We can also take the ratio of the fifth and sixth equations. This gives

$$
\begin{aligned}
(1-\xi+\mu)-\frac{\chi}{\alpha}(1-\xi) &= (1-\xi+\mu)\frac{-\theta\pi_1 Y_1}{\left(1-\frac{\theta}{2}(\pi)^2\right)}\frac{\mathcal{PC}_Y(Y_1,\pi_1)}{\mathcal{PC}_\pi(Y_1,\pi_1)} \\
\xi-\mu-\frac{\chi}{\alpha}\xi+\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2} &= \frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left((\xi-\mu)\frac{-\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right) \\
\xi_{N_G}/\xi_{K_G} &= \frac{K\beta_N N_G}{\beta_G K_G} \\
\xi_{K_{H2}}/\xi_{K_G} &= \frac{K\beta_K K_{H2}}{\beta_G K_G} \\
0 &= \mathcal{PC}(Y_1,\pi_1) = \mathcal{PC}\left(Y_1^*,\pi_1^*\right) \\
\xi_{K_G}\beta\log C_2^*/C_2 &= \frac{\beta_G K_G}{K}\left(\frac{\beta(1-\xi)}{C_2}+\frac{\xi\beta}{C_2^*}\right)+\frac{\beta_G K_G}{K}\left(\frac{1}{C_2^*}-\frac{1}{C_2}\right)\mu \\
\log C_1/C_1^* &= \log C_2/C_2^*
\end{aligned}
$$

We now use the second equation of the previous system to solve for $\xi-\mu$ and substitute back into the first equation. We also use this expression to substitute out the Lagrange multiplier μ in the fifth equation.

$$
\begin{aligned}
&\left(1-\frac{\frac{\chi}{\alpha}\xi-\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}\right)-\frac{\chi}{\alpha}(1-\xi) = \\
&\qquad\left(1-\frac{\frac{\chi}{\alpha}\xi-\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}\right)\frac{-\theta\pi_1 Y_1}{\left(1-\frac{\theta}{2}(\pi)^2\right)}\frac{\mathcal{PC}_Y(Y_1,\pi_1)}{\mathcal{PC}_\pi(Y_1,\pi_1)} \\
&\xi-\mu = \frac{\frac{\chi}{\alpha}\xi-\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)} \\
&\xi_{K_G}\beta\log C_2^*/C_2 = \frac{\beta_G K_G}{K}\left(\frac{\beta(1-\xi)}{C_2}+\frac{\xi\beta}{C_2^*}+\left(\frac{1}{C_2^*}-\frac{1}{C_2}\right)\left(\xi-\frac{\frac{\chi}{\alpha}\xi-\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}\right)\right) \\
&\xi_{K_{H2}}/\xi_{K_G} = \frac{\beta_N N_G}{K\beta_G K_G}
\end{aligned}
$$

In the first equation below, we next factorize by the term inside brackets, and then multiply both sides by $\left(1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^*Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)\right)$. In the fourth equation below we factorize by $\left(\frac{1}{C_2^*}-\frac{1}{C_2}\right)$:

$$\left(1+\frac{\theta\pi_1Y_1}{\left(1-\frac{\theta}{2}(\pi)^2\right)}\frac{\mathcal{PC}_Y(Y_1,\pi_1)}{\mathcal{PC}_\pi(Y_1,\pi_1)}\right)\left(1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^*Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)-\frac{\chi}{\alpha}\xi+\xi'Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}\right)$$

$$=\left(1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^*Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)\right)\frac{\chi}{\alpha}(1-\xi)$$

$$\xi-\mu=\frac{\frac{\chi}{\alpha}\xi-\xi'Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^*Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}$$

$$\xi_{K_G}\beta\log C_2^*/C_2=\frac{\beta_GK_G}{K}\left(\frac{\beta}{C_2}+\left(\frac{1}{C_2^*}-\frac{1}{C_2}\right)\left(\beta\xi+\xi-\frac{\frac{\chi}{\alpha}\xi-\xi'Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^*Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}\right)\right)$$

$$\xi_{K_{H2}}/\xi_{K_G}=\frac{\beta_NN_G}{K\beta_GK_G}$$

Dividing both sides of the first equation by $\left(1+\frac{\theta\pi_1Y_1}{\left(1-\frac{\theta}{2}(\pi)^2\right)}\frac{\mathcal{PC}_Y(Y_1,\pi_1)}{\mathcal{PC}_\pi(Y_1,\pi_1)}\right)$ and $\left(1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^*Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)\right)$ gives

$$1+\frac{-\frac{\chi}{\alpha}\xi+\xi'Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{\left(1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^*Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)\right)}=\frac{\frac{\chi}{\alpha}(1-\xi)}{\left(1+\frac{\theta\pi_1Y_1}{\left(1-\frac{\theta}{2}(\pi)^2\right)}\frac{\mathcal{PC}_Y(Y_1,\pi_1)}{\mathcal{PC}_\pi(Y_1,\pi_1)}\right)}$$

Gathering all the terms on the right-hand side and using market clearing $Y_1=\frac{C_1}{1-\frac{\theta}{2}\pi_1^2}=\frac{C_2}{\beta R_2\left(1-\frac{\theta}{2}\pi_1^2\right)}$,

$$1=\frac{R_2\frac{\chi}{\alpha}(1-\xi)}{\left(R_2-\frac{\theta\pi_1C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}\right)}+\frac{R_2\frac{\chi}{\alpha}\xi-\xi'(1+\beta)\frac{C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\log\frac{C_2^*}{C_2}}{\left(R_2-\frac{\theta\pi_1^*C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\right)}$$

Multiplying both sides by $R_2 - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_Y}{\mathcal{P}\mathcal{C}_\pi}$

$$R_2 - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_Y}{\mathcal{P}\mathcal{C}_\pi} = R_2\frac{\chi}{\alpha}(1-\xi) + \left(R_2\frac{\chi}{\alpha}\xi - \xi'(1+\beta)\frac{C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\log\frac{C_2^*}{C_2}\right)\frac{R_2 - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_Y}{\mathcal{P}\mathcal{C}_\pi}}{\left(R_2 - \frac{\theta\pi_1^* C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_{Y^*}}{\mathcal{P}\mathcal{C}_{\pi^*}}\right)}$$

Moving $R_2\frac{\chi}{\alpha}$ from the right to the left hand side and having a common denominator for both terms $R_2\xi\frac{\chi}{\alpha}$:

$$R_2\left(1-\frac{\chi}{\alpha}\right) - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_Y}{\mathcal{P}\mathcal{C}_\pi} = \frac{R_2\frac{\chi}{\alpha}\xi\left(\frac{\theta\pi_1^* C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_{Y^*}}{\mathcal{P}\mathcal{C}_{\pi^*}} - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_Y}{\mathcal{P}\mathcal{C}_\pi}\right) - \xi'(1+\beta)\frac{C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\log\frac{C_2^*}{C_2}}{\left(R_2 - \frac{\theta\pi_1^* C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_{Y^*}}{\mathcal{P}\mathcal{C}_{\pi^*}}\right)}$$

Factorizing the entire fraction on the right hand side by $\xi\frac{\chi}{\alpha}$

$$R_2\left(1-\frac{\chi}{\alpha}\right) - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_Y}{\mathcal{P}\mathcal{C}_\pi} = \xi\frac{\chi}{\alpha}\frac{R_2\left(\frac{\theta\pi_1^* C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_{Y^*}}{\mathcal{P}\mathcal{C}_{\pi^*}} - \frac{\theta\pi_1 C_2}{\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_Y}{\mathcal{P}\mathcal{C}_\pi}\right) - \frac{\xi'\alpha}{\xi\chi}(1+\beta)\frac{C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\log\frac{C_2^*}{C_2}}{\left(R_2 - \frac{\theta\pi_1^* C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_{Y^*}}{\mathcal{P}\mathcal{C}_{\pi^*}}\right)}$$

Dividing both sides by $\left(1-\frac{\chi}{\alpha}\right)$

$$R_2 - \frac{\theta\pi_1 C_2}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_Y}{\mathcal{P}\mathcal{C}_\pi} = \xi\frac{\chi}{\alpha}\frac{R_2\left(\frac{\theta\pi_1^* C_2^*}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_{Y^*}}{\mathcal{P}\mathcal{C}_{\pi^*}} - \frac{\theta\pi_1 C_2}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_Y}{\mathcal{P}\mathcal{C}_\pi}\right) - \frac{\xi'\alpha}{\xi\chi}\frac{(1+\beta)}{(1-\frac{\chi}{\alpha})}\frac{C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\log\frac{C_2^*}{C_2}}{\left(R_2 - \frac{\theta\pi_1^* C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{P}\mathcal{C}_{Y^*}}{\mathcal{P}\mathcal{C}_{\pi^*}}\right)}$$

Denoting the shadow interest rate in the good and in the run equilibrium respectively $\bar{R}_2 = \frac{\theta\pi_1 C_2}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}$ and $\underline{R}_2 = \frac{\theta\pi_1^* C_2^*}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}$ we obtain

$$R_2 - \bar{R}_2 = \xi\frac{\chi}{\alpha}\frac{R_2\left(\underline{R}_2 - \bar{R}_2\right) - \frac{\xi'\alpha}{\xi\chi}\frac{(1+\beta)}{(1-\frac{\chi}{\alpha})}\frac{C_2^*}{\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\log\frac{C_2^*}{C_2}}{R_2 - (1-\frac{\chi}{\alpha})\underline{R}_2}$$

Dividing the numerator and the denominator by R_2 gives

$$R_2 = \bar{R}_2 + \xi\frac{\chi}{\alpha}\frac{\underline{R}_2 - \bar{R}_2 - \frac{\xi'\alpha}{\xi\chi}\frac{(1+\beta)}{(1-\frac{\chi}{\alpha})}Y_1^*\log\frac{C_2^*}{C_2}}{1-(1-\frac{\chi}{\alpha})\frac{\underline{R}_2}{R_2}}$$

We can rewrite this as follows:

$$\begin{aligned}
R_2 &= \bar{R}_2 - \xi\Omega_1\left(\bar{R}_2 - \underline{R}_2\right) + \xi'\Omega_2\log\frac{C_2^*}{C_2} \\
\text{with}\quad \bar{R}_2 &= \frac{\theta\pi_1 C_2}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi)^2\right)^2}\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi} \\
\underline{R}_2 &= \frac{\theta\pi_1^* C_2^*}{(1-\frac{\chi}{\alpha})\beta\left(1-\frac{\theta}{2}(\pi^*)^2\right)^2}\frac{-\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}} \\
\Omega_2 &= \frac{(1+\beta)}{(1-\frac{\chi}{\alpha})}Y_1^*\frac{1}{1-(1-\frac{\chi}{\alpha})\frac{\underline{R}_2}{R_2}} \\
\Omega_1 &= \frac{\frac{\chi}{\alpha}}{1-(1-\frac{\chi}{\alpha})\frac{\underline{R}_2}{R_2}} \\
\xi' &= \xi_Y + \xi_\pi\frac{-\mathcal{PC}_Y}{\mathcal{PC}_\pi}
\end{aligned}$$

Recall that the FOC for credit policy is

$$\xi_{K_G}\beta\log C_2^*/C_2 = \frac{\beta_G K_G}{K}\left(\frac{\beta}{C_2} + \left(\frac{1}{C_2^*} - \frac{1}{C_2}\right)\left(\beta\xi + \xi - \frac{\frac{\chi}{\alpha}\xi - \xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2}}{1+\frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}\right)\right).$$

Rearranging gives:

$$\frac{K_G}{K} = \Omega_3 \frac{\xi_{K_G}}{\beta_G} \log C_2^*/C_2$$

$$\Omega_3 = \frac{\beta C_2}{\beta + \left(\frac{C_2}{C_2^*} - 1\right)\left(\xi(1+\beta) + \frac{\xi' Y_1^*(1+\beta)\log\frac{C_2^*}{C_2} - \frac{\chi}{\alpha}\xi}{1 + \frac{\mathcal{PC}_{Y^*}}{\mathcal{PC}_{\pi^*}}\left(\frac{\theta\pi_1^* Y_1^*}{\left(1-\frac{\theta}{2}(\pi^*)^2\right)}\right)}\right)}$$

We can also express the optimal equity injection as a function of the households purchases of capital:

$$\xi_{N_G}/\xi_{K_G} = \frac{K\beta_N N_G}{\beta_G K_G}$$

Similarly we can get the optimal level of asset purchases:

$$\xi_{K_{H2}}/\xi_{K_G} = \frac{K\beta_K K_{H2}}{\beta_G K_G}$$

C.4 Large Financial Crisis

In this Appendix, we consider a large financial crisis which we model as a case in which the run happens with probability one, $\xi = 1$. In equilibrium, banks lose all their net worth $N_1 = 0$, and absent government's interventions all the capital is intermediated by households which results in a drop in consumption in period 2, and hence in period 1.

Given that the run, the implied disruption of financial intermediation and the large drop in output occur independently of the stance of monetary policy, the only trade-off the central bank faces is between inflation stabilization and preserving output - the same one it faced outside of the constrained and of the run zones.

Lemma 10 (Large crisis - interest rate policy)**.** *When the other tools are costly $\beta_K, \beta_N > 0$, the optimal rate is given by*

$$R_2 = -\frac{\theta\pi_1^*}{\left(1 - \frac{\theta}{2}(\pi_1^*)^2\right)\left(1 - \frac{\chi}{\alpha}\right)} \frac{\mathcal{PC}_Y^*}{\mathcal{PC}_\pi^*} C_2^*$$

Proof. Lemma 10 is just a version of Lemma 3 since the problem of the CB is to maximize welfare subject to no constraint. The only difference is that, by assumption, households hold all assets. The definition of C_2^* is also a direct implication of the assumption that households hold all assets.

□

Two effects push the optimal rate in opposite directions. Higher inflation pushes the central bank to increase its rate, which is captured by the term $\frac{\mathcal{PC}_Y^*}{\mathcal{PC}_\pi^*}$. But the large financial crisis also leads to a drop in C_2^*, which pushes the central bank to decrease its rate. If the latter effect is stronger than the former, the central bank should decrease its rate. The drop in C_2^* depends on the use of other tools. In the extreme case where tools are prohibitively costly and are not used, the drop is largest and given by $C_2^* = Y_2 + K - \frac{\beta_K}{2}K - \frac{\beta_B}{2}L$. In this case, the central bank may have to cut rates despite the cost push shock.

When credit policy are available, they should be used to intermediate part of the private assets in the economy and minimize the drop in C_2^*.

Lemma 11 (Large crisis - other tools)**.** *Optimal credit policy and the implied C_2^* are given by*

$$K_G = \frac{\beta_K}{\beta_G + \beta_K}K$$
$$C_2^* = Y_2 + K - \frac{\beta_K \beta_G}{(\beta_K + \beta_G)2}K - \frac{\beta_B}{2}L$$

Full separation: In the case where $\beta_G = 0$, credit policy can fully address financial disruptions:

$$K_G = K$$
$$C_2^* = Y_2 + K$$

Proof. In Lemma 11, the optimal interest rate is given by the same first-order condition as in Lemma 10. The optimal asset purchase stems from taking the derivative of $C_2 = Y_2 + K - \frac{\beta_B}{2}B - \left(\frac{\beta_K}{2}\frac{K-K_G}{K}\right)(K - K_G) - \left(\frac{\beta_G}{2}\frac{K_G}{K}\right)K_G$ with respect to K_G and equalizing it to 0. □

Intuitively, the more efficient the government is at intermediating relative to households β_K/β_G the higher the share of private assets it should hold. In the extreme case where the government can intermediate assets without any cost $\beta_G = 0$, then it should hold all assets in the economy $K_G = K$. In addition, if there is no government debt $L = 0$, consumption can be fully stabilized, and the central bank faces only the inflation/output trade-off (and should raise the interest rate). This is a case of full separation.

D Strict Inflation targeting

Baseline

Lemma 12. *The optimal interest rate is such that*

$$\mathcal{PC}\left(\frac{C_2}{\beta R_2\left(1-\frac{\theta}{2}\bar{\pi}^2\right)}, \bar{\pi}\right) = 0 \quad with \quad C_2 = Y_2 + K \tag{53}$$

The higher the markup shock $\frac{\epsilon_1}{\epsilon_1 - 1}$, the steeper the Phillips curve $-\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}$ and the higher future consumption C_2, the higher the optimal interest rate.

Inside the constrained zone. We now derive the combination of interest rate policy and other tools when the economy is in the constrained zone. The central bank strictly targets inflation and the other part of the government chooses other tools to maximize households welfare. The following lemma formalizes the result.

Lemma 13. *The optimal interest rate is lower in the "constrained" zone than outside and strictly decreasing in K_{H2} and B_{H2}. It is given by*

$$\mathcal{PC}\left(\frac{C_2}{\beta R_2\left(1-\frac{\theta}{2}\bar{\pi}^2\right)}, \bar{\pi}\right) = 0$$

$$C_2 = Y_2 + K - \left(\frac{\beta_B}{2}\frac{P_2 B_{H2}}{B}\right) B_{H2} - \left(\frac{\beta_K}{2}\frac{P_2 K_{H2}}{K}\right) K_{H2} - \left(\frac{\beta_G}{2}\frac{K_G}{K}\right) K_G$$

$$K_G = \underbrace{\frac{\beta_K}{\beta_G}}_{\text{Efficiency of CB intermediation}} \underbrace{K_{H2}}_{\text{HH holding}} \frac{\mathcal{BSC}_{K_G}}{\mathcal{BSC}_{K_H}}$$

The stronger the cost-push shock $\frac{\epsilon_1}{\epsilon_1 - 1}$, the steeper the Phillips curve $-\frac{\mathcal{PC}_Y}{\mathcal{PC}_\pi}$ and the higher future consumption C_2, the higher the optimal interest rate.

Inside the run zone. Given that the inflation rate is random, the central bank cannot achieve perfect stabilization, it can only minimize average deviation from target. The objective is to minimize the expected squared deviations from target

$$\max_{R_2} -(1-\xi)\frac{(\pi_1 - \bar{\pi})^2}{2} - \xi\frac{(\pi_1^* - \bar{\pi})^2}{2} \quad s.t. \quad \mathcal{PC}(C_1, \pi_1) = 0 \quad \text{and} \quad \mathcal{PC}\left(C_1 \frac{C_2^*}{C_2}, \pi_1^*\right) = 0$$

The FOCs are given by

$$(1-\xi)(\pi_1-\bar{\pi}) = \lambda \mathcal{PC}_\pi$$
$$\xi(\pi_1^*-\bar{\pi}) = \lambda^* \mathcal{PC}_\pi^*$$
$$\lambda \mathcal{PC}_Y + \frac{C_2^*}{C_2}\lambda^* \mathcal{PC}_Y^* = 0$$

Substituting for the lagrange multipliers gives

$$(1-\xi)\frac{\pi_1-\bar{\pi}}{\mathcal{PC}_\pi}C_2\mathcal{PC}_Y + \xi\frac{\pi_1^*-\bar{\pi}}{\mathcal{PC}_\pi^*}C_2^*\mathcal{PC}_Y^* = 0$$

This FOC implies that in a run period, inflation is below target, and in a no zone situation it is above target. The interest rate must thus be below its level without the risk of a run.

When other tools are available, they can be used to decrease ξ and increase C_2^*, which then allows the central bank to raise its interest rate to stabilize inflation.

Large financial crisis. The results are qualitatively similar to those inside the "run" zone: when the central bank uses only the interest rate, the rate that stabilizes inflation is lower than the one prevailing outside of the run and of constrained zones. But quantitatively the rate is lower than in both zones given the larger drop in consumption implied by the large financial crisis, $C_2 = Y_2 + K - \frac{\beta_B}{2}L - \left(\frac{\beta_K}{2}\frac{K-K_G}{K}\right)(K-K_G) - \left(\frac{\beta_G}{2}\frac{K_G}{K}\right)K_G$

When governments use credit policy and $\beta_G = 0$ and $B = 0$ households don't hold any assets, $C_2 = Y_2 + K$ which implies that the policy rate is the same as the one prevailing outside of the constrained and run zones. Otherwise, when $\beta_G > 0$, or $L > 0$, or both, credit policy help offset the drop in consumption in period 2 and the optimal policy rate can be higher than in the case when only interest rate policy are used.

E Empirical relationship between monetary policy and financial instability

This appendix presents empirical evidence of the link between monetary policy and financial instability. In particular, we show that monetary policy tightening can exacerbate financial stability risks, particularly when fluctuations are driven by supply shocks. We also find that rate hikes increase the likelihood of both equity crashes and banking panics, indicating that both intermediary constraints and bank runs are important sources of financial instability. Moreover, rate hikes predict lower real stock prices, house prices, and bank lending, suggesting that these are important channels through which monetary policy impact financial stability risk.

E.1 Methodology.

Our baseline specification for evaluating the systematic impact of monetary policy on financial stability is given by:

$$C_{i,t+h} = \alpha_{i,h} + \beta_h \times \Delta r_{i,t} + \sum_{l=0}^{L} \mathbf{\Gamma}_{h,l} \mathbf{X}_{i,t-l} + \epsilon_{i,t+h} \tag{54}$$

where $C_{i,t+h}$ is indicates whether country i experienced a financial crisis in year t or in any of the following two years, $\alpha_{i,h}$ are country fixed effects and $\Delta r_{i,t}$ is the change in nominal short-term interest rates, which measure yields on three-month government securities and money market rates

The set of control variables $\mathbf{X}_{i,t}$ include four lags of the following variables: per capita real GDP growth, per capita real consumption growth, per capital real investment growth, CPI inflation, world GDP growth, changes in short-term and long-term interest rates, growth in real stock prices, real house prices, and real bank loans, the current account-to-GDP ratio, as well as the crisis dummy. The set of controls include contemporaneous values of these variables, except for the crisis dummy, and changes in short and long-term interest rates.

The rich set of controls aims to hold fix other channels that explain the correlation between short-term rates and incidence of financial crisis. For example, the growth rate in real stock prices, real house prices, and real bank loans, control for the role of risky credit build-ups, which could explain the tightening of short-term rates and subsequent rise in financial crisis risk. Similarly, real per capita GDP growth and real per capita consumption growth hold fix differences in real economic activity that could explain both changes in the policy rate and the probability of a financial crisis.

Another source of endogeneity concern is the possibility that central banks internalize financial stability risk when setting the policy rate, which would lead to a downward bias on the impact of rate hikes on financial stress. To this end, we instrument changes in nominal rates with the Trilemma instrument from Jordà et al. (2017). The instrument is based on the economic intuition that, under perfect capital mobility, maintaining an exchange rate peg requires a country to adjust their domestic interest rates to match those of the base country's. Changes in the country's interest rates would thus be plausibly exogenous to local economic and financial conditions.

The trilemma IV from Jordà et al. (2017) is given by

$$z_{i,t} \equiv \left(\Delta r_{b(i,t),i,t} - \Delta \hat{r}_{b(i,t),i,t}\right) \times PEG_{i,t} \times PEG_{i,t-1} \times KOPEN_{i,t}$$

where $r_{b(i,t),i,t}$ is short-term nominal rate of the base country for country i in period t, $\hat{r}_{b(i,t),i,t}$ is the predicted value of interest rates based on macroeconomic observables, $PEG_{i,t}$ is an indicator for whether the country's currency is fixed with respect to base b, and $KOPEN_{i,t}$ is an index of financial openness. The effect of monetary policy tightening on financial crisis is estimated using the LP-IV approach.[21] This involves estimating specification (54), but instrumenting $\Delta r_{i,t}$ with $z_{i,t}$.

E.2 Estimation of supply and demand shocks

To formalize the notion of supply and demand shocks, consider the following system of equations:

$$\text{Aggregate Demand: } \tilde{y}_t = -\alpha \pi_t + d_t$$
$$\text{Aggregate Supply: } \pi_t = \beta \tilde{y}_t + \eta_t$$

where $\tilde{y}_t$ is the output gap and π_t is the inflation rate, and $\alpha, \beta > 0$, and d_t and η_t are aggregate demand and supply shocks. This system can be micro-founded in a standard three-equation NK model, assuming that structural shocks are zero-mean white noise (Jump and Kohler, 2022). Aggregate demand implies a negative relationship between $(\tilde{y}_t, \pi_t)$. On the other hand, aggregate supply implies a positive relationship between $(\tilde{y}_t, \pi_t)$.

Re-arranging the system of equation yields the following:

$$\tilde{y}_t = \frac{1}{1+\alpha\beta}\left(d_t - \alpha\eta_t\right)$$
$$\pi_t = \frac{1}{1+\alpha\beta}\left(\beta d_t + \eta_t\right)$$

[21] We prefer linear probability models given that it allows for including a larger set of controls in the specification, particularly discrete time and country fixed effects.

We can see that the demand shock d_t moves output gap and inflation in the same direction, whereas the supply shock η_t moves output gap and inflation in the opposite direction.[22]

To bring this model to data, we first express the system of equation as an SVAR. We proxy the output gap with the change in real GDP Δy_t and inflation with the change in CPI. The SVAR specification is given by

$$Az_t = \sum_{j=1}^{p} A_j z_{t-j} + \epsilon_t \tag{55}$$

where

$$A = \begin{bmatrix} 1 & \alpha \\ -\beta & 1 \end{bmatrix}, z_t = \begin{bmatrix} \Delta y_t \\ \pi_t \end{bmatrix}, \epsilon_t = \begin{bmatrix} d_t \\ \eta_t \end{bmatrix}$$

The relationship between reduced form residuals $v_t = [v_t^y, v_t^\pi]'$ and the structural shocks $\epsilon_t = [d_t, \eta_t]'$ is given by

$$v_t \equiv z_t - E\left[z_t | z_{t-1}, ..., z_{t-p}\right] = A^{-1}\epsilon_t$$

Jump and Kohler (2022) show that the restrictions on the slope of supply and demand curves imply the following restrictions on the signs of the reduced form shocks:

Positive demand shock	$d_t > 0$	$\rightarrow v_t^y > 0, v_t^\pi > 0$
Negative demand shock	$d_t < 0$	$\rightarrow v_t^y < 0, v_t^\pi < 0$
Positive supply shock	$\eta_t > 0$	$\rightarrow v_t^y > 0, v_t^\pi < 0$
Negative supply shock	$\eta_t < 0$	$\rightarrow v_t^y < 0, v_t^\pi > 0$

To estimate the demand and supply shocks, we use data on real GDP and CPI inflation rate from the Jorda-Schularick-Taylor Macrohistory Database (Jordà et al., 2017), which provides annual data on real and financial sector variables for 18 advanced economies from 1870 to 2016. The countries included in the sample are Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

[22] Demand shocks include discount rate factor shocks, fiscal spending shocks, and monetary policy shocks, whereas supply shocks include cost-push shocks and produuctivity shocks. The negative impact of cost-push shocks on output relies on the reaction of interest rate policy to inflation, which is likely to be muted (but still not zero) when the exchange rate is pegged.

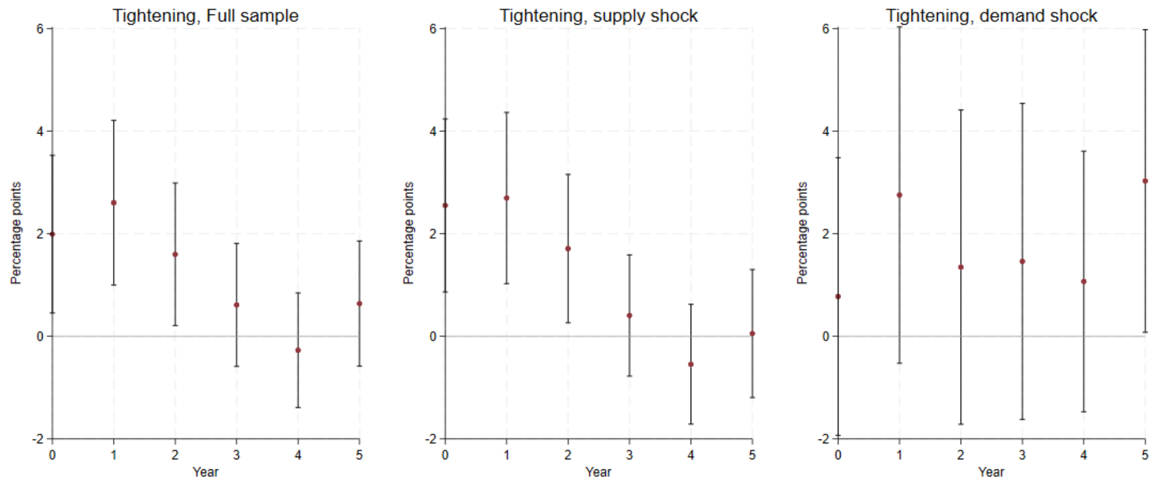

Figure 7: Annual probability of financial crisis following a one percentage point increase in short-term nominal rates. Solid bars denote 90 percent confidence intervals. The full sample depicts the unconditional effect. The other two panels show the effect of a tightening in period t conditional on a supply or demand shock in period $t-1$.

The reduced form specification is given by

$$\Delta y_t = \delta_i^y + \gamma_{d(t)}^y + \sum_{j=1}^{L} \sum_{\tau}^{d(T)} \beta_{j,\tau}^{yy} y_{t-j} \times 1\{d(t) = \tau\} + \sum_{j=1}^{L} \sum_{\tau}^{d(T)} \beta_{j,\tau}^{y\pi} \pi_{t-j} \times 1\{d(t) = \tau\} + v_t^y \tag{56}$$

$$\pi_t = \delta_i^\pi + \gamma_{d(t)}^\pi + \sum_{j=1}^{L} \sum_{\tau}^{d(T)} \beta_{j,\tau}^{\pi y} y_{t-j} \times 1\{d(t) = \tau\} + \sum_{j=1}^{L} \sum_{\tau}^{d(T)} \beta_{j,\tau}^{\pi\pi} \pi_{t-j} \times 1\{d(t) = \tau\} + v_t^\pi$$

where $(\delta_i^y, \delta_i^\pi)$ are country fixed effects, $(\gamma_{d(t)}^y, \gamma_{d(t)}^\pi)$ are decade fixed effects. By interacting the lagged variables by decade indicators, the specification allows for the auto-regressive coefficients to vary over time. This accommodates structural changes in the relationship between inflation and real GDP growth over the long sample period. The number of distributive lags in the reduced form VAR is set to 4.

E.3 Impact of monetary policy on financial instability

The second and third panels show the impact of rate hikes in periods following supply shocks versus demand shocks. We see that the effect of rate hikes are primarily observed in the periods following supply shocks. The magnitude of the impact is similar to the

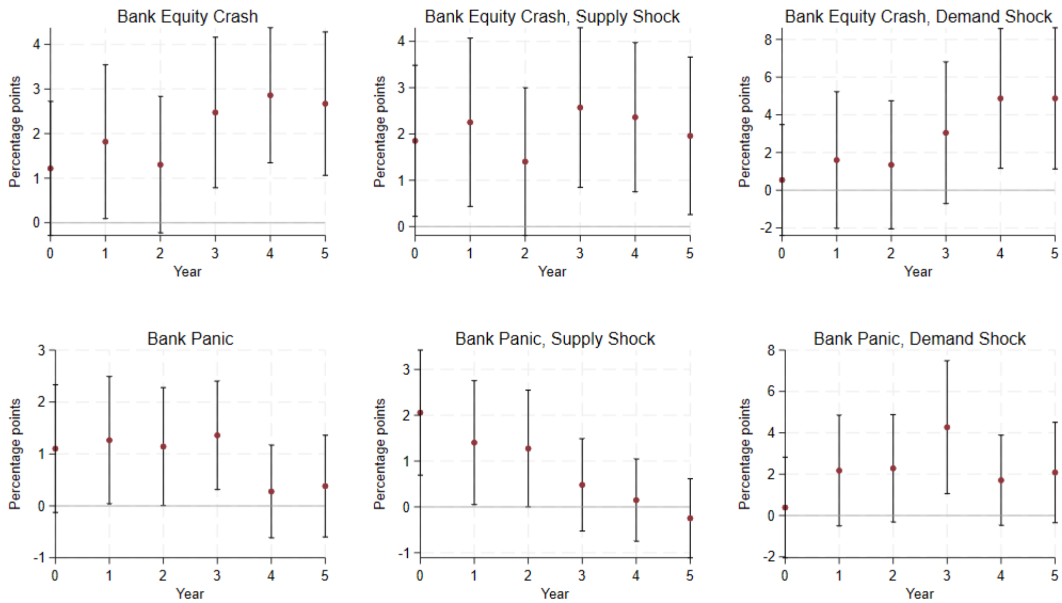

Figure 8: Annual probability of bank equity crashes (top panel) and banking panics (bottom panel) following a one percentage point increase in short-term nominal rates. Solid bars denote 90 percent confidence intervals. The second and third columns show the effect of a tightening and loosening in period t conditional on a supply or demand shock in period $t-1$.

unconditional sample, with financial crisis risk peaking at 2.5 percent one year after the rate hike. On the other hand, in the periods following demand shocks, financial crisis risk rises but is largely insignificant.[23]

E.4 Channels of transmission

We now investigate the potential channels through which monetary policy affects financial stability. First, we show that monetary policy tightening leads to both heightened risk of bank equity crashes and banking panics. The significance of these two channels suggest important roles of intermediary capacity constraints and the risk of depositor runs as sources of financial instability. Second, we show that tightening leads to large declines in real stock prices, house prices, and bank credit. The impacts of rate hikes

[23] This finding complements those from Boissay et al. (2023), who document that rate hikes exacerbate financial stress in the presence of supply-driven inflation, whereas it dampens financial stress following demand-driven inflation. While we do not find that rate hikes lead to lower financial crisis risk following demand-driven shocks, our findings indicate that monetary policy has significant financial stability implications following supply-driven shocks.

on house prices and bank credit are not reversed even five years after the initial hike. These findings highlight that monetary policy can have negative implications for asset prices and bank profitability, which in turn matter for financial stability.

Equity crashes and banking panics – Figure 8 shows the impact of a one percent increase in short-term nominal rates on the probability of bank equity crashes and banking panics.[24] The empirical specification is similar to (54), except with lagged controls for the crisis substituted with lagged controls for both indicators of financial stress.

The figure shows that rate hikes lead to a significant increases in the probability of both bank equity crashes and banking panics. Interestingly, the timing for which both sources of financial stress become elevated differs. The likelihood of banking panics rises by 1 percent point in the same year of the rate increase. On the other hand, the likelihood of bank equity crashes become significant three years after the initial rate hike. Given an average unconditional annual probability of a bank equity crash and banking panics of 3.5 percent and 3.4 percent respectively, these represent sizable increases in financial crisis risk.

The second and third panels show that the effects of rate hikes on these two measures of financial stress is more immediate in the periods following supply shocks. In the years following demand shocks, the probability of bank equity crashes and bank panics is elevated for four and five years, respectively, after the initial rate hike. By contrast, supply shocks are associated with increases in bank equity crash risk three years after the rate hike, and with bank panics in the same year of the rate hikes.

Bank credit and asset prices – Next, we examine the impact of rate hikes on bank credit and asset prices. The empirical specification is similar to (54), but with the left-hand variables replaced with cumulative log changes of these variables. The specification excludes contemporaneous values of the dependent variables as controls, but includes contemporaneous values of the financial crisis indicator to isolate the direct impact of short-term rate hikes on these variables.

Figure 9 shows that a short-term rate hike leads to significant declines in bank credit, stock prices, and house prices. The effect of rate hikes on stock prices tends to be largest one year after the initial rate hike, and dissipates in the subsequent years. On the other hand, short-term rate hikes continue to have an impact on bank credit and housing prices several years after the initial rate hike, with a sizable cumulative impact five years after the initial rate hike. The model described in the subsequent section shows that declines in asset prices and lending capacity are key channels through which rate

[24]The correlation between these two indicators of financial stress is 0.34.

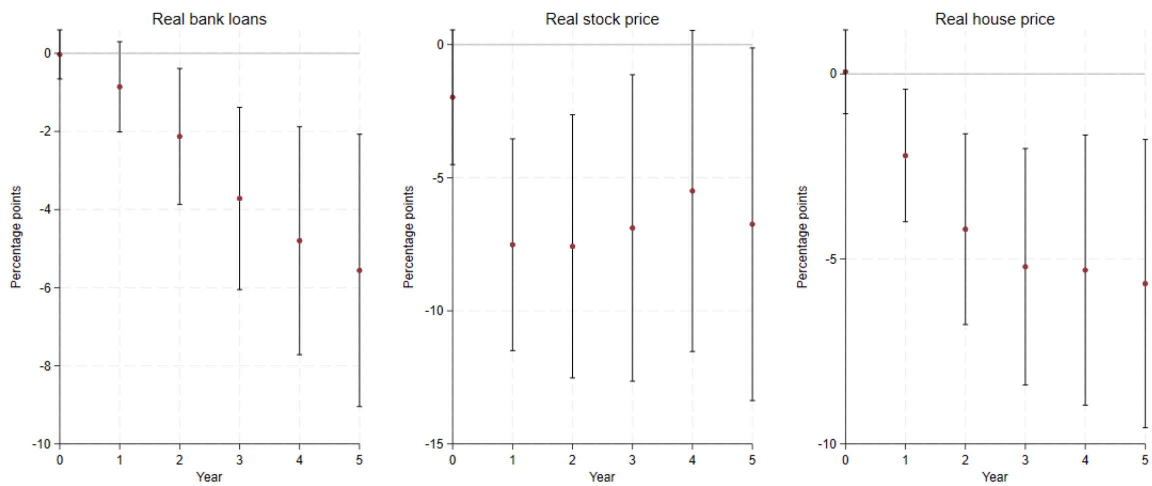

Figure 9: Cumulative log percentage change in real bank loans, real stock price, and real house price following a one percentage point increase in short-term nominal rates. Solid bars denote 90 percent confidence intervals. The second and third columns show the effect of a tightening and loosening in period t conditional on a supply or demand shock in period $t-1$.

hikes exacerbate financial instability.

E.5 Regression results

Table 4: Impact of Rate Hikes on Probability of Financial Crisis, Bank Equity Crash, and Bank Panics

	OLS			IV		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Financial Crisis (JST)						
$\Delta r(i,t)$	0.496***	0.495**		1.785***	1.970**	
	(2.76)	(2.41)		(2.59)	(2.24)	
$\Delta r(i,t) \times \text{DD}(i,t-1)$			0.410			1.973
			(1.31)			(0.98)
$\Delta r(i,t) \times \text{SS}(i,t-1)$			0.447			2.012**
			(1.61)			(2.24)
Observations	1243	1243	1243	1243	1243	1243
Panel B: Bank Equity Crash (BVX)						
$\Delta r(i,t)$	0.275	0.271		1.595**	1.757*	
	(1.17)	(1.06)		(2.07)	(1.69)	
$\Delta r(i,t) \times \text{DD}(i,t-1)$			0.382			2.101
			(0.96)			(0.92)
$\Delta r(i,t) \times \text{SS}(i,t-1)$			0.0490			1.891*
			(0.14)			(1.80)
Observations	1243	1243	1243	1243	1243	1243
Panel C: Bank Panic (BVX)						
$\Delta r(i,t)$	0.507***	0.460**		1.702***	1.298*	
	(3.00)	(2.31)		(2.64)	(1.74)	
$\Delta r(i,t) \times \text{DD}(i,t-1)$			0.598*			2.111
			(1.85)			(1.19)
$\Delta r(i,t) \times \text{SS}(i,t-1)$			0.268			1.255
			(1.09)			(1.62)
Observations	1243	1243	1243	1243	1243	1243

Notes: OLS and IV local projection estimates for the specified outcome. Outcomes are one year ahead indicators of financial crisis, bank equity crash, and bank panics respectively. Columns 1 and 4 report estimates from a specification with just lagged outcome variables as control; Columns 2 and 5 report estimates from a specification with the full set of controls; and Columns 3 and 6 report estimates with short term interest rates and all controls interacted with dummies for supply shocks and demand shocks. Stars denote significance at * 10%, ** 5%, and *** 1% levels.

Table 5: Impact of Rate Hikes on Stock Prices, House Prices, and Bank Credit

	OLS			IV		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Real Stock Price						
$\Delta r(i,t)$	-5.382***	-3.347***		-8.584***	-4.501	
	(-4.85)	(-3.07)		(-2.60)	(-1.27)	
$\Delta r(i,t) \times \text{DD}(i,t-1)$			-3.501**			-4.310
			(-2.46)			(-0.76)
$\Delta r(i,t) \times \text{SS}(i,t-1)$			-3.333**			-4.578
			(-2.23)			(-1.25)
Observations	1195	1195	1195	1195	1195	1195
Panel B: Real House Price						
$\Delta r(i,t)$	-1.058*	-1.136**		-3.188*	-5.067**	
	(-1.87)	(-2.08)		(-1.83)	(-2.33)	
$\Delta r(i,t) \times \text{DD}(i,t-1)$			-0.580			-1.156
			(-0.80)			(-0.32)
$\Delta r(i,t) \times \text{SS}(i,t-1)$			-1.956***			-6.914***
			(-2.59)			(-3.32)
Observations	1183	1183	1183	1183	1183	1183
Panel C: Real Bank Loans						
$\Delta r(i,t)$	-0.649	-1.448***		-2.105	-4.264**	
	(-1.48)	(-3.43)		(-1.52)	(-2.50)	
$\Delta r(i,t) \times \text{DD}(i,t-1)$			-1.878***			-2.338
			(-3.35)			(-0.84)
$\Delta r(i,t) \times \text{SS}(i,t-1)$			-1.282**			-5.192***
			(-2.12)			(-2.98)
Observations	1187	1187	1187	1187	1187	1187

Notes: IV local projection estimates for the specified outcome. Outcomes are four years ahead cumulative change in log real stock price, real house prices, and real bank credit, respectively. Columns 1 and 4 report estimates from a specification with just lagged outcome variables as control; Columns 2 and 5 report estimates from a specification with the full set of controls; and Columns 3 and 6 report estimates with short term interest rates and all controls interacted with dummies for supply shocks and demand shocks. Stars denote significance at * 10%, ** 5%, and *** 1% levels.

Tables 4 and 5 report OLS and IV local–projection estimates for each outcome. Across outcomes, the effect of rate hikes is systematically larger in magnitude under IV than under OLS, consistent with attenuation bias due to the endogenous response of monetary policy to domestic economic conditions (see Jordà et al. (2020)). Intuitively, the OLS regression uses policy-rate changes that partly reflect information about the state of the economy (e.g., easing in bad times, tightening in good times), which biases the estimated effect toward zero; the IV strategy isolates the unanticipated component of policy shocks, recovering larger responses.

For the probability of a financial crisis (Panel A), IV coefficients on the policy-rate change are larger than OLS and remain positive and statistically significant across specifications. For bank equity crashes and bank panics (Panels B–C), OLS coefficients are small or only marginally significant, whereas IV estimates are notably larger and statistically significant. Interaction terms with demand and supply conditions are generally positive but imprecisely estimated; where significant, they suggest that the impact of rate hikes on stress indicators is stronger in environments dominated by adverse demand or supply conditions, respectively.

Real stock prices (Panel A) and house prices (Panel B) fall on impact after a rate hike. The IV estimates are meaningfully larger in magnitude than OLS and remain precisely estimated across specifications, consistent with a stronger discount–rate channel. For real bank loans (Panel C), point estimates are less stable across specifications and interactions, which is plausible given the slower pass-through of monetary policy to credit volumes.

E.6 Additional figures

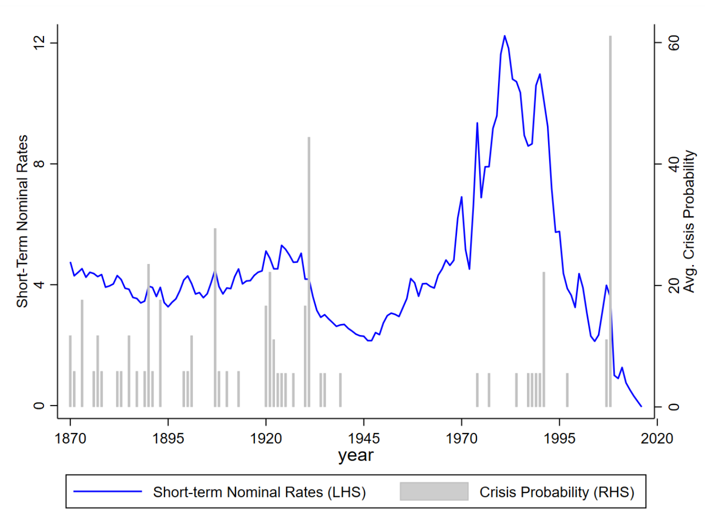

Figure 10: The figure shows the equally-weighted averages of short-term nominal rates (LHS) and probability of a financial crisis as defined in Jordà et al. (2017) (RHS). The sample consists of 18 advanced economies from 1870 to 2016 (see main-text for full list of countries).

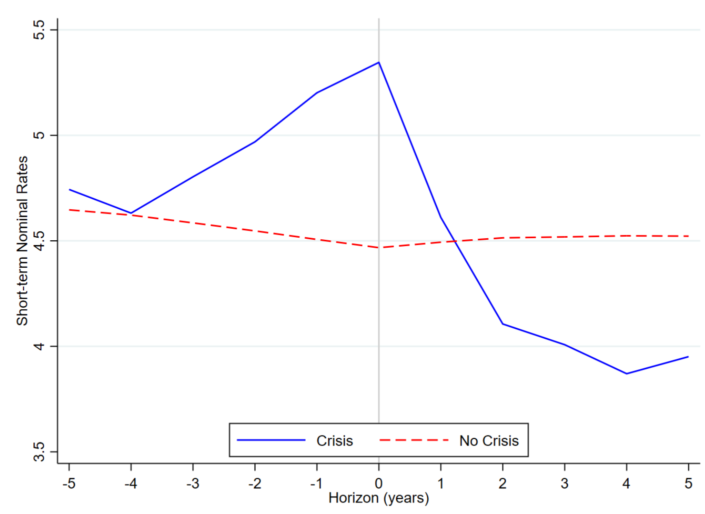

Figure 11: The figure shows the average short-term nominal rate in a 5 year window around a crisis event (blue dashed line) and a non-crisis event (red dashed line). Crisis (non-crisis) events are country-year observations where a crisis occurs (does not occur).