The Great Gatsby Goes to College: Tuition, Inequality and Intergenerational Mobility[*]

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Abstract

This paper studies how higher education shapes intergenerational mobility, income inequality, and aggregate income. We develop a tractable overlapping-generations model in which heterogeneous households sort across an endogenous distribution of heterogeneous colleges under borrowing constraints. We show that the contribution of higher education to intergenerational persistence (IGE) decomposes between an efficient ability- and an inefficient income-sorting channel. Calibrating to U.S. micro-data, they account for 2.7 and 4.4 points of the IGE (34.9), respectively. The increase in the returns to human capital since 1980 strengthens the income-sorting channel and rationalizes key trends, including widening dispersion in college spending, rising tuition, higher inequality, and an increase in the IGE. The optimal policy mix features more progressive need-based aid—to compress income sorting and reduce misallocation—and less progressive subsidies to colleges—to maintain the level and differentiation of financial resources across colleges—thereby raising GDP and intergenerational mobility.

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1. Introduction

How higher education shapes intergenerational mobility, income inequality, and aggregate income is central to U.S. education policy debates. College is traditionally viewed as a pathway to upward mobility, yet access is highly unequal, especially at top-tier, well-resourced institutions. Children from the top 1% of the income distribution are 77 times more likely to attend an elite college than those from the bottom quintile (Chetty, Friedman, Saez, Turner, and Yagan, 2020). These disparities may be concerning if they contribute to the strong and rising intergenerational persistence of earnings (Davis and Mazumder, 2026), and even more so if they reflect the role of parental income—rather than ability—in determining access to high-quality colleges. Understanding how ability and parental income shape college sorting and intergenerational persistence is therefore central for higher education policy.

To shed light on this question, we build a tractable model of the sorting of students and financial resources across heterogeneous colleges, embedded in a general equilibrium dynastic framework. The model delivers an analytical decomposition of the contribution of higher education to intergenerational persistence into an ability- and an income-sorting channel across colleges. Ability-based sorting is efficient but income-based sorting plays a dual role: it generates misallocation by tying college access to parental income due to the borrowing constraint, but it also sustains the level and differentiation of resources across colleges which is valuable. We show how each sorting channel maps to technology and policy parameters. We use a calibrated version of the model to quantify both channels, run counterfactuals and analyze the optimal mix of progressive subsidies to colleges and need-based aid.

The household side of the model builds on the literature on intergenerational transmission and inequality of human capital (Benabou, 2002). A continuum of heterogeneous households differ in their human capital, transmit ability to their children with some randomness and invest in higher education subject to a borrowing constraint. Households face an equilibrium tuition schedule that depends on college quality, student ability, and parental income. After college, each child becomes an adult whose human capital reflects their ability, college quality, and a labor market shock; they then supply labor in a competitive market and have a child, and the process repeats across generations.

A key novelty of our framework is to embed an endogenous distribution of heterogeneous colleges—the supply side of the higher education market—into this general equilibrium dynastic model. Colleges seek to maximize the quality they provide to their students, which depends on both resources per student and the average ability of the student body—the "peer effect." Because student ability raises college quality, colleges discount tuition for high-ability students; they also value students from richer families, who bring in additional resources. These forces generate an equilibrium in which students sort across colleges along two dimensions, ability and parental income, and in which colleges endogenously differ in resources.[1] As in Cai and Heathcote (2022), colleges are price-takers and the tuition schedule clears each segment of the market. We close the model with a government that implements non-linear merit and need-based aid to students and non-linear transfers to colleges.

A central contribution is to analytically decompose the role of higher education in intergenerational persistence into an ability- and an income-sorting channel. The ability-sorting margin is the relationship between college quality and student ability, holding parental income fixed, and the income-sorting margin is the analogous relationship with parental income, holding ability fixed. The ability-sorting margin depends on the strength of peer effects and is amplified by merit-based aid. The income-sorting margin depends on the importance of financial resources in the higher education technology and the returns to human capital, and is compressed by need-based aid and progressive transfers to colleges. We further show how both margins shape inequality and aggregate output.

While sorting on ability reflects efficient matching of students, sorting on income plays a dual role. On the one hand, it reflects student misallocation by tying college quality to parental income due to the borrowing constraint. On the other hand, because higher-income families can pay more, income sorting also determines how revenues are distributed across colleges. When richer students are concentrated in top colleges, those colleges command more resources, generating a more differentiated allocation of expenditures across the college hierarchy. Some differentiation in resources is valuable given complementarities between resources and student ability. This gives rise to a novel trade-off: policies that reduce income sorting improve the allocation of students but simultaneously compress the dispersion of resources across colleges.

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[1] This microfoundation builds on a literature that estimates equilibrium models of higher education (Rothschild and White, 1995; Epple, Romano, and Sieg, 2006; Cai and Heathcote, 2022; Blandin and Herrington, 2022).

We then show analytically that an increase in the returns to education—a primitive of the model—increases the strength of the income-sorting channel and rationalizes key U.S. trends since 1980: (a) disparities in expenditures per student across colleges have widened (Capelle, 2019); (b) the share of students from the lowest income quintile at top colleges has stagnated (Bailey and Dynarski, 2011; Chetty, Friedman, Saez, Turner, and Yagan, 2020); (c) tuition fees before financial aid have risen in real terms; (d) the intergenerational elasticity of income (IGE) has likely increased (Lee and Solon, 2009; Davis and Mazumder, 2026); and (e) returns to education and income inequality have both risen (Autor, Katz, and Kearney, 2008; Piketty and Saez, 2003). Fact (b) implies that rising returns to skill worsens the misallocation of students. Trends (d) and (e) together correspond to a gradual shift of the U.S. economy to the right side of the Great Gatsby curve—the positive empirical relationship between income inequality and the intergenerational transmission of economic status across countries (Corak, 2013).

Intuitively, rising returns to education increase the dispersion of labor earnings for a given distribution of human capital, raising income inequality. Richer households respond by demanding higher-quality education, allowing top colleges to raise tuition and widening the dispersion of resources across institutions. Under borrowing constraints, high-ability students from low-income backgrounds are priced out of top colleges, leading to a stagnation in their representation and a weakening of the ability-sorting channel relative to income. The rise of the income-sorting channel, together with greater resources available at top colleges, increases intergenerational persistence, which in turn feeds back into greater inequality in human capital in the next generation. Higher education thereby contributes to the gradual shift of the U.S. economy along the Great Gatsby curve.

We then calibrate a quantitative version of the model. We allow parents to choose the size of their financial bequest and give individuals the option of not attending college. The model is calibrated using several microdata sources: (i) the restricted-use National Longitudinal Survey of Youth of 1997 (NLSY), a representative panel of high-schoolers with detailed information on parental background, children's abilities, their experience in higher education, and their income in their early thirties; (ii) the National Postsecondary Student Aid Study (NPSAS) from the National Center for Education Statistics (NCES), a student-level dataset on net tuition and financial aid; and (iii) the Integrated Postsecondary Education Data System (IPEDS) from the

NCES, a panel covering the universe of colleges. We validate the model in two ways. First, we show that it quantitatively matches the empirical distribution of parental income across college-quality quintiles in the micro-data of Chetty, Friedman, Saez, Turner, and Yagan (2020). Second, we show that it is consistent with trends (a)–(e) between 1980 and 2010.

We first use the calibrated model to quantify the contribution of the two student sorting margins to intergenerational persistence. In the baseline 2010 economy, the income- and ability-sorting channels account for 4.4 and 2.7 points of the IGE of 34.9, respectively. The overall contribution of higher education has increased due to the rise in the returns to human capital, driven by a rise in the income-sorting channel. In a 1980 steady state, the two channels account for 2.2 and 2.1 points of a total persistence of 30.0. Taken together, these results indicate that the college channel was roughly balanced between ability and income sorting in 1980, whereas income-based sorting now accounts for the larger share of intergenerational persistence.

We then run counterfactuals showing that, despite substantial student misallocation, both the sorting of students by ability and the differentiation of resources across colleges, play an important role in raising GDP. Our first counterfactual equalizes higher education by randomly allocating students to colleges and equalizing spending across institutions, thereby neutralizing both sorting channels as well as the differences in financial resources. This counterfactual thus quantifies the overall contribution of college heterogeneity. Relative to the baseline, the IGE falls by 24.5%, the income Gini by 3.7%, and GDP by 10.5%. The second counterfactual isolates the ability-sorting channel by equalizing spending while perfectly sorting students across colleges by ability. Relative to the baseline, the IGE falls by 15.9% and the income Gini by 1.9%. Despite perfect sorting by ability, GDP declines by 8.0%, indicating that flattening the allocation of resources entails substantial economic costs.

Finally, we solve for the welfare-maximizing mix of need-based aid to students and transfers to colleges. Need-based aid can mitigate the misallocation of students across heterogeneous colleges generated by income-based sorting under financial constraints.[2] However, doing so tends to weaken incentives for households to invest in higher education and to compress the dispersion of financial resources across colleges, which

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[2] Expansions of federal need-based programs have been debated in recent U.S. national elections (e.g., the "College for All" proposal). Similarly, increases in institutional need-based aid have been a focus of recent tuition policies at elite colleges.

lowers aggregate output given complementarities between student ability and financial resources. Transfers to colleges provide a second instrument that can offset this resource effect by shaping the steepness of the expenditure ladder directly.

The optimal policy mix combines significantly more progressive need-based aid but less progressive subsidies to colleges than in the status-quo. Need-based aid has a more direct effect on the allocation of students by compressing income sorting and strengthening ability sorting, whereas transfers to colleges shape more directly the level and dispersion of resources. The optimal policy mix can therefore act on both margins and decouple the allocation of students from the allocation of resources. Together, these policy reforms raise GDP by 9.2%, reduce intergenerational persistence by 26.6% and leave inequality nearly unchanged.

Literature. This paper contributes to the literature on educational choice, mobility, and inequality in intergenerational frameworks (Benabou, 1996; Fernandez and Rogerson, 1996; Abbott, Gallipoli, Meghir, and Violante, 2013; Guvenen, Kuruscu, and Ozkan, 2014; Hanushek, Leung, and Yilmaz, 2014; Krueger and Ludwig, 2016; Kotera and Seshadri, 2017; Caucutt and Lochner, 2017; Hendricks and Leukhina, 2017; Guerrieri and Fogli, 2017; Colas, Findeisen, and Sachs, 2018; Durlauf and Seshadri, 2018; Blandin and Herrington, 2022; Lee and Seshadri, 2019; Eckert and Kleineberg, 2019; Hubmer, Krusell, and Smith Jr, 2021; Wright, 2023; Hendricks, Koreshkova, and Leukhina, 2024; Fabre, 2025; Krueger, Ludwig, and Popova, 2025; Gu and Zhang, 2024). Our closest precedent is Restuccia and Urrutia (2004), which studies the role of early and higher education for intergenerational persistence, in a setting with a single college with exogenous tuition and quality. We open the box of the role of higher education, providing a decomposition of its role in intergenerational persistence into an ability- and an income-sorting channel. Our framework allows for rich heterogeneity of colleges, endogenizes the ladder of quality and tuition fees, and accounts for the complex set of government interventions in higher education. The model can replicate the untargeted distribution of parental income across heterogeneous colleges and trends (a)–(e). We find that considering the heterogeneity in colleges is quantitatively important for understanding how higher education shapes intergenerational persistence, aggregate income and inequality.[3]

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[3] Jovanovic (2014) studies an economy where long-term growth depends on the quality of assignment between workers and managers. In our model, (i) aggregate income depends on the quality of sorting of students to college qualities, an endogenous bundle of teaching expenditures and students'

Our paper builds on the literature that models the admission and tuition decisions of colleges and the equilibrium of the higher education market. Prior work has studied financial aid policies (Epple, Romano, and Sieg, 2006; Fillmore, 2016), expansions in the supply of public college seats (Fu, 2014), affirmative action (Kapor, 2015), and the effect of rising inequality on tuition (Cai and Heathcote, 2022). Our contribution is to embed the sorting of heterogeneous students across heterogeneous colleges into an intergenerational setting, to analytically characterize how the ability-sorting and income-sorting channels shape intergenerational persistence, and to quantify their importance. The analytical characterization of the equilibrium allocation transparently links technology and policy parameters to the two sorting channels, and we can establish existence and uniqueness of the dynamic equilibrium—two challenging issues in the literature on clubs.

This paper also contributes to the literature that studies the determinants of tuition fees. Existing work emphasizes how rising income inequality (Cai and Heathcote, 2022), the expansion of credit supply (Lucca, Nadauld, and Shen, 2015), financial aid (Gordon and Hedlund, 2017), and the rising cost of university inputs, or Baumol's disease (Jones and Yang, 2016), can rationalize rising tuition. We stress the role of the increase in the returns to education: higher returns raise household demand for college quality, especially at the top of the distribution, pushing up both the average and the dispersion of tuition. This mechanism is close in spirit to the revenue theory of cost of Bowen (1980), in which universities charge as much as they can and spend on inputs that increase education quality. We find that this channel accounts for two thirds of the tuition spending increase relative to GDP, consistent with Martin, Hill, and Waters (2017). Importantly, we show that the same increase in the returns to education rationalizes the rising dispersion in resources per student across colleges.[4]

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mean ability; (ii) students are heterogeneous in two dimensions (abilities and parental income) and not just in ability, and (iii) the source of the misallocation is a financial friction, not exogenous noise in the assignment process.

[4] A rich empirical literature provides evidence on the returns to college quality and selectivity. Most papers find significant returns on the labor and marriage markets as well as for children's achievements (Black and Smith, 2006; Long, 2010; Hoekstra, 2009; Zimmerman, 2014; Bleemer, 2019). Another stream of the literature has found weak to no effects of selectivity (Dale and Krueger, 2011; Hickman and Mountjoy, 2019). Our results suggest moderate amplification effects of higher education. Another literature has shown that parental background matters a lot for achievements and access in top colleges (Bailey and Dynarski, 2011; Chetty, Friedman, Saez, Turner, and Yagan, 2020; Hoxby and Turner, 2019) and that financial aid policy has a significant impact on college decisions (Dynarski, 2003; Angrist, Autor, Hudson, and Pallais, 2016).

The rest of the paper is organized as follows. Section 2 presents the baseline model. Section 3 characterizes its equilibrium in closed form and shows that the college-stage intergenerational persistence splits into an ability-sorting channel and an income-sorting channel. Section 4 shows that an increase in the return to human capital strengthens the income-sorting channel and generates trends (a)–(e). Section 5 augments the model with several policy instruments. Section 6 calibrates a quantitative extension with enrollment and intergenerational financial transfers and validates the model against untargeted moments. Section 7 runs counterfactuals that quantify the channels and solve for the optimal mix of progressive subsidies to colleges and need-based student aid. Section 8 concludes.

2. A Baseline Model of Human Capital Transmission through a Hierarchy of Heterogeneous Colleges

The economy is populated by dynastic households and colleges. At each generation, parents imperfectly transmit human capital to their child and choose which college to send them to. Colleges, in turn, choose educational spending and the composition of their student body to maximize the quality they deliver. The distribution of quality and the tuition schedule are determined in the equilibrium of the market for higher education. This section develops a parsimonious laissez-faire economy with only the minimal ingredients to generate heterogeneous colleges and two-dimensional sorting of students. Taxes, transfers and financial aid are introduced in Section 5.[5]

2.1. Households

There is a continuum of dynasties indexed by $i \in \mathscr{I}$. Each individual lives for two periods—one as a child, one as an adult—and each adult has one child.[6] A generation-t household of dynasty i is characterized by its parents' human capital h_{it} and the child's ability at the end of high school z_{it}. It chooses consumption c_{it}, labor supply ℓ_{it}

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[5] We extend the model in Benabou (2002) along several dimensions: heterogeneous colleges, a rich set of higher education policies, and a birth shock. In Benabou (2002), households buy an educational good at a constant unit price—independent of household or student characteristics—and there is no quality ladder.

[6] We show in Appendix B that this two-period model is a reduced-form expression of a full life-cycle model in which households can borrow and save at an exogenous interest rate r to smooth consumption during their lifetime.

and the college quality q_{it} to send the child to. Dropping the generation and dynasty subscripts and denoting next-generation variables with a prime, the value function $\mathscr{U}(h, z)$ solves

$$\mathscr{U}(h, z) = \max_{c,\ell,q} \left\{ \ln(c) - \ell^{\eta} + \beta E\left[\mathscr{U}(h', z')\right] \right\} \tag{1}$$

where β is the intergenerational discount factor. The child's ability at the end of high school is a log-linear combination of parental human capital and a birth shock ξ_z capturing the randomness of transmission:[7]

$$z = (\xi_z h)^{\alpha_z}. \tag{2}$$

Lifetime income y depends on human capital h and hours ℓ:

$$y = h^{\lambda} \ell, \tag{3}$$

where λ is the elasticity of output to human capital—the "returns to human capital." This parameter plays a central role below: in Section 4 we show that an increase in λ rationalizes the trends in higher education discussed in the introduction.[8]

Income is spent on consumption and on tuition. The tuition schedule $e(q, z, y)$ is an equilibrium object that depends on college quality q, child ability z and parental income y. Normalizing the price of the final good to one,

$$c + e(q, z, y) = y. \tag{4}$$

Households face an intergenerational borrowing constraint: parents cannot leave a bequest or pass debt on to their child. A large literature documents that borrowing constraints matter for college choices (Lochner and Monge-Naranjo, 2012). This

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[7] Given our focus on higher education, we abstract from parents' endogenous pre-college investments—a channel studied extensively in prior work (e.g. Restuccia and Urrutia (2004), Lee and Seshadri (2019), Eckert and Kleineberg (2019)). Our estimates of the contribution of higher education to persistence and aggregate income should therefore be interpreted as lower bounds: equalizing higher education would likely reduce incentives for pre-college investment, dampening aggregate income gains.

[8] Although simple, this functional form is also the reduced form of a more elaborate production function with physical capital, or of a household's payoff in an aggregate production process with complementarity across heterogeneous tasks.

assumption rules out net financial transfers across generations, but not gross transfers—e.g., children borrowing from their parents early in life and repaying them later, or student loans exactly offset by a parental transfer. The quantitative model in Section 6 relaxes this constraint.

Post-college human capital is a log-linear combination of pre-college ability z, college quality q and a labor market shock ξ_y:[9]

$$h' = z q^{\alpha_q} \xi_y. \tag{5}$$

The two shocks enter at different stages. The birth shock ξ_z is realized—and observed by the household—before the college choice is made, while the labor market shock ξ_y is realized only once the child enters the labor market. Both are i.i.d. across generations and households and log-normally distributed:

$$\ln \xi_z \sim \text{i.i.d. } \mathscr{N}\left(-\sigma_z^2/2,\ \sigma_z^2\right) \tag{6}$$

$$\ln \xi_y \sim \text{i.i.d. } \mathscr{N}\left(-\sigma_y^2/2,\ \sigma_y^2\right). \tag{7}$$

2.2. Colleges

Technology. There is a mass one of ex-ante identical colleges indexed by $j \in [0,1]$. Each college operates a technology that converts educational services per student I_j and the average ability of its student body $\tilde{z}_j$—the "peer effect"—into quality q_j:

$$q_j = I_j^{\omega_I} \tilde{z}_j^{\omega_z}, \qquad \omega_I, \omega_z > 0. \tag{8}$$

Colleges are therefore clubs: because $\tilde{z}_j$ enters the quality delivered to every student, the composition of admitted students affects all students. Empirical work supports this formulation. Sacerdote (2011), Smith and Stange (2016) and Mehta, Stinebrickner, and Stinebrickner (2018) find evidence of peer effects—particularly from roommates—on in-college achievement; Zimmerman (2019) documents long-run labor-market returns to college networks; and the fact that colleges actively compete for high-ability students (Hoxby, 2009, 2013) is also consistent evidence.

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[9] The Cobb-Douglas functional form embeds a unit elasticity of substitution between pre-college ability and college quality. Consistent with this assumption, Dillon and Smith (2018) document complementarities between ability and college quality in long-term earnings, and Lee and Seshadri (2019) estimate a unit elasticity across periods of the human capital accumulation process.

We let peer effects reflect both the ability and the socioeconomic composition of the student body:

$$\ln \tilde{z}_j = E_{\phi_j(z,y)}[\ln z] - \sigma^2_{u,j}, \tag{9}$$

where $\phi_j(z,y)$ is the endogenous distribution of admitted students. The first term is the geometric mean of abilities. The second term, $\sigma^2_{u,j}$, captures the cost of socioeconomic heterogeneity within the college: greater dispersion hampers peer interactions and instruction, a mechanism supported by empirical studies such as Figlio and Page (2002) and Duflo et al. (2011). Following Alon et al. (2025), we define it as the within-college variance of a weighted log combination of ability and parental income,

$$\sigma^2_{u,j} \; = \; \frac{\Omega}{2} \, V_{\phi_j(z,y)}\Big(\ln z^{\omega_z/\omega_I} y^{-\omega_y/\omega_I}\Big) \, ,$$

with the constant Ω given in Appendix A. This parameterization makes the quality production function effectively a geometric aggregator of tuition revenue and ability, which is what preserves tractability: it keeps the equilibrium tuition schedule and the matching of students to colleges log-linear.[10]

Educational services I_j are financed entirely by tuition revenue, giving the static budget constraint

$$I_j = E_{\phi_j(z,y)}[e(q,z,y)]. \tag{10}$$

Objective and Positioning Game. Colleges maximize the quality q they deliver to their students, a standard objective for non-profit and public institutions (Epple, Romano, and Sieg, 2006; Fu, 2014; Alon, Capelle, and Matsuda, 2025). They are competitive and take the equilibrium tuition schedule as given (Cai and Heathcote, 2022).[11]

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[10] Without this specification the tuition function and the assignment mechanism would not be log-linear, breaking the linearity of the laws of motion of the mean and variance of log human capital, and with them the log-normality of the distribution. The college problem under this specification is similar in flavor to Fu (2014), where colleges maximize a weighted combination of average ability and a quadratic function of net tuition.

[11] An alternative objective is profit-maximization as in Cai and Heathcote (2022). Quality maximization is consistent with the fact that non-profit and public colleges account for 90–95% of enrollment. Both formulations deliver very similar tuition schedules—a point already noted by Epple et al. (2006) in a setting with imperfectly-competitive colleges. In Appendix C we show that, to first order, the equilibrium tuition schedule with quality-maximizing colleges coincides with that under profit maximization.

Because the technology is constant returns to scale, college size is indeterminate; we normalize all colleges to the same mass of students.

Each period, the college's problem has two stages. In the second stage, taking the tuition schedule and the outcome of the first stage as given, a college chooses educational services I_j and the student body $\phi_j(z,y)$—which pins down the peer effect $\tilde{z}_j$—to solve

$$\max_{I_j, \tilde{z}_j, \phi_j(z,y)} q_j \quad \text{subject to (8), (9), (10),} \tag{11}$$

together with a positioning constraint inherited from the first stage.[12]

In the first stage, colleges play a positioning game on the quality line. Colleges choose their quality sequentially according to their index $j \in [0,1]$; the payoff to operating at quality q is (11). Since colleges are ex-ante identical, the ordering is without loss of generality. In the subgame-perfect Nash equilibrium, colleges choose in descending order of quality, so the positioning constraint takes the form $q \leq \chi^{-1}(j)$, where $\chi(q)$ is the equilibrium density of students across qualities. A full game-theoretic treatment is in Appendix A.7. The fixed-size assumption rules out agglomeration at the top—in its absence, every college would want to be the best college, which only one can be.[13] We denote by $\phi(j,z,y)$ and $I(j)$ the student distribution and spending per student in the college of order j.

2.3. Equilibrium

An equilibrium path is a sequence of tuition schedules $e_t(q,z,y)$, household policy functions $\{c_t(h,z), \ell_t(h,z), q_t(h,z), y_t(h,z)\}$, college policy functions $\{\phi_t(j,z,y), I_t(j)\}$, a distribution of human capital $f_t(h)$ and a distribution of students across qualities $\chi_t(q)$ such that: (i) given e_t, households solve (1); (ii) given e_t and χ_t, colleges solve (11); (iii) the market for higher education clears—the sorting rule $q_t(h,z)$ is consistent with $\chi_t(q)$—and the final-good market clears; (iv) $f_t(h)$ evolves according to the intergenerational law of motion implied by the sorting rule and the human capital accumulation technology.

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[12] This formulation abstracts from within-institution heterogeneity (tracks, fields of study), congestion forces, fixed factors such as endowments, and imperfect information about applicant types—an important inefficiency studied by Fu (2014). We focus instead on a different inefficiency: the borrowing constraint.

[13] The positioning game plays, for quality-maximizing colleges, the role that free entry plays for profit-maximizing ones. A key difference is that in the subgame-perfect equilibrium, colleges at different qualities receive heterogeneous payoffs, whereas free entry would equalize profits to zero.

3. Equilibrium and the Two Margins of Students' Sorting

This section analytically characterizes the model's equilibrium and derives a central result of the paper: a decomposition of the intergenerational elasticity (IGE) of human capital into a before-college component and two distinct college-stage components, an ability-sorting margin and an income-sorting margin. Equilibrium features a hierarchy of colleges differing in their financial resources and two-dimensional sorting of students by ability and family income. Sorting on ability is efficient and reflects the colleges' desire to attract strong students for peer-effect reasons. Sorting on income reflects the desire of richer families to send their children to better colleges, the borrowing constraint as well as the colleges' need for financial resources. On the one hand income-sorting leads to the misallocation of students. On the other it sustains the level and the differentiation of resources across colleges in equilibrium, which is valuable.

3.1. Equilibrium Tuition Schedule

A college choosing to supply quality q must select the combination of educational services I and student ability distribution that delivers q most efficiently. Because ability substitutes for resources in the quality technology, colleges trade off lower tuition against higher student ability. The proposition below gives the unique equilibrium tuition schedule consistent with all colleges at an interior optimum. It is log-linear, which keeps the rest of the model tractable.[14] All proofs are in Appendix A.

Proposition 3.1. *The equilibrium tuition schedule is given by*

$$e_t(q,z,y) = q^{\frac{1}{\omega_I}} z^{-\frac{\omega_z}{\omega_I}}. \tag{12}$$

Tuition is increasing in quality q with elasticity $1/\omega_I$ and decreases in ability z with elasticity $-\omega_z/\omega_I$. The two elasticities have the following interpretation. Higher quality requires financing higher spending per student, so higher-quality colleges must charge more; if spending is a strong driver of quality (large ω_I), an increase in revenue brings large quality gains so the elasticity of tuition to quality $1/\omega_I$ can be small. The negative elasticity of tuition to ability—a discount for high-ability students—reflects

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[14] We construct an equilibrium in which the distribution of human capital remains log-normal across generations. A necessary and sufficient condition is that the tuition schedule is log-linear. We cannot rule out the existence of other, non-interior, equilibria, but such equilibria would defeat the analytical purpose of this section.

the colleges' desire to attract strong students for peer-effect reasons. Tuition does not depend on parental income since colleges have no reason to price-discriminate on background (this changes once institutional need-based aid is introduced in Section 5).

3.2. Household Policy Functions

Given the tuition schedule (12), each household chooses which college to send its child to. Since tuition is monotonic in q, the decision reduces to choosing how much of income to spend on higher education. Let $s_t(q,z,y) \equiv e_t(q,z,y)/y$ denote the spending rate. With log preferences, log-normal shocks and log-linear technologies, the spending rate and labor supply admit closed-form expressions.[15]

Proposition 3.2. *Define* $U = \frac{\partial \ln \mathscr{U}}{\partial \ln h}$*, the elasticity of the value function to human capital. In equilibrium, all households' spending rate, labor supply and the elasticity of the value to human capital* U *are given by:*

$$s_t = \frac{\beta \alpha_q \omega_I U_{t+1}}{1 + \beta \alpha_q \omega_I U_{t+1}} \tag{13}$$

$$\ell_t = \left[\frac{1}{\eta} \left(1 + \beta \alpha_q \omega_I U_{t+1}\right) \right]^{\frac{1}{\eta}} \tag{14}$$

$$\text{with} \quad U_t = \sum_{k=0}^{\infty} \beta^k \lambda_{t+k} \prod_{m=0}^{k-1} \rho_{t+m} \tag{15}$$

where ρ_t is the IGE of human capital at generation t, which we derive below. Two features are worth noting. First, s_t and ℓ_t are independent of household type: under log preferences and no financial transfers across generations, rich and poor households spend the same fraction of income on higher education.[16] Second, both depend positively on the current-generation return of higher education spending $\alpha_q \omega_I$, which is the elasticity of the child's next-period human capital to current college spending. Both s_t and ℓ_t also depend on the forward-looking object U_{t+1}, which is increasing in all future IGEs $\{\rho_{t+m}\}_{m=1}^{\infty}$ and in all future returns to education $\{\lambda_{t+k}\}_{k=0}^{\infty}$: the higher future persistence ρ, the stronger the transmission of today's investment through the dynasty, and the higher future λ, the higher the payoff to

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[15] This setting builds on a long tradition, e.g. Glomm and Ravikumar (1992).

[16] In the quantitative model of Section 6, where bequests and an outside option are added, there will be heterogeneity in spending rates across households.

human capital accumulated now. The dependence on λ plays a central role in Section 4 where we argue that a current and anticipated rise in λ raises s_t.

3.3. Equilibrium Sorting Rule

Combining the equilibrium tuition schedule—the supply side—with the household spending rule—the demand side—yields the equilibrium student sorting rule—a mapping from household types (z, y) to college qualities q. For future reference, we denote the elasticity of quality to ability ε_{qz} and to income ε_{qy}. The proposition below gives the analytical expression for this mapping and these elasticities in terms of the technology parameters.

Proposition 3.3. *In equilibrium, the elasticity of quality to ability is given by* $\varepsilon_{qz} = \omega_z$ *and the elasticity to income by* $\varepsilon_{qy} = \omega_I$. *The student sorting rule is given by*

$$q_t(z, y) = (s_t y)^{\omega_I} z^{\omega_z}. \tag{16}$$

The sorting on ability corresponds to the fact that high-ability students sort in higher-quality colleges because colleges want them for peer-effect reasons and attract them through tuition discounts with elasticity ω_z. It is the efficient margin of sorting.

The sorting on income stems from the fact that richer households buy higher-quality colleges because they can spend more, with elasticity ω_I. This channel arises because of the borrowing constraint combined with colleges' need for revenue. Absent financial constraints, low-income but high-ability students could borrow and parental income would not matter, *i.e.* $\varepsilon_{qy} = 0$.

3.4. Law of Motion of Human Capital and the IGE Decomposition

We now show that the IGE of human capital which measures the degree of persistence of economic status across generations, can be decomposed into a pre-college component and two distinct college-stage components, an ability-sorting margin and an income-sorting margin. In a steady-state equilibrium, the IGE of human capital is equal to the IGE of earnings, which is a common empirical measure of intergenerational persistence. Substituting the sorting rule (16) into the human capital law (5) and collecting terms in $\ln h$ yields $\ln h_{t+1} = \rho_t \ln h_t + \ln \xi_y + (\alpha_z + \alpha_q \omega_z) \ln \xi_z + \alpha_q \omega_I \ln (s_t \ell)$.

Proposition 3.4 (IGE Decomposition.)**.** *Denoting ρ_t the IGE of human capital, it admits the following decomposition independently of the equilibrium*

$$\rho_t = \underbrace{\alpha_z}_{\textit{Before College}} + \underbrace{\underbrace{\alpha_q \varepsilon_{qz} \alpha_z}_{\textit{Ability-Sorting}} + \underbrace{\alpha_q \varepsilon_{qy} \lambda}_{\textit{Income-Sorting}}}_{\textit{College}} . \tag{17}$$

In equilibrium, it is given by $\rho_t = \alpha_z + \alpha_q \omega_z \alpha_z + \alpha_q \omega_I \lambda$.

This decomposition gives a clear mapping from the parameters governing the higher education technology and the law of human capital accumulation to intergenerational persistence. The before-college term α_z summarizes all transmission operating before college. This paper opens the box of the transmission of economic status through college. The transmission during college happens through the two channels introduced above. Both channels are multiplied by α_q which captures the elasticity of the future adult's human capital to college quality.

The ability-sorting channel $\alpha_q \omega_z \alpha_z$ reflects the importance of peer-effects in the production of college quality and the sorting of students by ability ω_z as well as the strength of the transmission of parental human capital to student ability α_z. The income-sorting channel $\alpha_q \omega_I \lambda$ reflects the sorting by parental income and the strength of the labor market returns to human capital for parents λ. Because higher-income families can pay more, sorting by income implies that better colleges command more resources. This generates a more differentiated allocation of expenditures across the college hierarchy and accentuate intergenerational persistence. However this income-sorting channel also creates an inefficiency by misallocating students across colleges. We next focus on this dual role of income sorting.

3.5. The Dual Role of Income Sorting: Misallocation *vs.* Resource Allocation.

To see how income sorting shapes both the allocation of resources across colleges and the misallocation of students, the proposition below gives the distribution of college quality and of within-college student types. These distributions depend on the elasticity of college quality to parental human capital indirectly through student ability $\varepsilon_{qz} \alpha_z$, directly through parental income $\varepsilon_{qy} \lambda_t$ and on the first and second moments of the aggregate distribution of human capital (m_{ht}, Σ_{ht}).

Proposition 3.5. *1. The distribution of college quality is given by*

$$\ln q \sim \mathcal{N}\left(\mu_{1t}(m_{ht}, \Sigma_{ht}), (\varepsilon_{qz}\alpha_z)^2\sigma_z^2 + (\varepsilon_{qz}\alpha_z + \varepsilon_{qy}\lambda_t)^2\Sigma_{ht}^2\right).$$

2. Within a college of quality q, the distribution of parents' (log) human capital is:

$$\ln h|q \sim \mathcal{N}\left(\mu_{2t}(q, m_{ht}, \Sigma_{ht}), \frac{(\varepsilon_{qz}\alpha_z)^2\sigma_z^2}{(\varepsilon_{qz}\alpha_z)^2\sigma_z^2 + (\varepsilon_{qz}\alpha_z + \varepsilon_{qy}\lambda_t)^2\Sigma_{ht}^2}\Sigma_{ht}^2\right)$$

with μ_{2t} increasing in q.

3. Within a college of quality q, the distribution of students' (log) abilities is:

$$\ln z|q \sim \mathcal{N}\left(\mu_{3,t}(q, m_{ht}, \Sigma_{ht}), \frac{(\varepsilon_{qy}\lambda_t)^2\Sigma_{ht}^2}{(\varepsilon_{qz}\alpha_z)^2\sigma_z^2 + (\varepsilon_{qz}\alpha_z + \varepsilon_{qy}\lambda_t)^2\Sigma_{ht}^2}\alpha_z^2\sigma_z^2\right)$$

with $\mu_{3,t}$ increasing in q.

The cross-college dispersion of quality, $(\varepsilon_{qz}\alpha_z)^2\sigma_z^2 + (\varepsilon_{qz}\alpha_z + \varepsilon_{qy}\lambda_t)^2\Sigma_{ht}^2$, is increasing in both sorting elasticities $\varepsilon_{qz}\alpha_z, \varepsilon_{qy}\lambda_t$ as well as in the dispersion of human capital Σ_{ht}. More heterogeneous households generate more heterogeneous demands for quality, which the equilibrium accommodates—since the ladder of colleges is endogenous—by wider dispersion in tuition, expenditures per student, and quality. The mean μ_{1t} rises with the aggregate mean m_{ht}: as average human capital grows, so does the average quality demanded, and equilibrium delivers it through higher tuition and higher average student ability (details in Appendix A.4).

The role of the income-sorting elasticity $\varepsilon_{qy}\lambda_t$ in shaping the dispersion of quality reflects a tight equilibrium link between the income-based sorting and the allocation of educational resources. Under borrowing constraints, dispersion in parental income directly translates into dispersion in spending across colleges.[17] Higher income inequality raises demand for top-quality colleges, which in equilibrium increases tuition and expenditures at the top of the distribution. This mechanism implies that changes in the labor market returns to human capital λ endogenously expand the dispersion of educational resources and stretch the upper tail of the college quality distribution, as emphasized in Section 4.

The strength of the income-sorting channel relative to the ability-sorting channel $\varepsilon_{qz}\alpha_z/\varepsilon_{qy}\lambda_t$ governs the misallocation of students and the within-college distribution.

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[17] We give the expression for the distribution of resources across colleges in Appendix A.4.

Assortative matching by ability is positive but imperfect: the mean $\mu_{3,t}$ rises with q, confirming that higher-ability students cluster in higher-quality colleges, but the within-college variance of ability decreases with $\varepsilon_{qz}\alpha_z/\varepsilon_{qy}\lambda_t$ while the within-college variance of parental income moves in the opposite direction. Hence the ratio $\varepsilon_{qz}\alpha_z/\varepsilon_{qy}\lambda_t$ measures the quality of student allocation: when it is high, colleges are economically diverse but ability-homogeneous; when it is low, colleges are ability-diverse but economically segregated.

The dual role of the income-sorting channel in the equilibrium of our economy leads to an important novel trade-off: policies that compress the income-sorting channel reduce student misallocation but simultaneously compress the dispersion of expenditures, which may decrease GDP due to their complementarities with students' ability.

3.6. Aggregate Law of Motion of Human Capital.

Using the assumption of log-normality of both shocks, (6) and (7), if the economy starts from a log-normal distribution then human capital stays log-normally distributed along the equilibrium path:

Proposition 3.6. *If* $\ln h_t \sim \mathcal{N}\left(m_{ht}, \Sigma^2_{ht}\right)$ *then*

$$\ln h_{t+1} \sim \mathcal{N}\left(m_{ht+1}, \Sigma^2_{ht+1}\right) \tag{18}$$

$$m_{ht+1} = \rho_t m_{ht} + X_{1t} \tag{19}$$

$$\Sigma^2_{ht+1} = \rho_t^2 \Sigma^2_{ht} + X_2 \tag{20}$$

$$\text{where} \quad \rho_t = \alpha_z + \alpha_q(\varepsilon_{qz}\alpha_z + \varepsilon_{qy}\lambda_t)$$

$$X_{1t} = -\frac{\sigma_y^2}{2} - (\alpha_z + \alpha_q\varepsilon_{qz}\alpha_z)\frac{\sigma_z^2}{2} + \alpha_q\omega_I \ln\left(\ell_t s_t\right)$$

$$X_2 = \sigma_y^2 + (\alpha_z + \alpha_q\varepsilon_{qz}\alpha_z)^2\sigma_z^2.$$

Two aspects of the recursion are worth noting. The shifter X_{1t} in (19) is increasing in the spending rate s_t and in labor supply ℓ_t. The law of motion of the variance (20) is the mathematical expression of the Great Gatsby curve—the positive relationship between the level of inequality Σ_h and the strength of the intergenerational transmission of status ρ_t. The ability- and income-sorting elasticities enter through ρ_t, while X_2 depends only on the ability-sorting margin $\alpha_q\varepsilon_{qz}\alpha_z$—an asymmetry we return to below.

3.7. Aggregate Output and the Two Margins.

From (3) and the log-normal property, aggregate output satisfies

$$\ln Y_t = \lambda_t \left(m_{ht} + \tfrac{1}{2}\Sigma_{ht}^2 \right) + \ln \ell_t .$$

Output loads on both the mean and the variance of $\ln h$, because the right tail of a log-normal distribution of human capital disproportionately contributes to aggregate production. Keeping the median m_t fixed, higher inequality in human capital is thus associated with higher aggregate output—which entails an equity-efficiency trade-off.

The two margins of sorting do not play a symmetric role in this trade-off. Both raise the IGE and inequality through ρ_t, but only the ability-sorting margin $\alpha_q \varepsilon_{qz} \alpha_z$ increases X_2 and thus the dispersion of human capital, since $X_2 = \sigma_y^2 + (\alpha_z + \alpha_q \varepsilon_{qz} \alpha_z)^2 \sigma_z^2$ depends exclusively on $\varepsilon_{qz} \alpha_z$. As a result, the ability-sorting channel has a stronger impact on aggregate output, reflecting the complementarities across peers, and between the peer-effects and educational resources.

4. Rationalizing Trends in Higher Education

In this section we show that shifts in the returns to human capital λ can rationalize long-run U.S. trends in higher education and that they increase the strength of the income-sorting channel. More specifically, such shifts can generate: (a) wider dispersion in expenditures per student across colleges (Capelle, 2019); (b) stagnation of the share of students from the lowest income quintile in top colleges despite increases in financial aid (Bailey and Dynarski, 2011; Chetty, Friedman, Saez, Turner, and Yagan, 2020); (c) rising real tuition before and after financial aid; (d) a likely increase in the IGE (Davis and Mazumder, 2026); and (e) rising income inequality (Piketty and Saez, 2003; Autor, Katz, and Kearney, 2008). Focusing on λ is natural given the broad consensus that rising returns to skill are a primary driver of inequality (Katz and Murphy, 1992; Autor, Katz, and Kearney, 2008).[18] Proposition 4.1 formalizes the

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[18] We do not take a stand on the exact source of the increase in the return to human capital. Many forces have contributed: skill-biased technical change, capital-skill complementarity, better assortative matching of workers and firms, rising assortative mating and the number of single households, and greater substitutability across skills due to international trade or communication technology.

key comparative static. The proof in Appendix A.9 is derived for the more general policy-augmented environment introduced in Section 5.

Proposition 4.1. *Assume the economy starts from a steady state at* $t = 0$ *in the baseline model of Section 2. Consider a weakly increasing sequence* $\{\lambda_t\}_0^{+\infty}$*. Then along the equilibrium path,*

a) The Gini coefficient of colleges' log expenditures per student (and quality) increases.

b) The ratio of the variance of log income within a college over the variance of log income in the economy decreases.

c) The average expenditure for colleges as a share of income increases.

d) The intergenerational elasticity increases.

e) The Gini coefficients of human capital and income increase.

Higher Inequality, More Dispersion across Colleges and a Stronger Income-Sorting Channel: Facts (a), (d), (e). A higher return to human capital widens the distribution of income for a given distribution of human capital, raising the Gini of earnings [fact (e)]. Because all households in the baseline spend the same share of income on higher education, dispersion in income translates one-for-one into dispersion in the willingness to pay for college. The equilibrium tuition schedule adjusts accordingly: top colleges attract higher-income families, raise spending, and equilibrium tuition at the top rises, widening the dispersion of expenditures per student [fact (a)]. The same mechanism strengthens the income-sorting channel: $\omega_I \lambda_t$ rises one-for-one with λ_t, and with it the income-sorting margin $\alpha_q \omega_I \lambda_t$ that enters the IGE. Because $\rho_t = \alpha_z + \alpha_q \omega_z \alpha_z + \alpha_q \omega_I \lambda_t$ and only $\omega_I \lambda_t$ moves, the IGE rises [fact (d)]. This makes the economy move along the Great Gatsby curve: higher inequality strengthens the college-stage transmission of economic status, which feeds back into yet higher inequality (Corak, 2013).

Misallocation and Within-College Sorting: Fact (b). While $\omega_I \lambda_t$ rises in absolute terms with λ_t, the ability-sorting elasticity $\omega_z \alpha_z$ remains unchanged. The income-sorting channel therefore strengthens relative to the ability-sorting channel as well, and the ratio $\omega_z \alpha_z / \omega_I \lambda_t$ that governs within-college sorting (Proposition 3.5) falls. Top colleges become more homogeneous in parental income—high-ability students from low-income families are priced out while students from rich families buy their way in [fact (b)]. This is the misallocation effect of higher λ_t. This increase in misallocation implies that the rise in quality at top colleges due to increasing concentration of resources is partially mitigated by the decline in the share of high-ability students due to the weakening of the ability-sorting channel.

Spending on Higher Education and Tuition: Fact (c). The share of income spent on higher education rise because higher returns strengthen households' incentives to invest [fact (c)]. As a result average tuition rises, together with average spending in higher education. The same demand-driven mechanism simultaneously raises the level and the dispersion of tuition.

Why Colleges Accommodate the Widening Dispersion. To obtain fact (a) the endogeneity of the college quality distribution is essential and constitutes a key feature of this paper. In response to changes in the equilibrium tuition schedule caused by shifts in λ_t, quality-maximizing colleges respond endogenously. A top college that failed to raise quality in response to the higher willingness to pay of richer households would be displaced by another offering higher quality, pushing up expenditures at the top in equilibrium. The mechanism is akin to the revenue theory of cost of Bowen (1980), applied to a hierarchy of colleges rather than to a single institution.[19]

5. Taxes, Transfers and Financial Aid in Higher Education

The model of Section 2 abstracted from government intervention. We now allow for progressive income taxation and several higher education policies: merit- and need-based student aid, and progressive public transfers to colleges. We also allow colleges themselves to engage in price-discrimination based on parental income. We use the log-linear tax and transfer schedules introduced by Persson (1983), Benabou (2002) and Heathcote, Storesletten, and Violante (2017): they fit the empirical schedules well and preserve the tractability of the baseline framework. The contribution of this

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[19] Bowen summarizes his theory (p. 19): (1) The dominant goals of institutions are educational excellence, prestige, and influence. (2) In quest of excellence, prestige, and influence, there is virtually no limit to the amount of money an institution could spend for seemingly fruitful educational ends. (3) Each institution raises all the money it can. (4) Each institution spends all it raises. (5) The cumulative effect is toward ever-increasing expenditure.

section is to show how policy parameters shape the sorting elasticities, $\varepsilon_{qz}\alpha_z$ and $\varepsilon_{qy}\lambda_t$.

5.1. Government

The government implements three instruments specific to higher education—non-linear merit- and need-based financial aid to college students, and non-linear transfers to colleges—and two that not are not—a linear consumption tax and a progressive income tax.

Progressive Income Tax. The household labor income is subject to a progressive tax schedule with the average tax rate, a_y, and its progressivity, τ_y. The after-tax-and-transfer lifetime earnings y is given by

$$y = (1 - a_y) y_m^{1-\tau_y} T_y \tag{21}$$

where y_m is before-tax-and-transfer lifetime earnings and T_y is a normalizing aggregate endogenous factor ensuring that a_y parametrizes the average tax rate. The non-linear schedules for financial aid and the college subsidy are in the same spirit as this income tax schedule.

Merit and Need-Based Financial Aid. Financial aid is allowed to be progressive with income and merit-based with abilities:

$$e(q, z, y) = T_n z^{-\tau_m} y^{\tau_n} \frac{e_u(q, z, y)}{(1 + a_n)} \tag{22}$$

where $e(q, z, y)$ is the after financial aid net tuition faced by households, as specified in (4) and $e_u(q, z, y)$ is the before financial aid price, commonly referred to as the sticker price. τ_m is the slope of the merit-based subsidy, τ_n is the rate of progressivity of the need-based subsidy and T_n ensures that a_n is the average financial aid to students.

Transfers to Colleges. Financial transfers to colleges by states and the federal government are large and highly progressive, in the sense that colleges that spend less per student receive relatively more subsidies, as is documented in a companion paper Capelle (2019). This progressivity is closely related to the location of public and private colleges in the distribution of quality. Other papers modeling the higher education sector differentiate between public and private colleges. In contrast, we do not specify any ex ante differences across colleges.[20] In our model, the bottom and middle of the distribution of qualities, i.e. the colleges that receive relatively more transfers from the government, can be interpreted as public colleges. This way of modeling government transfers allows us to keep the model tractable while capturing most of the heterogeneity in government transfers along the quality distribution. Taking into account these transfers, the budget constraint of a college is:

$$I = T_u(1 + a_u) \left(E_{z,y}[e_u(q, z, y)]\right)^{1-\tau_u} \tag{23}$$

where τ_u is the degree of progressivity of subsidies to universities and T_u ensures that a_u is the average amount of transfers per student received by colleges. Capelle (2019) shows that the functional form in (23) is a good approximation of the data.

5.2. College Need-Based Aid and Social Objective

We also allow colleges themselves to internalize a social objective and offer need-based aid, motivated by the fact that universities increasingly emphasize efforts to recruit low-income students. We endogenize this behavior by giving colleges a social objective.[21] The social objective is modeled as follows. A college's payoff is increasing in the quality of higher education, as in the previous section, and decreasing in the geometric average of parental incomes, $\bar{y}_j$, and this penalty is parametrized by $\omega_y > 0$:

$$\ln q_j - \omega_y \ln \bar{y}_j \tag{24}$$

$$\text{with} \quad \ln \bar{y}_j = E_{\phi_j(z,y)}[\ln(y)] \tag{25}$$

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[20] It is unclear what differentiates public colleges' objectives and constraints from non-profit private ones beyond the fact that the former receive public subsidies but not the latter. One common additional assumption in the literature is that tuition fees at public universities are subject to specific constraints. For example, Epple, Romano, and Sieg (2006) and Cai and Heathcote (2022) assume that tuition fees at public colleges are exogenous, which corresponds to the notion that tuition fees are fixed by states' legislatures. But decentralization policies have given public colleges significant autonomy in their tuition and hiring policies (Mc Guinness, 2011). For-profit colleges do have a clearly different objective, but they make up a very small part of total enrollment.

[21] Colleges could give need-based aid to students not because of a social objective but because of parents with higher income are less elastic to prices and therefore higher mark-up, as in Epple, Romano, and Sieg (2006). Colleges do not discriminate by parental income in Cai and Heathcote (2022).

A college maximizes (24) subject to the technology for quality (8), the definition of the peer effect (9), the after-subsidy budget constraint (23) and the definition of average parental income (25).

Government Budget Constraints. Revenues (income tax and consumption tax) must equal spending (transfers to colleges and students) at all periods. In addition, the variables T_u, T_y, T_n are pinned down by the constraints that a_y, a_n, a_u are respectively the average rate of income tax, financial aid and transfers to college. We give more details in Appendix A.3.1.

5.3. Properties of the Policy-Augmented Equilibrium

We now extend the closed-form characterization of Section 3 to the policy-augmented environment. The IGE can still be decomposed into a before-college component, an ability-sorting margin, and an income-sorting margin, and we show how each margin is shaped by policy parameters.

Sorting Rule and Intergenerational Persistence. The log-linear form of the tuition schedule, the sorting rule, and the aggregate log-normal law of motion all carry over from Section 3 (full expressions can be found in Appendix A). The IGE decomposition takes the same form as before

$$\tilde{\rho}_t = \alpha_z + \alpha_q \varepsilon_{qz} \alpha_z + \alpha_q \varepsilon_{qy} \lambda_t, \tag{26}$$

but the income- and ability-sorting elasticities now incorporate policy parameters

$$\varepsilon_{qy} = \Big(\vartheta_{It}(1-\tau_{ut})(1-\tau_{nt}) - \vartheta_{yt}\Big)(1-\tau_{yt}), \qquad \varepsilon_{qz} = \Big(\vartheta_{zt} + \vartheta_{It}(1-\tau_{ut})\tau_{mt}\Big) \tag{27}$$

with

$$\vartheta_{l,t} = \frac{\omega_l}{1-\nu_t \omega_y} \quad \text{for } l = I, z, y, \tag{28}$$

where ν_t is the endogenous elasticity of mean parental income within a college to quality. The factor $1/(1-\nu_t\omega_y)$ that transforms ω_l into $\vartheta_{l,t}$ reflects the cross-subsidization from high-income to low-income families within a college implied by the social objective. When colleges have a social objective, tuition fees for a family with a given income y increase less steeply with college quality: a family with income y becomes poorer relative to the within-college mean parental income as one climbs the quality ladder since parental income rises in equilibrium with quality. The effect is stronger when ω_y and ν_t are large.[22]

Finally, the equilibrium household spending rate on college takes the form

$$s_t = \frac{\beta\alpha_q\vartheta_{It}(1-\tau_{ut})U_{t+1}}{1+\beta\alpha_q\vartheta_{It}(1-\tau_{ut})U_{t+1}}, \tag{29}$$

which is decreasing in τ_{ut}.

Existence and uniqueness of the steady state and equilibrium path are established in Appendix A.8: local stability obtains under an intuitive sufficient condition, and global stability obtains for ω_y small enough.

Discussion on Trade-offs. The three policies that can directly affect income-sorting—progressive transfers to colleges (τ_u), progressive need-based federal aid (τ_n), and institutional need-based aid implied by the colleges' social objective (ω_y)—face trade-offs. On the one hand, these three instruments dampen the income-sorting elasticity $\varepsilon_{qy}\lambda_t$ and address the misallocation of students. These instruments relax the borrowing constraint for high-ability but constrained students and reallocate them toward higher-quality colleges, improving the sorting of students as reflected in a decline in the within-college variance of ability, governed by $\varepsilon_{qz}\alpha_z/\varepsilon_{qy}\lambda_t$. In principle, $\varepsilon_{qy}\lambda_t$ could even turn negative if ω_y, τ_n or τ_u are large enough that $\vartheta_{It}(1-\tau_u)(1-\tau_n) < \vartheta_{yt}$.

On the other hand, these policies also entail economic costs. We highlight a novel trade-off: because the income-sorting channel is tightly linked to the allocation of financial resources, these instruments simultaneously compress the dispersion of resources across colleges, which may reduce aggregate output given complementarities between ability and expenditures. These instruments also imply a more traditional cost: they all lower the household spending rate s_t, thereby reducing aggregate resources available in higher education. The net effect on efficiency is therefore ambiguous and depends on the relative strength of these opposing forces.

While all three instruments operate through similar channels, they are not equivalent. Progressive transfers to colleges directly flatten the distribution of resources across colleges and therefore lower the return to private spending, in addition to

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[22] If inequality increases for exogenous reasons—as in our comparative statics with respect to λ_t—the endogenous increase in ν_t provides a partial mitigating force by making colleges willing to endogenously redistribute more across students, provided $\omega_y > 0$.

their indirect effect through $\varepsilon_{qy}\lambda_t$. A flatter distribution of resources may entail large efficiency costs given the complementarities between resources and student ability while a lower return decreases incentives to invest in higher education (s_t). There is a slight difference between ω_y and τ_n. Only ω_y amplifies the ability-sorting elasticity $\varepsilon_{qz}\alpha_z$, since $\vartheta_{zt} = \omega_z/(1 - \nu_t\omega_y)$ increases with ω_y.

This asymmetry is useful for policy design: need-based aid has a more direct effect on the allocation of students by compressing income sorting, whereas transfers to colleges shape the level and dispersion of resources and incentives. The optimal policy mix can therefore act on both margins and decouple the allocation of students from the allocation of resources.

Finally, merit-based aid (τ_m) also faces trade-offs. On the one hand it can directly improve the sorting of students by ability and indirectly allocate additional resources to better colleges. On the other hand, it amplifies intergenerational persistence through the ability-sorting channel, it increases in inequality and it doesn't address the key source of misallocation: the income-sorting channel due to the borrowing constraint.

While we discuss ω_y given its increasing role in colleges' stated need-based aid policies in recent years and τ_m for analytical completeness, the quantitative analysis in Section 7 focuses on the two instruments that map directly to government policies that are used in practice: need-based aid (τ_n) and transfers to colleges (τ_u).

6. Quantitative Model: Calibration and Validation

The model of the previous section delivered a sharp analytical decomposition of intergenerational persistence into a before-college component and two college-stage margins. We now extend this framework to incorporate several realistic features, including an enrollment margin and intergenerational financial transfers (bequests and student loans). We calibrate the model to match important moments of the data. We show that it replicates the untargeted within-college distribution of parental income by college quality quintile reported by Chetty, Friedman, Saez, Turner, and Yagan (2020), and can generate a significant portion of the five long-term changes described in Section 4 following an increase in the returns to human capital λ when comparing the 1980 and 2010 steady states.

6.1. Quantitative Extension

We extend the baseline model along three dimensions. First, we relax the restrictions on intergenerational financial transfers: both positive (parental bequests) and negative transfers (student debt) are allowed up to a limit:

$$\mathscr{U}(h,z,a) = \max_{c,\ell,q,a'} \{\ln c - \ell^{\eta} + \beta E(\mathscr{U}(h',z',a'))\} \tag{30}$$

$$y + (1+r)a = c(1+a_c) + e(q,z,y) + a' \tag{31}$$

$$a' \geq \underline{a} \tag{32}$$

where r denotes the interest rate and $\underline{a}$ is the exogenous borrowing limit.

Households now face a portfolio problem: they trade off bequests against higher education for their children. High-ability children from low-income families take up loans; low-ability children in rich families are more likely to receive a financial bequest. Allowing these transfers weakens, but does not eliminate, the link between parental income and the child's position on the college ladder, since the borrowing limit $\underline{a}$ remains binding for a meaningful share of households. Relative to the baseline, the share of income spent on tuition is now heterogeneous across households and increasing in child ability.

Second, we add an outside option delivering quality $\underline{q}$ for free:

$$e(\underline{q},y,z) = 0 \quad \forall\,(z,y). \tag{33}$$

Individuals for whom $q = \underline{q}$ is optimal take the outside option, giving rise to an enrollment margin that was absent from the baseline. If $\underline{q} > 0$, no household chooses $q < \underline{q}$ in equilibrium, so the quality distribution has a Dirac mass at $\underline{q}$ and the enrollment rate is the share of households with $q > \underline{q}$. This margin matters for policy: by lowering the out-of-pocket cost for low-income families, need-based aid affects not only which college enrolling students attend but also whether they enroll at all.

Third, we generalize the law of human-capital accumulation (5) to allow a direct transmission term from h to h' that is not mediated by ability or college quality, and to allow a non-unitary elasticity on z:

$$h' = z^{\zeta} q^{\alpha_q} h^{\alpha_y} \xi_y. \tag{34}$$

We find in our estimation that this generalization is supported by and better matches the micro-data. All other technological constraints are inherited from the policy-augmented environment of Section 5.[23] The baseline is the special case $a' = \underline{q} = 0$, $\zeta = 1$, $\alpha_y = 0$.

With this generalization, household policy functions and distributions lose their closed forms: the outside option introduces a mass point, and the portfolio choice makes the share of income spent on tuition a non-trivial function of (z, y). In the decomposition of the IGE, the contribution of the ability- and income-sorting channels no longer have closed-form expressions.

6.2. Data and Calibration

The core dataset is the restricted-use version of the NLSY-1997, a representative annual panel of individuals who were 12 to 17 years-old in 1997. It features data on parental income, abilities measured by a common comprehensive test score, the Armed Services Vocational Aptitude Battery (ASVAB), a detailed description of their path through the higher education system—each college they attended, the time spent and the degree obtained—and their labor earnings. The ability measure is normalized to follow a standard normal distribution.[24]

To calibrate the parameters related to financial aid, we use the restricted-use NCES-NPSAS in 2000, which is the closest survey to the average year when individuals in the NLSY go to college. It is a representative survey of students that features detailed information about parental income, out-of-pocket college costs and financial aid disaggregated by source—federal government, state and institutional.

The publicly available NCES-IPEDS annual surveys provide college-level information on expenditures, revenues, enrollment and the distribution of test scores within each college. We use the 2000 to 2004 surveys. Finally we complement these data with statistics on enrollments from the NCES and measures of aggregate spending for higher education from the OECD.

External Calibration. Out of the 21 parameters to calibrate, we set 12 without solving the model. The list of externally calibrated parameters is given in the first

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[23] In Appendix A.10, we discuss the colleges' problem in the quantitative framework.

[24] The test score is given on an arbitrary scale. Our renormalization is a convenient and model-consistent rescaling.

column of Table 1.

From the OECD, we compute a_u by dividing the total amount of public subsidies by the total revenues before public aid. According to the specification for subsidies to university, τ_u can be estimated in a weighted least-square regression of log total revenues per student on log revenues before public transfers in the cross-section of colleges, where the weights are given by students enrollment. We run this regression in a companion paper Capelle (2019) and find $\tau_u = 0.35$ at the beginning of the 2000s.

The income tax schedule parameters a_y, τ_y are informed by the average tax rate and the slope of the income tax schedule estimated by Heathcote, Storesletten, and Violante (2017).[25] We calibrate a_y using the ratio of federal, state and local governments revenues over GDP, $a_y = 0.3$ (Office of Management and Budget, 2010; Urban Institute and Brookings Institution Tax Policy Center, 2010).

In the NCES-NPSAS dataset, one observes parental income $y_{m,i}$, test score, institutional aid, government aid as well as out-of-pocket costs. Regressing the (log) ratio of after-government aid payment on before-government aid payment over parental income and student ability gives τ_n and τ_m. We use the average financial aid received by students from their state and the federal government to calibrate a_n. To estimate the progressivity of institutional financial aid ω_y—"the social objective parameter"—we run a regression of before-government aid payment on college fixed effects, parental income and student ability. The parameter is identified by the elasticity of before-government aid tuition to parental income.

We use estimates of the Frisch elasticity of labor supply from the literature to calibrate η (Chetty, Guren, Manoli, and Weber, 2011).[26] The returns to education is set to $\lambda = 0.67$ following the value used in Benabou (1996) based on empirical estimates of the elasticity of output to human capital. The generation length is set to 30 years. We set the interest rate to 3.5% consistent with long-term averages in the U.S. For $\underline{a}$, we target the official limit on student loans, as a percentage of lifetime GDP per capita, which amounts to 3%.[27]

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[25] In order to calibrate τ_y, we take the midpoint between the value estimated by Heathcote, Storesletten, and Violante (2017) (0.2) and the one matching the ratio between the market income and after tax and transfers Gini in the U.S in 2000 (0.26), which gives $\tau_y = 0.23$.

[26] The Frisch elasticity of labor supply is given by $\frac{1}{\eta - 1}$. Micro estimates range from 0.2 to 0.8 and macro estimates range from 0.8 to 3 (Chetty, Guren, Manoli, and Weber, 2011). Our preferred estimate is the median value 1 which implies $\eta = \left(1 + \frac{1}{1}\right) = 2$.

[27] We compute the borrowing limit from the aggregate undergraduate Direct Loan limits reported by the U.S. Department of Education: \$31,000 for dependent undergraduates and \$57,500 for

Table 1: Externally Calibrated Parameters

Parameter	Description	Value	Source
a_u	Average transfer to colleges	0.40	OECD
τ_u	Elasticity of transfers to colleges	0.35	IPEDS
a_y	Average income tax rate	0.30	OMB (2010), Brookings (2010)
τ_y	Progressivity of income tax	0.23	Heathcote et al. (2017)
τ_n	Progressivity of need-based subsidies	0.11	NPSAS
τ_m	Progressivity of merit-based subsidies	0.00	NPSAS
a_n	Average financial aid	0.20	NPSAS
ω_y	Social objective of colleges	0.00	NPSAS
r	Interest rate	3.5%	Standard
λ	Return to human capital	0.67	Benabou (1996)
$\underline{a}$	Borrowing limit	3.0%	U.S. Dpt. of Education
η	Inverse elasticity of labor supply	2.0	Chetty et al. (2011)

Internal Calibration The remaining parameters are jointly calibrated by simulated method of moments, treating the U.S. economy as being in steady state in 2010. Results are reported in Table 2. While identification of the structural parameters is joint, we give a heuristic identification argument by linking each parameter to the moment most informative for it.

To calibrate $\underline{q}$—the outside option to college—it is natural to target the enrollment rate: the lower $\underline{q}$, the stronger the incentives to go to college. The immediate enrollment rate, provided by the NCES, in the U.S. in the 2000s is about 70%.

We calibrate the standard deviation of the birth shock σ_z so that the model-implied equilibrium standard deviation of log ability, which also depends on the dispersion of human capital h, is one, consistent with our normalization of ability in the data. The Gini coefficient of income is used to inform the variance of labor market shocks, σ_y^2. The Congressional Budget Office estimates that the Gini on U.S. household income after taxes and transfers is about 0.45 (Congressional Budget Office, 2024).[28]

The intergenerational rate of preference β governs altruism toward children and is connected to the share of young adults who borrow, since more altruistic parents transmit more bequests. The parameter ω_I governs the income-sorting margin: it is the elasticity of college quality to parental income in the sorting rule (16). Since college quality is unobserved, we identify it by matching the regression coefficient of

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independent undergraduates. Taking the midpoint and dividing by U.S. GDP per capita in 2010 over a 30-year model period gives 3%.

[28] Using panel administrative data, Kopczuk, Saez, and Song (2010) find a similar estimate for the Gini coefficient of lifetime labor earnings.

log teaching expenditures I on log parental income y (controlling for ability) in the model and the data. The ability-sorting margin primitive ω_z is pinned down by the normalization $\omega_z = 1 - \omega_I$, which is without loss of generality given that the scaling of overall returns to college quality is absorbed by α_q.

We finally turn to the coefficients governing the law of motion of human capital (34). To estimate the elasticity of child ability to parental income α_z, we match the regression coefficient of child ability z on family income y_m in the NLSY97. To estimate the intergenerational elasticity of child earning to parents' income α_y, we match the regression coefficients of the child's labor earnings on parents' income (IGE).[29] To estimate the elasticity of child earnings to their own before-college ability ζ, we match the regression coefficient of the child's future earnings on their high-school ability. Finally, the coefficient on college quality α_q in the production of future child human capital governs the returns to education and thus the willingness to pay for education, and is connected to higher education spending as a share of GDP. We target the share of private spending for higher education in GDP in the U.S. over the period 2000-2004, which is 1.3% (OECD).

Table 2: Internally Calibrated Parameters

Parameter	Description	Value
$\underline{q}$	Quality of outside option	0.002
σ_z	Standard deviation of birth shock	2.96
σ_y	Standard deviation of labor market shock	1.22
β	Intergenerational rate of preference	0.22
ω_I	Expenditure effect in colleges ($\omega_z = 1 - \omega_I$)	0.78
α_z	Elasticity of child ability to parents human capital	0.37
α_y	Elasticity of human capital to parental human capital	0.31
ζ	Elasticity of human capital to ability	0.21
α_q	Elasticity of human capital to college quality	0.31

6.3. Model Fit and Validation

Table 3 shows that the model matches the targeted moments closely. We now validate the model in two different ways.

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[29] Throughout the quantitative sections, the IGE refers to the intergenerational elasticity of earnings—the slope of the child's $\log y'_m$ on the parent's $\log y_m$.

Table 3: Targets for Internal Calibration

Description	Source	Data	Model
Enrollment rate	NCES	70%	71%
Standard deviation of ability	Standardization	1.00	1.14
Income Gini coefficient	Congressional Budget Office (2024)	0.45	0.41
Share of students who borrow	NCES	65%	65%
Elasticity of I to y in sorting rule	NLSY & IPEDS	0.15	0.15
Elasticity of z to y_m	NLSY	0.43	0.44
Elasticity of y'_m to z	NLSY & IPEDS	0.31	0.33
Elasticity of y'_m to y_m	NLSY	0.35	0.35
Private spending on higher education	OECD	1.3%	1.3%

Notes: Private spending on higher education is in percent of GDP. Enrollment rate is the first time enrollment rate. The elasticity of I to y is the regression coefficient on $\ln(y)$ in the regression of $\ln(I)$ on a constant, $\ln(y)$ and $\ln(z)$.

Validation 1: Within-College Parental Income Distributions. The first validation asks whether the calibrated model generates realistic within-college distributions of parental income across the college quality ladder, an object that directly reflects the two sorting margins. We use the distributions of parental income within college-quality quintiles reported by Chetty, Friedman, Saez, Turner, and Yagan (2020), where colleges are sorted by average parental income.[30] Figure 1 shows a reasonably close match quintile by quintile.

Validation 2: Increasing the Returns to Human Capital (1980 vs. 2010). The second validation is the quantitative counterpart of the comparative static in Section 4: can a realistic rise in λ generate changes consistent with the untargeted facts (a)–(e)? Our approach is to compare a steady-state in 1980 to our baseline equilibrium in 2010.[31] We denote x_{1980} and x_{2010} the value of a parameter x in each steady state. For 1980, we recalibrate three parameters: λ_{1980} to match the 2010–1980 change in the market-income Gini reported by the CBO (+0.07); $\underline{q}_{1980}$ to match the 50% enrollment rate reported by NCES in 1980; and r_{1980} to match the 22% lower wealth-to-output ratio reported by BEA. We also set $\tau_{u,1980} = 0.5$ to match the decline in the progressivity of college transfers (Capelle, 2019). The resulting 1980-steady-state parameters are $\lambda_{1980} = 0.56$, $\underline{q}_{1980} = 0.002$, $r_{1980} = 3.85\%$.

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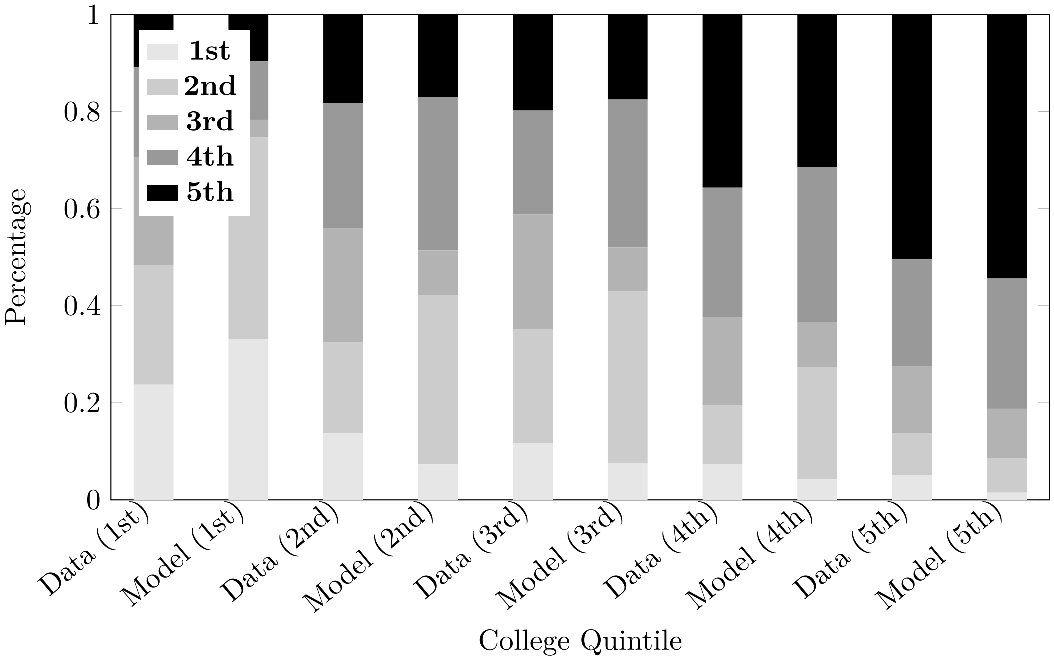
[30] The ranking of colleges is robust to using median parental income, average kid income, or average spending per student.

[31] Convergence to the steady state is fast: Proposition 3.6 implies a half-life of $\ln(1/2)/\ln(0.35) = 0.65$ generations given the IGE of 0.35.

Figure 1: Income Distribution by College Quintile (Data and Model)



Notes: The figure reports the distribution of parental income within each quintile of college quality. The underlying data source is Chetty, Friedman, Saez, Turner, and Yagan (2020).

Comparing steady states, we find that the the model can rationalize a substantial portion of the five untargeted changes. Table 4 summarizes the comparison: the Gini of college expenditures per student rises by 3.4 percentage points (pp) *vs.* 6pp in the data; the share of bottom-quintile students at top colleges falls; tuition spending as a share of GDP rises by 0.70pp *vs.* 0.9pp in the data (National Center for Education Statistics, 2023); and the IGE rises by 16.2%, within the range of empirical estimates.[32] By design of the exercise, we perfectly match the change in the Gini of income.

7. Quantitative Exercises

We now use the calibrated model to do three exercises. First we quantify the contribution of the ability- and income-sorting channels to the IGE and how they have changed with the increase in the returns to human capital. Second we run two counterfactuals that quantify how the sorting of students and expenditures shape not

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[32] Estimates of how the IGE has changed over time vary widely, from essentially no change (Lee and Solon, 2009) to a 60% increase (Davis and Mazumder, 2026).

Table 4: Change in λ: 1980 vs. 2010 Steady-state Comparison

Moment	**1980**	**2010**
Gini of college expenditures per student	0.083	0.118
Bottom-income-quintile share at top-quintile colleges (%)	5.2	1.5
Tuition spending / GDP (%)	0.61	1.31
IGE of earnings (×100)	30.0	34.9
Ability-sorting contribution (×100)	2.13	2.70
Income-sorting contribution (×100)	2.16	4.43
Ability-sorting share (%)	49.6	37.9
Income-sorting share (%)	50.4	62.1

Notes: "IGE of earnings (×100)" is the slope of the child's $\log y'_m$ on the parent's $\log y_m$ in the simulated panel, ×100. The ability- and income-sorting contributions and shares are computed via the chain-rule decomposition (35)–(36) on the simulated panel; contributions are reported ×100 on the same scale as the IGE.

only the IGE, but also income inequality and aggregate output. Third we analyze the welfare-maximizing mix of progressive need-based aid and progressive subsidies to colleges to address misallocation of students while preserving incentives to invest in higher education and the differentiation of expenditures across colleges. The strengths of the sorting channels are reported in Table 5 and the aggregate outcomes are in Table 6. All quantitative analyses compare stationary equilibria.

7.1. Contributions of Margins to the IGE

Because the IGE no longer admits a closed-form decomposition, we estimate the contribution of the ability- and income-sorting channels to the IGE by running three OLS regressions on a large panel of households drawn from the stationary distribution of the model. First from a regression of $\log q$ on $\log y_{m,p}$ and $\log z$ we obtain the partial elasticities of college quality to parental market income and to ability, denoted $\widehat{\varepsilon}_{q,y_m}$ and $\widehat{\varepsilon}_{qz}$. Second regressing $\log z$ on $\log y_{m,p}$ yields $\widehat{\varepsilon}_{z,y_m}$, the relationship between parental income and child ability. Third regressing $\log y_m$ on $\log h$ yields $\widehat{\varepsilon}_{y_m,h}$, the average elasticity of market income to human capital. Using the chain rule, the contribution of each sorting channel is given by

$$\text{Ability-sorting contribution:} = \widehat{\varepsilon}_{y_m,h} \cdot \alpha_q \cdot \widehat{\varepsilon}_{qz} \cdot \widehat{\varepsilon}_{z,y_m}, \tag{35}$$

$$\text{Income-sorting contribution:} = \widehat{\varepsilon}_{y_m,h} \cdot \alpha_q \cdot \widehat{\varepsilon}_{q,y_m}. \tag{36}$$

Table 5: IGE and Contribution of Margins

	Policy	**IGE**	**Contribution (Level)**		**Contribution (Share, %)**	
Counterfactual	**Value**	**Level**	**Ability**	**Income**	**Ability**	**Income**
Baseline (status quo)	–	34.9	2.7	4.43	37.9	62.1
Counterfactual Sorting						
Random Admission	–	26.3	0.0	0.0	–	–
Equal Resources	–	29.3	1.92	0.0	100.0	0.0
Welfare-Maximizing Policies						
Optimal τ_n	0.238	34.0	2.84	3.56	44.4	55.6
Optimal τ_u	0.105	36.9	2.85	6.14	31.7	68.3
Optimal (τ_n, τ_u)	$0.721 / -0.087$	25.6	3.42	-1.63	191.0	-91.0

[1] Notes: All level values (IGE and contributions) are reported ×100, on the same scale as the IGE level. The IGE column shows the intergenerational elasticity of earnings, the regression slope of the child's $\log y'_m$ on the parent's $\log y_m$ in the simulated panel (consistent with the calibration target in Table 3). The status quo policy values are $\tau_n = 0.11$ and $\tau_u = 0.35$. The ability- and income-sorting contributions are computed via the chain-rule decomposition (35)–(36) on the simulated panel. Shares report each contribution as a fraction of the sum.

Table 6: Equilibrium Outcomes

	Policy	% Change from Status Quo				
Counterfactual	**Value**	**Gini Earnings**	**Gini Exp./Stud.**	**IGE**	**GDP**	**Welfare**
Counterfactual Sorting						
Random Admission	–	-3.7	-100.0	-24.5	-10.5	-6.4
Equal Resources	–	-1.9	-100.0	-15.9	-8.0	-4.8
Welfare-Maximizing Policies						
Optimal τ_n	0.238	-0.5	-6.0	-2.5	0.8	1.0
Optimal τ_u	0.105	0.9	13.2	5.8	8.8	9.0
Optimal (τ_n, τ_u)	$0.721 / -0.087$	-0.1	25.8	-26.6	9.2	15.0
Status quo levels	–	41.1	11.8	34.9	–	–

[1] Notes: Status quo policy values: $\tau_n = 0.11$ and $\tau_u = 0.35$. See Section 7.2 for details on the counterfactuals. The Optimal (τ_n, τ_u) row is the joint welfare-maximizing pair of policies.

In the baseline 2010 economy, we find that the income- and ability-sorting channels account for 4.4 and 2.7 points of the total intergenerational persistence of 34.9, respectively (Table 5). The overall contribution of college has increased due to the rise in the returns to human capital, driven by a rise in the income-sorting channel. In a 1980 steady state, the two channels account for 2.2 and 2.1 points of a total persistence of 30.0. Taken together, these results indicate that the college channel was roughly balanced between ability and income sorting in 1980, whereas income-based sorting now accounts for the larger share of intergenerational persistence.

7.2. Counterfactuals: Sorting of Students and Resources Across Colleges

Next, we run two counterfactuals that quantify student sorting through the ability- and income-sorting margins, together with resource allocation across colleges, shape intergenerational persistence, inequality and aggregate income. They directly quantify the importance of college heterogeneity, relative to a setting with homogeneous colleges and help us understand the key trade-offs faced by the policies analyzed in Section 7.3.

Random Admission. The first exercise randomly allocates students across colleges. As a result, spending per student and average student ability are equalized across all colleges and every student receives the same higher education. By construction, this exercise eliminates both the ability- and income-sorting margins of students as well as the differentiation of expenditures across colleges.

In the counterfactual, the common college quality is given by the production function of quality (8), the average child ability in society and average government transfers per student.[33] Because households' optimal spending rate for higher education drops to zero, all the resources spent in the higher education system have to be financed through taxes and transfers to colleges. We assume government subsidies adjust so that the share of GDP going to higher education remains the same as in the status quo allocation.[34]

Quantitatively, the exercise reduces the IGE by 24.5% (Table 6). As Table 5 shows, the ability- and income-sorting contributions are both zero and the only remaining driver of the IGE is the before-college component $(\alpha_z \zeta + \alpha_y)$. Inequality falls by 3.7% and aggregate income by 10.5%. These sizable effects illustrate the importance of heterogeneous colleges and the sorting of students and resources across them in shaping

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[33] The random allocation of students not only equalizes college experiences among college-goers but implies that everyone goes to college. It thus neutralizes both the extensive (enrolling or not) and the intensive (quality) margin.

[34] The choice for the level of subsidies does not influence inequality or mobility, but it does have a first-order effect on the aggregate level of production. This assumption allows to focus on the effect of misallocation on aggregate production. In practice it means that the level of government subsidies should increase to offset the decline in private spending for higher education.

intergenerational persistence, inequality and output. Importantly, it also highlights the trade-off between output on the one hand and inequality and intergenerational persistence on the other hand discussed in Section 3 and at the core of Section 7.3.

Equal Resources. The second exercise equalizes financial resources across all colleges and perfectly sort students by ability. This neutralizes the income-sorting margin, as parental income no longer affects college quality conditional on ability, but also removes the dispersion of expenditures—recall that these two dimensions are tightly linked in the decentralized equilibrium. By construction, the college-stage channel operates entirely through ability sorting. As in the previous counterfactual, government policies offset the decline in private spending so that the aggregate higher-education spending rate is held fixed.

In the counterfactual, the IGE falls by 15.9% and the Gini of earnings by 1.9% (Table 6). Despite perfect sorting by ability, GDP declines by 8.0%, indicating that flattening the distribution of resources across colleges entails large output losses. This highlights the central trade-off between compressing the income-sorting channel and preserving differentiation in educational resources, and its quantitative importance for the design of optimal policy, to which we now turn.

7.3. Welfare-Maximizing Policies

Finally, we analyze the optimal design of two instruments that act on the income-sorting/resource trade-off: federal need-based aid (τ_n) on the demand side, and progressive transfers to colleges (τ_u) on the supply side. Both policies compress income sorting and improves the allocation of students. But need-based aid have a more direct impact on the sorting of students while transfers to colleges more directly shape the resource ladder, and therefore the incentives and expenditure dispersion. This asymmetry will prove important. To see that, we next solve for the optimal degree of progressivity of each instrument and then for the optimal mix.

Optimal Progressivity of Federal Need-Based Aid. We first solve for the degree of progressivity τ_n that maximizes welfare, holding all other policy parameters fixed. The optimum level, $\tau_n = 0.238$, is about twice as high as in the baseline. At the optimum, the income-sorting contribution falls from 4.4 to 3.6 (Table 5), while the ability-sorting contribution rises modestly, from 2.7 to 2.8. Mechanically, τ_n relaxes the financial constraint for low-income high-ability students, thereby weakening income sorting. As a result, the ability-sorting share rises to 44.4% of the total contribution of colleges to intergenerational persistence from a base of 37.9%. The IGE declines by 2.5%. Although the Gini of expenditures per student falls by 6.0%, GDP increases by 0.8%, reflecting the improvement in student sorting and increases in enrollment of low-income students. Overall welfare increases by 1.0%.

Optimal Progressivity of Transfers to Colleges. In contrast with need-based student aid, and perhaps surprisingly, the optimal progressivity of transfers to colleges is $\tau_u = 0.105$, well below its baseline level of 0.35. This implies a steeper and more dispersed allocation of resources across colleges: the Gini of expenditures across colleges increases by 13.2%. Because $\varepsilon_{qy}\lambda_t$ increases in $(1 - \tau_u)$—it is exactly proportional to it in the baseline model—reducing τ_u raises the income-sorting margin from 4.4 to 6.1, increasing its share from 62.1% to 68.3%. The ability-sorting contribution also rises but more modestly, from 2.7 to 2.8, driven by the steeper resource ladder. As a result, the IGE rises by 5.8% and inequality rises by 0.9%.

Despite the deterioration in the quality of student sorting, efficiency gains are large with GDP rising by 8.8%. The household spending rate s_t increases, as the steeper quality ladder strengthens incentives to invest in higher education. In addition, resources are reallocated toward high-quality colleges, improving the production of human capital given complementarities between ability and resources. Overall welfare rises by 9.0%.

The contrast with need-based student aid τ_n reflects the more direct effect of subsidies to colleges τ_u on the allocation of resources. Increasing the progressivity of transfers to colleges flattens the distribution of resources directly, while reducing it steepens the resource ladder. Need-based student aid instead operates indirectly through student sorting. This difference explains why lowering τ_u raises household incentives to invest in higher education and increases the resources available to colleges, especially at the top of the quality distribution.

Joint Optimum: (τ_n, τ_u). Given the single-instrument results, it is natural that combining a more progressive need-based student aid with less progressive college subsidies improves welfare. The welfare-maximizing mix sets federal need-based aid to $\tau_n = 0.721$—well above the single-instrument optimum—and public transfers to colleges to $\tau_u = -0.087$, corresponding to a slightly regressive policy. Relative to the baseline, the ability-sorting contribution rises from 2.7 to 3.4, while the income-sorting contribution turns negative, falling from 4.4 to -1.6. The IGE itself falls by 26.6%. The negative income-sorting contribution means that, conditional on ability, parental income is no longer associated with access to higher quality. Aggregate output rises by 9.2% and inequality changes little (-0.1%). As a result, welfare rises by 15.0%.

Using both instruments jointly attenuates the trade-offs each instrument faces in isolation. Highly progressive need-based student aid τ_n mitigates student misallocation by compressing the income-sorting margin and strengthening the ability-sorting margin, but tends to reduce the level and differentiation of resources in higher education, which is valuable given complementarities between ability and resources. The slightly regressive subsidies to colleges offset these effects by steepening the ladder of resources—the Gini of college expenditures increases by 25.8%—and sustaining incentives to invest. By lowering τ_u while raising τ_n, the planner breaks the link between the allocation of resources and the sorting of students by parental income and thereby improves the allocation of students while increasing the differentiation of resources across colleges.

8. Conclusion

This paper studies how higher education shapes intergenerational mobility, income inequality, and aggregate income. We develop a tractable general equilibrium model in which heterogeneous households sort across an endogenous distribution of heterogeneous colleges under borrowing constraints. A central contribution is to decompose the role of higher education in intergenerational persistence into two margins: an ability-sorting channel, which reflects efficient matching driven by peer effects, and an income-sorting channel, which captures misallocation arising from financial constraints. A key insight of the paper is that the income-sorting channel also governs the dispersion of financial resources across colleges, which is valuable given complementarities between resources and student ability. This gives rise to a novel trade-off for policies that reduce income sorting between improving the allocation of students and compressing the dispersion of resources across colleges.

Quantitatively, we find that the income-sorting channel accounts for the majority and growing share of intergenerational persistence in the United States. The rise in the returns to education since 1980 has strengthened this channel and rationalizes key trends in higher education, including rising tuition, widening dispersion in college spending, and increasing inequality. Counterfactuals show that both the sorting of students by ability and the differentiation of resources across colleges are important for aggregate outcomes, and that eliminating income-based sorting reduces intergenerational persistence but can entail sizable output losses.

The optimal policy mix balances these forces by combining highly progressive need-based aid to students with less progressive—and even slightly regressive—transfers to colleges. This combination improves the allocation of students by weakening the link between college access and parental income, while preserving the differentiation of resources across institutions that sustains productive complementarities. As a result, it raises both intergenerational mobility and aggregate output, while decreasing inequality.

There are three areas of investigation for future research. First the allocation of students across colleges in the model works through a system of clearing markets, while the real world displays a mix of price mechanism and quantity restrictions. Second, beyond the accumulation of human capital and labor market returns, higher education has non-pecuniary returns and there is evidence that households derive direct consumption value from going to college. The implications for welfare and policy analysis are likely to be far-reaching. Third, the paper has focused on the role played by tuition and public subsidies in shaping inequality in higher education, but the analysis should be extended to account for donations and endowments, which are likely to be an additional source of inequality.

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A. Analytical Model - Details

We solve the model using a guess and verify. We guess that the tuition function before government financial aid are given respectively by:

$$e_u(q,z,y) = \left(\frac{1}{(1+a_u)T_u} q^{\frac{1}{\vartheta_I}} z^{-\frac{\vartheta_z}{\vartheta_I}} \left(\frac{y}{\kappa}\right)^{\frac{\vartheta_y}{\vartheta_I}} \right)^{\frac{1}{1-\tau_u}} \tag{37}$$

A.1. Solution to the Household Problem

Proposition A.1. *In equilibrium, the sorting rule is given by*

$$q_t = \left(\frac{s_t y_t^{1-\tau_{nt}} z_t^{\tau_{mt}} (1+a_{ht})}{T_{nt}} \right)^{\vartheta_{It}(1-\tau_{ut})} ((1+a_{ut})T_{ut})^{\vartheta_{It}} z_t^{\vartheta_{zt}} \left(\frac{y_t}{\kappa_t}\right)^{-\vartheta_{yt}} \tag{38}$$

with the household spending rate

$$s_t = \frac{\beta\alpha_q\vartheta_I(1-\tau_u)U_{t+1}}{1+\beta\alpha_q\vartheta_I(1-\tau_u)U_{t+1}}. \tag{39}$$

The IGE is $\tilde{\rho}_t = \alpha_z + \alpha_q(\varepsilon_{qz}\alpha_z + \varepsilon_{qy}\lambda_t)$ *with* $\varepsilon_{qy}\lambda_t = (\vartheta_{It}(1-\tau_{ut})(1-\tau_{nt}) - \vartheta_{yt})(1-\tau_{yt})\lambda_t$ *and* $\varepsilon_{qz}\alpha_z = \alpha_z(\vartheta_{zt} + \vartheta_{It}(1-\tau_{ut})\tau_{mt})$*.*

Using the guess (37) and the expression for financial aid (22), the problem of the Households is

$$\mathscr{U} = \max_{s,\ell} \left[\ln \frac{(1-s)(1-a_y)T_y}{1+a_c} + \ln \left(h^\lambda \ell\right)^{1-\tau_y} - \ell^\eta \right] + \beta E \mathscr{U}' \tag{40}$$

$$\begin{aligned} &\ln h' = \ln \xi_y + \alpha_z(1+\alpha_q(\vartheta_z + \tau_m(1-\tau_u)\vartheta_I)) \ln \xi_z + \tilde{\rho} \ln h \\ &+ \alpha_q\vartheta_I \left(\ln \left(s(1+a_n)/T_n\right)^{1-\tau_u} (1+a_u)T_u \right) + \alpha_q(\vartheta_I(1-\tau_u)(1-\tau_{nt}) - \vartheta_y) \ln \ell^{(1-\tau_y)} \\ &+ \alpha_q\left(\vartheta_I(1-\tau_u)(1-\tau_n) - \vartheta_y\right)\left((1-\tau_{yt})\ln(1-a_y)T_y)\right) + \alpha_q\vartheta_y \ln \kappa_t \end{aligned} \tag{41}$$

with $\tilde{\rho} = \alpha_z + \alpha_y + \alpha_z\alpha_q(\vartheta_z + \tau_m(1-\tau_u)\vartheta_I) + \alpha_q(\vartheta_I(1-\tau_u)(1-\tau_n) - \vartheta_y)(1-\tau_y)\lambda$ and s the spending rate, i.e. the amount of spending for college over income. We then guess that $\mathscr{U}_t = U_t \ln h_t + Z_t \ln \xi_{zt} + B_t$. Replacing this guess into (40), then using

(41) to substitute for $\ln h_{t+1}$ and using (6) and (7)

$$\begin{aligned}
U_t \ln h_t + Z_t \ln \xi_{zt} + B_t = \max_{s,\ell} & \left[\ln \frac{(1-s)(1-a_y)T_y}{1+a_c} + (1-\tau_y)\ln h^{\lambda}\ell - \ell^{\eta} \right] \\
& + \beta \Bigg[U_{t+1} \Bigg(\mu_y + \alpha_z(1+\alpha_q(\vartheta_z + \tau_{mt}(1-\tau_u)\vartheta_I)) \ln \xi_{zt} + \tilde{\rho}_t \ln h_t \\
& + \alpha_q \vartheta_I \left(\ln \left(s(1+a_n)/T_n\right)^{1-\tau_u} (1+a_u)T_u \right) + \alpha_q(\vartheta_I(1-\tau_u)(1-\tau_{nt}) - \vartheta_y) \ln \ell^{(1-\tau_y)} \\
& + \alpha_q(\vartheta_I(1-\tau_u)(1-\tau_{nt}) - \vartheta_y)\left((1-\tau_{yt})\ln(1-a_{yt})T_{yt}\right) + \alpha_q \vartheta_y \ln \kappa_t \Bigg) + Z_{t+1}\mu_b + B_{t+1} \Bigg]
\end{aligned}$$

Gathering all the terms in $\ln h_t$ one gets that U_t has to verify
$U_t = \sum_{k=0}^{\infty} \beta^k (1-\tau^y_{t+k})\lambda_{t+k} \prod_{m=0}^{k-1} \tilde{\rho}_{t+m}$
Gathering all the terms in $\ln \xi_{zt}$, one gets $Z_t = \Big(U_t - (1-\tau_{yt})\lambda\Big) \frac{\alpha_z(1+\alpha_q(\vartheta_z + \vartheta_I(1-\tau_u)\tau_{mt}))}{\alpha^h_t}$.
Finally gathering the independent terms, the F.O.C for s and ℓ give

$$s_t = \frac{\beta \alpha_q \vartheta_I (1-\tau_u) U_{t+1}}{1 + \beta \alpha_q \vartheta_I (1-\tau_u) U_{t+1}} \tag{42}$$

$$\ell = \left[(1-\tau_{yt}) \frac{1}{\eta} \left(1 + \beta \alpha_q \left(\vartheta_I(1-\tau_u)(1-\tau_{nt}) - \vartheta_y\right) U_{t+1}\right) \right]^{\frac{1}{\eta}} \tag{43}$$

A.2. University Problem

Proposition A.2. *The equilibrium before-financial-aid tuition schedule is given by*

$$e_{ut}(q,z,y) = \left(\frac{1}{(1+a_{ut})T_{ut}} q^{\frac{1}{\vartheta_{It}}} z^{-\frac{\vartheta_{zt}}{\vartheta_{It}}} \left(\frac{y}{\kappa_t}\right)^{\frac{\vartheta_{yt}}{\vartheta_{It}}} \right)^{\frac{1}{1-\tau_{ut}}} \tag{44}$$

where $\vartheta_{l,t} = \dfrac{\omega_l}{1-\nu_t \omega_y} \quad \forall\, l = I, z, y$

with ν_t *the endogenous elasticity of mean parental income within a college to quality*

$\bar{y}_t(q) = \kappa_t q^{\nu_t}$.

We define

$$\Omega = \frac{\omega_I(1-\tau_u)}{\omega_z} \quad \text{and} \quad \ln \bar{z} = E_{\phi_j(.)}[\ln(z)] \tag{45}$$

We then provide a generalized definition of σ_u that takes into account government policies

$$\sigma_u^2 = \frac{\Omega}{2} E\left(\left(\ln\left(\bar{z}^{\frac{\omega_z}{\omega_I(1-\tau_u)}}\bar{y}^{-\frac{\omega_y}{\omega_I(1-\tau_u)}}\right) - \ln z^{\frac{\omega_z}{\omega_I(1-\tau_u)}} y^{-\frac{\omega_y}{\omega_I(1-\tau_u)}}\right)^2\right) \tag{46}$$

Using this definition and our guess for tuitions (37), one gets the following

$$\sigma_u^2 = \frac{\Omega}{2} E\left(\left(\ln e_u(q,z,y) - \ln\left(\frac{\tilde{I}}{(1+a_u)T_u}\right)^{\frac{1}{1-\tau_u}}\right)^2\right) = \frac{\Omega}{2}\tilde{\sigma}_u^2$$

where we define $\ln \tilde{I} = \ln I - \frac{\tilde{\sigma}_u^2}{2}$. We now show that $\tilde{\sigma}_u^2$ is the within-university variance of log tuition. We guess that tuition fees are log-normally distributed within the university. Denoting $\mu_{e,q}, \sigma_{e,q}$ the mean and standard deviation of log tuition within the university of quality q, the budget constraint of the university -given by (23)-becomes

$$\begin{aligned} I &= T_u(1+a_u)\left(E_{z,y}[e_u(q,z,y)]\right)^{1-\tau_u} = T_u(1+a_u)e^{(1-\tau_u)\mu_{eq}+(1-\tau_u)\frac{\sigma_{eq}^2}{2}} \\ \iff \frac{1}{(1-\tau_u)}\ln\frac{\tilde{I}}{(1+a_u)T_u} &+ \frac{\tilde{\sigma}_u^2}{2} - \frac{\sigma_{eq}^2}{2} = \mu_{e,q} = E\ln e_u(z,y) \end{aligned}$$

Substituting this last line into the expression of σ_u^2 above gives

$$\tilde{\sigma}_u^2 = \int \phi(z,y)\left(\ln e_u(z,y) - E\ln e_u(z,y) + \frac{\sigma_{eq}^2 - \sigma_u^2}{2}\right)^2 dzdy \tag{47}$$

$$\iff \tilde{\sigma}_u^2 = \sigma_{e,q}^2 + \left(\frac{\sigma_{eq}^2 - \sigma_u^2}{2}\right)^2 + 0 \quad \Rightarrow \tilde{\sigma}_u^2 = \sigma_{e,q}^2 \quad \text{or} \quad \tilde{\sigma}_u^2 = \sigma_{e,q}^2 + 4$$

$\tilde{\sigma}_u = \sigma_{e,q}$ is a solution to the quadratic equation. This verifies our guess. $\mu_{e,q} = E\ln e_u(z,y) = \ln\left(\frac{\tilde{I}}{(1+a_u)T_u}\right)^{\frac{1}{1-\tau_u}}$ and $\sigma_u^2 = \sigma_{eq}^2$ are respectively the mean and standard deviation of within-university log tuitions. Therefore we can now rewrite the problem of the university replacing I with $\tilde{I}$

$$\max_{\tilde{I},\bar{z},\bar{y},r(.)} \ln \tilde{I}^{\omega_I} \bar{z}^{\omega_z} \bar{y}^{-\omega_y} \tag{48}$$
$$\ln \tilde{I} \int_0^\infty r_{z,y} dz\, dy = \int r_{z,y} \Big((1-\tau_u) \ln(e_u)^i + \ln(1+a_u) T_u \Big)\, dz\, dy$$
$$\ln \bar{z} \int_0^\infty r_{z,y} dz\, dy = \int_0^\infty r_{z,y} \ln z\; dz\, dy \quad \text{and} \quad \ln \bar{y} \int_0^1 r_{z,y}\; dz\, dy = \int_0^1 r_{z,y} \ln y\; dz\, dy$$

where $r_{z,y}$ denotes the mass of individuals of type (z,y).

The F.O.Cs are $\quad \frac{\omega_I}{\tilde{I}} + \frac{\lambda_1}{\tilde{I}} = 0, \qquad \frac{\omega_z}{\bar{z}} + \frac{\lambda_2}{\bar{z}} = 0 \quad$ and $\quad -\frac{\omega_y}{\bar{y}} + \frac{\lambda_3}{\bar{y}} = 0$

$$r_{z,y} = \begin{cases} 0 \quad \text{if} \quad \left(\frac{1}{(1+a_u)T_u} q^{\frac{1}{\omega_I}} z^{-\frac{\omega_z}{\omega_I}} (y/\bar{y})^{\frac{\omega_y}{\omega_I}} \right)^{\frac{1}{1-\tau_u}} < e_u(q,z,y) \\ c \in \mathbb{R} \quad \text{if equality} \\ +\infty \quad \text{if strictly larger} \end{cases}$$

where we have solved for the Lagrange multipliers. We guess that in equilibrium, $\bar{y} = \kappa_t q^{\nu_t}$. Therefore whenever a college admits a certain student type, the tuition formula is:

$$e_u(q,z,y) = \left(\frac{1}{(1+a_u)T_u} q^{\frac{1-\nu\tilde{\omega_y}}{\omega_I}} z^{-\frac{\omega_z}{\omega_I}} y^{\frac{\omega_y}{\omega_I}} \kappa_t^{-\frac{\omega_y}{\omega_I}} \right)^{\frac{1}{1-\tau_u}} = \left(\frac{1}{(1+a_u)T_u} q^{\frac{1}{\vartheta_I}} z^{-\frac{\vartheta_z}{\vartheta_I}} y^{\frac{\vartheta_y}{\vartheta_I}} \kappa_t^{-\frac{\vartheta_y}{\vartheta_I}} \right)^{\frac{1}{1-\tau_u}}$$
$$\textit{with} \quad \vartheta_I = \frac{\omega_I}{1-\nu\omega_y} \qquad \vartheta_z = \frac{\omega_z}{1-\nu\omega_y} \qquad \vartheta_y = \frac{\omega_y}{1-\nu\omega_y}$$

We can solve for ν_t and κ_t using the equilibrium outcome given by the mean income in proposition 3.5. We do this later in appendix A.4.1. This confirms the guess for tuition fees (37). Given this guess for tuition, a university is always at the interior solution, therefore always indifferent between all types.

A.3. Other Equilibrium Conditions

A.3.1. Government Budget Constraints

There are two kinds of constraints. The first one is the aggregate budget constraint that states that revenues (income tax and consumption tax) must equal spending (transfers to colleges and students) at any period.

$$\int_0^1 a_y y(i) + a_c c(i) + e(i) di = \int_0^1 e(i)(1+a_u)(1+a_n) di \tag{49}$$

The other three constraints, (51), (50) and (52) pin down T_u, T_y, T_n such that a_y, a_n, a_u parametrize respectively the average rate of income tax, financial aid and transfers to college. Denoting f_q the mass of students in colleges of quality q

$$\int_0^1 y(i)^{1-\tau_y} T_y di = \int_0^1 y(i) di \tag{50}$$

$$(1+a_n)\int_0^1 e(i) di = \int_0^1 e_u(i) di \tag{51}$$

$$\int E_{z,y}[e_u(q,z,y)] f_q dq = \int T_u \left(E_{z,y}[e_u(q,z,y)]\right)^{1-\tau_u} f_q dq \tag{52}$$

Lemma 1. *Along the equilibrium path, the government budget constraints* (49),(50),(51) *and* (52) *are given by*

$$\frac{a_{c,t}(1-s_t)}{(1+a_{c,t})} = s_t(1+a_{ut})(1+a_{ht}) - \frac{a_{yt}}{1-a_{yt}} - s_t \tag{53}$$

$$\ln T_y = \tau_y \ln \ell + \tau_y \lambda m_h + \frac{\lambda^2}{2}(2-\tau_y)\tau_y \Sigma_{ht}^2 \tag{54}$$

$$\ln T_n = (-\tau_n \lambda + \alpha_z \tau_m) m_h + \frac{\alpha_z \tau_m}{2}(\alpha_z \tau_m - 1)\sigma_z^2 - \tau_n(\ln \ell(1-a_y)) \tag{55}$$

$$+ \left[\lambda^2(1-\tau_y)^2(\tau_n-2)\tau_n + 2\lambda(1-\tau_n)(1-\tau_y)\tau_m\alpha_z + (\alpha_z\tau_m)^2 - \tau_n\lambda^2(2-\tau_y)\tau_y\right]\frac{\Sigma_h^2}{2} \tag{56}$$

$$\ln T_u = \tau_u\left(\ln \ell s(1+a_n)(1-a_y) + \lambda m_h + \lambda^2 \frac{\Sigma_h^2}{2}\right) + \frac{\tau_u}{1-\tau_u}\frac{\sigma_I^2}{2} \tag{57}$$

1. Solving for the aggregate state budget constraint is immediate
2. Solving for T_y. Using (50), and the expression for market income y_m, (21), and using the guess that $\ln h$ is normally distributed one gets:

$$\int_0^1 (\ell h^\lambda)^{1-\tau_{yt}} T_y di = \int_0^1 \ell h^\lambda di \iff T_y e^{\lambda(1-\tau_y)m_h + \frac{(\lambda(1-\tau_y))^2}{2}\Sigma_h^2} = \ell^{\tau_y} e^{\lambda m_h + \frac{\lambda^2}{2}\Sigma_h^2}$$

3. Solving for T_n. Using (51), one gets:

$$(1+a_n)\int_0^1 e^i di = \int_0^1 (e_u)^i di \iff (1+a_n)\int_0^1 s y_I di = \int_0^1 \frac{sy(1+a_n)}{T_n z^{-\tau_m} y^{\tau_n}} di$$

$$T_n(1-a_y)^{\tau_n}(\ell)^{\tau_n(1-\tau_y)}(T_y)^{\tau_n} e^{\lambda(1-\tau_y)m_h + (\lambda(1-\tau_y))^2 \frac{\Sigma_h^2}{2}}$$
$$= e^{(\lambda(1-\tau_y)(1-\tau_n)+\tau_m\alpha_z)m_h - \alpha_z\tau_m\frac{\sigma_z^2}{2} + (\lambda(1-\tau_y)(1-\tau_n)+\tau_m\alpha_z)^2\frac{\Sigma_h^2}{2} + \frac{(\alpha_z\tau_m)^2}{2}\sigma_z^2}$$

4. Solving for T_u. Substituting (23) into (52), one gets

$$\int E_{z,y}[e_u(q,z,y)] f_q dq = \int T_u \left(E_{z,y}[e_u(q,z,y)]\right)^{1-\tau_u} f_q dq$$

$$\iff \int \left(\frac{I_q}{(1+a_u)T_u}\right)^{\frac{1}{1-\tau_u}} f_q dq = \int \frac{I_q}{(1+a_u)} f_q dq$$

$$\left(\frac{1}{(1+a_u)T_u}\right)^{\frac{1}{1-\tau_u}} \int I_i^{\frac{1}{1-\tau_u}} di = \frac{1}{(1+a_u)} \int I_i di$$

$$\iff \left(\frac{1}{1+a_u}\right)^{\frac{\tau_u}{1-\tau_u}} \int I_i^{\frac{1}{1-\tau_u}} di = (T_u)^{\frac{1}{1-\tau_u}} \int I_i di$$

where i indexes households. We then guess that I_i is log-normally distributed with mean μ_I and variance σ_I^2 - we give an expression for these variables in appendix A.6):

$$\left(\frac{1}{1+a_u}\right)^{\frac{\tau_u}{1-\tau_u}} e^{\frac{\mu_I}{1-\tau_u}+\frac{\sigma_I^2}{2(1-\tau_u)^2}} = (T_u)^{\frac{1}{1-\tau_u}} e^{\mu_I+\frac{\sigma_I^2}{2}}$$

$$\Rightarrow \ln T_u = \tau_u \ln\left(\frac{1}{1+a_u}\right) + \mu_I \tau_u + \frac{\sigma_I^2}{2}\frac{\tau_u(2-\tau_u)}{(1-\tau_u)}$$

Using the guess and from previous results, one gets

$$\ln E(I) = \mu_I + \frac{\sigma_I^2}{2} = \ln \ell s(1+a_n)(1+a_u)(1-a_y) + \lambda m_h + \lambda^2 \frac{\Sigma_h^2}{2}$$

$$\text{Hence} \quad \mu_I = \ln \ell s(1+a_n)(1+a_u)(1-a_y) + \lambda m_h + \lambda^2 \frac{\Sigma_h^2}{2} - \frac{\sigma_I^2}{2}$$

Substituting back into the expression for T_u gives

$$\ln T_u = \tau_u \left(\ln \ell s(1+a_n)(1-a_y) + \lambda m_h + \lambda^2 \frac{\Sigma_h^2}{2}\right) + \frac{\tau_u}{1-\tau_u}\frac{\sigma_I^2}{2}$$

I derive the expression for σ_I^2 in appendix A.6

A.4. Quality Distribution and Within-College Distributions

Parents' Education and Income. Taking the logarithm of (16): $\ln q = (\varepsilon_{qy}\lambda_t + \varepsilon_{qz}\alpha_z)\ln h + \varepsilon_{qz}\alpha_z \ln \xi_z + x$ with $x = \varepsilon_{qy}\lambda_t \left(\ln\left(s\frac{(1+a_n)}{T_n}\right)^{1-\tau_u}((1+a_u)T_u)\right) + (\varepsilon_{qy}\lambda_t(1-\tau_u)(1-\tau_n)-\vartheta_y)\ln(\ell)^{1-\tau_y}T_y(1-a_y) + \vartheta_y \ln \kappa_t$, where $\varepsilon_{qy}\lambda_t = (\vartheta_I(1-\tau_u)(1-\tau_{nt})-\vartheta_y)(1-\tau_y)\lambda$ and $\varepsilon_{qz}\alpha_z = \alpha_z(\vartheta_z + \tau_m(1-\tau_u)\vartheta_I)$. All pairs (h, ξ_y) that verify this condition will go to a university with quality q. The distribution of parents human capital within a given university of quality q can therefore be computed explicitly. The mass of individuals with $\ln h$ and going to $\ln q$ is given by:

$$
\begin{aligned}
& f\left(\frac{1}{\varepsilon_{qz}\alpha_z}\left(\ln q - x - \left(\varepsilon_{qy}\lambda_t + \varepsilon_{qz}\alpha_z\right)\ln h\right) \cap \ln h\right) = f_{\xi_z}\left(\frac{1}{\varepsilon_{qz}\alpha_z}\left(\ln q - x - \left(\varepsilon_{qy}\lambda_t + \varepsilon_{qz}\alpha_z\right)\ln h\right)\right) f_h\left(\ln h\right) \\
& = \phi\left(\frac{\ln q - x - \left(\varepsilon_{qy}\lambda_t + \varepsilon_{qz}\alpha_z\right)\ln h}{\varepsilon_{qz}\alpha_z}, \mu_b, \sigma_z^2\right)\phi\left(\ln h, m_h, \Sigma_h^2\right) \\
& = \phi\left(\ln h, \underbrace{\frac{\ln q - x - \varepsilon_{qz}\alpha_z\mu_b}{\varepsilon_{qy}\lambda_t + \varepsilon_{qz}\alpha_z}}_{\mu_1^q}, \underbrace{\left(\frac{\varepsilon_{qz}\alpha_z\sigma_z}{\varepsilon_{qz}\alpha_z + \varepsilon_{qy}\lambda_t}\right)^2}_{\sigma_1^2}\right)\phi\left(\ln h, m_h, \Sigma_h^2\right) \\
& = \phi\left(\ln h, \mu_1^q, \sigma_1^2\right)\phi\left(\ln h, m_h, \Sigma_h^2\right) = \phi\left(\mu_1^q, m_h, \sigma_1^2 + \Sigma_h^2\right)\phi\left(\ln h, \mu_2^q, \sigma_2^2\right)
\end{aligned}
$$

where the RHS is the mass of individuals going to quality q and the LHS is the density of people whose parents have human capital h conditional on college q.

$$
f(\ln h|q) \sim \mathscr{N}\left(\frac{\Sigma_h^{-2}m_h + \left(\frac{\varepsilon_{qz}\alpha_z}{\varepsilon_{qy}\lambda_t+\varepsilon_{qz}\alpha_z}\right)^{-2}\sigma_z^{-2}\frac{(\ln q - x - \varepsilon_{qz}\alpha_z\mu_b)}{\varepsilon_{qy}\lambda_t+\varepsilon_{qz}\alpha_z}}{\Sigma_h^{-2} + \left(\frac{\varepsilon_{qz}\alpha_z}{\varepsilon_{qy}\lambda_t+\varepsilon_{qz}\alpha_z}\right)^{-2}\sigma_z^{-2}}, \frac{\Sigma_h^2\left(\frac{\varepsilon_{qz}\alpha_z}{\varepsilon_{qy}\lambda_t+\varepsilon_{qz}\alpha_z}\right)^2\sigma_z^2}{\Sigma_h^2 + \left(\frac{\varepsilon_{qz}\alpha_z}{\varepsilon_{qy}\lambda_t+\varepsilon_{qz}\alpha_z}\right)^2\sigma_z^2}\right) \sim \mathscr{N}\left(\mu_2^q, \sigma_2^2\right)
$$

For future reference we introduce $\mu_2^q = \mu_{2,1}m_h + \mu_{2,2}(\ln q - x - \varepsilon_{qz}\alpha_z\mu_b)$

$$
\text{with} \quad \mu_{2,1} = \frac{\Sigma_h^{-2}}{\Sigma_h^{-2} + \left(\frac{\varepsilon_{qz}\alpha_z}{\varepsilon_{qy}\lambda_t+\varepsilon_{qz}\alpha_z}\right)^{-2}\sigma_z^{-2}} \quad \text{and} \quad \mu_{2,2} = \frac{\left(\frac{\varepsilon_{qz}\alpha_z}{\varepsilon_{qy}\lambda_t+\varepsilon_{qz}\alpha_z}\right)^{-2}\sigma_z^{-2}}{\left[\Sigma_h^{-2} + \left(\frac{\varepsilon_{qz}\alpha_z}{\varepsilon_{qy}\lambda_t+\varepsilon_{qz}\alpha_z}\right)^{-2}\sigma_z^{-2}\right]\left(\varepsilon_{qy}\lambda_t + \varepsilon_{qz}\alpha_z\right)}
$$

where the second line stems from independence of h and ξ_z. The mass of individuals studying in college of quality q is $\phi\left(\mu_1^q, m_h, \sigma_1^2 + \Sigma_h^2\right)$ and the density of $\ln h$ conditional on being in this college is $\phi\left(\ln h, \mu_2^q, \sigma_2^2\right)$. From the distribution of parents' human capital within a college, the distribution of parents' income is

$$
\ln y \sim \mathscr{N}\left(\ln(1-a_y) + (1-\tau_y)\left[\lambda\mu_2^q + \ln \ell\right] + \ln T_y, (1-\tau_y)^2\lambda^2\sigma_2^2\right).
$$

Distribution of College Quality The mass of students in college of quality q is $\phi\left(\mu_1^q, m_h, \sigma_1^2 + \Sigma_h^2\right)$, with

$$
\mu_1 = \tfrac{1}{\varepsilon_{qy}\lambda_t+\varepsilon_{qz}\alpha_z}\left(\ln q - x - \varepsilon_{qz}\alpha_z\mu_b\right) \quad \text{and} \quad \sigma_1^2 = \left(\tfrac{\varepsilon_{qz}\alpha_z}{\varepsilon_{qz}\alpha_z+\varepsilon_{qy}\lambda_t}\right)^2\sigma_z^2.
$$

The distribution of quality is therefore

$$\ln q \sim \mathscr{N}\Big((\varepsilon_{qy}\lambda_t + \varepsilon_{qz}\alpha_z)m_h + x + \varepsilon_{qz}\alpha_z\mu_b,\ (\varepsilon_{qz}\alpha_z)^2\sigma_z^2 + (\varepsilon_{qy}\lambda_t + \varepsilon_{qz}\alpha_z)^2\Sigma_h^2\Big)\,.$$

Students' Abilities From the definition of abilities $\ln z = \alpha_z \ln h + \alpha_z \ln \xi_z$ and the sorting rule used above $\ln q = (\varepsilon_{qy}\lambda_t + \varepsilon_{qz}\alpha_z)\ln h + \varepsilon_{qz}\alpha_z \ln \xi_z + x$, one gets

$$\ln z = \frac{\alpha_z}{\varepsilon_{qz}\alpha_z}\left(\ln q - \varepsilon_{qy}\lambda_t \ln h - x\right) \Rightarrow \ln z|q \sim \mathscr{N}\left(\frac{\alpha_z}{\varepsilon_{qz}\alpha_z}\left(\ln q - \varepsilon_{qy}\lambda_t\mu_2^q - x\right),\left(\frac{\alpha_z\varepsilon_{qy}\lambda_t}{\varepsilon_{qz}\alpha_z}\right)^2\sigma_2^2\right)$$

A.4.1. Solving for κ_t and ν

The initial guess was that $\bar{y} = \kappa_t q^{\nu_t}$. Recall that $\ln \bar{y}$ is the mean log (after tax) income within a college $\ln \bar{y} = \ln(1-a_y)\,(\ell)^{1-\tau_y}\,T_y + (1-\tau_y)\lambda\left(\mu_{2,1}m_h + \mu_{2,2}(\ln q - x - \varepsilon_{qz}\alpha_z\mu_b)\right)$

Identifying coefficients with the guess $\ln \bar{y} = \ln \kappa_t + \nu_t \ln q$, one gets:

$$\nu = (1-\tau_y)\lambda\mu_{2,2} = (1-\tau_y)\lambda\frac{\left(\frac{\varepsilon_{qz}\alpha_z}{\varepsilon_{qy}\lambda_t+\varepsilon_{qz}\alpha_z}\right)^{-2}\sigma_z^{-2}}{\left[\Sigma_h^{-2} + \left(\frac{\varepsilon_{qz}\alpha_z}{\varepsilon_{qy}\lambda_t+\varepsilon_{qz}\alpha_z}\right)^{-2}\sigma_z^{-2}\right]\left(\varepsilon_{qy}\lambda_t + \varepsilon_{qz}\alpha_z\right)}$$

$$\iff \nu = \frac{1}{\left[\Sigma_h^{-2}\left(\frac{\varepsilon_{qz}\alpha_z}{\varepsilon_{qy}\lambda_t+\varepsilon_{qz}\alpha_z}\right)^2\sigma_z^2 + 1\right]\left[(\omega_I(1-\tau_u)(1-\tau_n) - \omega_y) + \frac{\omega_z}{(1-\tau_y)\lambda}\right] + \omega_y}$$

ν is therefore only a function of Σ_h^2. Identifying the coefficient independent of $\ln q$, and recalling that x_t is a linear function of $\ln \kappa_t$, and defining $\tilde{x} = x - \vartheta_y \ln \kappa_t$, one gets:

$$\begin{aligned}\kappa_t &= (1-a_y)\,(\ell)^{1-\tau_y}\,T_y e^{(1-\tau_y)\lambda(\mu_{2,1}m_h - \mu_{2,2}(x+\varepsilon_{qz}\alpha_z\mu_y)}\\ &= \left((1-a_y)\,(\ell)^{1-\tau_y}\,T_y e^{(1-\tau_y)\lambda(\mu_{2,1}m_h - \mu_{2,2}(\tilde{x}+\varepsilon_{qz}\alpha_z\mu_b)}\right)^{1-\nu\omega_y}\end{aligned}$$

A.5. Law of Motion

Proposition A.3. *If* $\ln h_t \sim \mathscr{N}(m_{ht}, \Sigma_{ht}^2)$, *then* $\ln h_{t+1} \sim \mathscr{N}(m_{ht+1}, \Sigma_{ht+1}^2)$ *with*

$$m_{ht+1} = \rho_t\, m_{ht} + X_{1t}, \tag{58}$$

$$\Sigma_{ht+1}^2 = \tilde{\rho}_t^2\, \Sigma_{ht}^2 + X_{2t}, \tag{59}$$

where $\rho_t = \alpha_z + \alpha_y + \alpha_z\alpha_q\omega_z + \alpha_q\omega_I\lambda$ *and the expressions for* X_{1t} *and* X_{2t} *are derived below.*

Replacing κ_t, T_y, T_n and T_u obtained above in the law of motion for human capital

$$\begin{aligned}
&\ln h' = \ln \xi_y + \alpha_z(1 + \alpha_q(\vartheta_z + \tau_m(1-\tau_u)\vartheta_I)) \ln \xi_z + \tilde{\rho} \ln h \\
&+ \alpha_q\omega_I \left(\ln s + \ln(1+a_n) + \ln(1+a_u) + \tau_u \left(\ln \ell(1-a_y) + \lambda m_h + \lambda^2 \frac{\Sigma_h^2}{2} \right) + \frac{\tau_u}{1-\tau_u}\frac{\sigma_I^2}{2} \right) \\
&+ \alpha_q\omega_I(1-\tau_u)(1-\tau_n) \left(\ln \ell + \ln(1-a_y) + \tau_y\lambda m_h + \frac{\lambda^2}{2}(2-\tau_y)\tau_y\Sigma_h^2 \right) \\
&+ \alpha_q\omega_y\nu\varepsilon_{qz}\alpha_z\frac{\sigma_z^2}{2} + \alpha_q\omega_y(1-\tau_y)\lambda\mu_{2,1}m_h \\
&+ \alpha_q\omega_I(1-\tau_u)\Bigg[(\tau_n\lambda - \alpha_z\tau_m)m_h + \frac{\alpha_z\tau_m}{2}(1-\alpha_z\tau_m)\sigma_z^2 + \tau_n(\ln \ell(1-a_y)) \\
&- \left[\lambda^2(1-\tau_y)^2(\tau_n-2)\tau_n + 2\lambda(1-\tau_n)(1-\tau_y)\tau_m\alpha_z + (\alpha_z\tau_m)^2 - \tau_n\lambda^2(2-\tau_y)\tau_y \right] \frac{\Sigma_h^2}{2} \Bigg]
\end{aligned}$$

We now take the expectation, we factorize out all the terms in m_h as well as all the terms in σ_z^2. The next steps consist in simplifying the coefficient in front of σ_z^2, of factorizing out all the terms in Σ_h^2 and in using the expression in (A.6) for σ_I^2. We also use the fact that $\mu_{2,1} = 1 - \mu_{2,2}(\varepsilon_{qy}\lambda_t + \varepsilon_{qz}\alpha_z)$. One obtains

$$\begin{aligned}
&m_h' = \rho m_h - \frac{\sigma_y^2}{2} \\
&\quad + \left[\frac{\tau_u}{1-\tau_u} \left(\frac{\alpha_z(1-\tau_u)}{(1-\nu\omega_y)} (\tau_m + \omega_z(1-\tau_n)\nu) \right)^2 - \alpha_z \left(\alpha_q \left(\omega_z + \omega_I(1-\tau_u)(\tau_m)^2\alpha_z \right) + 1 \right) \right] \frac{\sigma_z^2}{2} \\
&\quad + \alpha_q\omega_I \left(\ln \ell(1-a_y)s(1+a_u)(1+a_n) \right) \\
&\quad + \alpha_q\omega_I \Bigg[\lambda^2 + \frac{\tau_u}{1-\tau_u} \left(\frac{\alpha_z(1-\tau_u)}{(1-\nu\omega_y)} (\tau_m + \omega_z(1-\tau_n)\nu) \right)^2 \left(\frac{\omega_I}{\omega_z} + 1 \right)^2 \\
&\qquad\qquad\qquad - (1-\tau_u) \left[\lambda(1-\tau_y)(1-\tau_n) + (\alpha_z\tau_m) \right]^2 \Bigg] \frac{\Sigma_h^2}{2}
\end{aligned}$$

with $\rho = \alpha_z + \alpha_y + \alpha_z\alpha_q\omega_z + \alpha_q\omega_I\lambda$. Finally taking the variance gives the expression for the law of motion of Σ_h^2: $\Sigma_h^{2'} = \sigma_y^2 + \left(\alpha_z(1 + \alpha_q(\vartheta_z + \tau_m(1-\tau_u)\vartheta_I))\right)^2 \sigma_z^2 + \tilde{\rho}^2\Sigma_h^2$

Gathering all our results, the law of motion of human capital is given by

$$\begin{aligned}
&\ln h_{t+1} \sim \mathscr{N}\left(m_{ht+1}, \Sigma^2_{ht+1}\right)\\
&m_{ht+1} = \rho_t m_{ht} + X_1\\
&\Sigma^2_{ht+1} = \bar{\rho}_t^2 \Sigma^2_{ht} + X_{2t}(\Sigma_{ht})\\
&\rho_t = \alpha_z + \alpha_y + \alpha_z\alpha_q\omega_z + \alpha_q\omega_I\lambda_t\\
&X_{1t} \ = -\frac{\sigma_y^2}{2} + \left[\frac{\tau_{ut}}{1-\tau_{ut}}\left(\frac{\alpha_z(1-\tau_{ut})}{(1-\nu_t\omega_y)}\left(\tau_{mt}+\omega_z(1-\tau_{nt})\nu_t\right)\right)^2\right.\\
&\qquad\qquad\qquad \left. -\alpha_z\left(\alpha_q\left(\omega_z+\omega_I(1-\tau_{ut})(\tau_{mt})^2\alpha_z\right)+1\right)\right]\frac{\sigma_z^2}{2}\\
&+\alpha_q\omega_I \ln \ell_t(1-a_{yt})s_t(1+a_{ut})(1+a_{ht})\\
&+\alpha_q\omega_I\left[\lambda_t^2 + \frac{\tau_{ut}}{1-\tau_{ut}}\left(\frac{\alpha_z(1-\tau_{ut})}{(1-\nu_t\omega_y)}\left(\tau_{mt}+\omega_z(1-\tau_{nt})\nu_t\right)\right)^2\left(\frac{\varepsilon_{qy}\lambda_t}{\varepsilon_{qz}\alpha_z}+1\right)^2\right.\\
&\qquad\qquad\qquad \left. -(1-\tau_{ut})\left[\lambda(1-\tau_{yt})(1-\tau_{nt})+(\alpha_z\tau_{mt})\right]^2\right]\frac{\Sigma^2_{ht}}{2}\\
&X_{2t} = \sigma_y^2 + \left(\alpha_z(1+\alpha_q(\vartheta_z+\tau_m(1-\tau_u)\vartheta_I))\right)^2\sigma_z^2
\end{aligned}$$

A.6. From the Distribution of $\ln q$ to the Distribution of $\ln I$.

Using the definition of q and the expression for $\bar{z}$ obtained earlier,

$$\ln q = \ln \tilde{I}^{\omega_I}\bar{z}^{\omega_z} = \omega_I \ln \tilde{I} + \omega_z\left(\frac{\alpha_z}{\varepsilon_{qz}\alpha_z}(\ln q - \varepsilon_{qy}\lambda_t\mu_2^q - x)\right),$$

which implies

$$\ln \tilde{I} = \frac{1}{\omega_I}\left(\ln q\left(1-\alpha_z\frac{\omega_z}{\varepsilon_{qz}\alpha_z}\right)+\alpha_z\frac{\omega_z}{\varepsilon_{qz}\alpha_z}(\varepsilon_{qy}\lambda_t\mu_2+x)\right).$$

Using $\mu_2^q = \mu_{2,1}m_h + \mu_{2,2}(\ln q - x - \varepsilon_{qz}\alpha_z\mu_b)$ and letting $\chi \equiv \alpha_z\omega_z/\varepsilon_{qz}\alpha_z$, one gets

$$\ln \tilde{I} = \frac{1}{\omega_I}\left[\ln q\,(1-\chi+\chi\,\varepsilon_{qy}\lambda_t\mu_{2,2}) + \chi\,(\varepsilon_{qy}\lambda_t\mu_{2,1}m_h + (1-\varepsilon_{qy}\lambda_t\mu_{2,2})x - \varepsilon_{qy}\lambda_t\mu_{2,2}\varepsilon_{qz}\alpha_z\mu_b)\right].$$

Hence from the distribution $\ln q$ we can recover the distribution of $\ln \tilde{I} \sim \mathscr{N}\left(\mu_{\tilde{I}}, \sigma^2_{\tilde{I}}\right)$ with

$$\mu_{\tilde{I}} = \frac{1}{\omega_I}\left[\mu_q\left(1-\chi+\chi\,\varepsilon_{qy}\lambda_t\mu_{2,2}\right)+\chi\left(\varepsilon_{qy}\lambda_t\mu_{2,1}m_h+(1-\varepsilon_{qy}\lambda_t\mu_{2,2})x-\varepsilon_{qy}\lambda_t\mu_{2,2}\varepsilon_{qz}\alpha_z\mu_b\right)\right],$$
$$\sigma^2_{\tilde{I}} = \left(\tfrac{\alpha_z(1-\tau_u)}{1-\nu\omega_y}\left(\tau_m+\omega_z(1-\tau_n)\nu\right)\right)^2\left(\sigma_z^2+\left(\tfrac{\omega_I}{\omega_z}+1\right)^2\Sigma_h^2\right).$$

The last line stems from

$$\begin{aligned}\frac{1}{\omega_I}\left(1-\alpha_z\frac{\omega_z}{\varepsilon_{qz}\alpha_z}+\alpha_z\frac{\omega_z}{\varepsilon_{qz}\alpha_z}\varepsilon_{qy}\lambda_t\mu_{2,2}\right) &= \frac{1}{\omega_I}\left(1-\alpha_z\frac{\omega_z}{\varepsilon_{qz}\alpha_z}+\alpha_z\frac{\omega_z}{\varepsilon_{qz}\alpha_z}\varepsilon_{qy}\lambda_t\mu_{2,2}\right)\\ &= \frac{\alpha_z(1-\tau_u)}{\varepsilon_{qz}\alpha_z(1-\nu\omega_y)}\left(\tau_m+\omega_z(1-\tau_n)\nu\right)\end{aligned}$$

where we used $\nu = (1-\tau_y)\lambda\mu_{2,2}$ and $\varepsilon_l = \frac{\omega_l}{1-\nu\omega_y}$. Finally

$$\begin{aligned}\sigma^2_{\tilde{I}} &= \left(\frac{\alpha_z(1-\tau_u)}{\varepsilon_{qz}\alpha_z(1-\nu\omega_y)}\left(\tau_m+\omega_z(1-\tau_n)\nu\right)\right)^2\sigma_q^2\\ &= \left(\frac{\alpha_z(1-\tau_u)}{(1-\nu\omega_y)}\left(\tau_m+\omega_z(1-\tau_n)\nu\right)\right)^2\left(\sigma_z^2+\left(\frac{\omega_I}{\omega_z}+1\right)^2\Sigma_h^2\right)\end{aligned}$$

Since $\ln\tilde{I} = \ln I - (1-\tau_u)\frac{\sigma_u^2}{2}$ and σ_u^2 is common across all colleges, we have $\ln I \sim \mathscr{N}\left(\mu_I, \sigma_I^2\right)$with $\mu_I = \mu_{\tilde{I}} + (1-\tau_u)\frac{\sigma_u^2}{2}$ and $\sigma_I^2 = \sigma_{\tilde{I}}^2$

Expression for σ_u^2 Given that all households save a fraction s of their disposable income and the selection equation into college, one gets

$$\begin{aligned}\ln e_u = \ln\frac{(1+a_n)s}{T_n}+\tau_m\frac{\alpha_z}{\varepsilon_{qz}\alpha_z}\left(\ln q - x\right)+\ln h^{(1-\tau_n)(1-\tau_y)\lambda-\tau_m\frac{\alpha_z}{\varepsilon_{qz}\alpha_z}\varepsilon_{qy}\lambda_t}\\ +(1-\tau_n)\ln T_y(1-a_y)\ell^{(1-\tau_y)}\end{aligned}$$

Hence the within-university variance of tuitions is given by:

$$\sigma_u^2 = \left((1-\tau_n)(1-\tau_y)\lambda-\tau_m\frac{\alpha_z}{\varepsilon_{qz}\alpha_z}\varepsilon_{qy}\lambda_t\right)^2\sigma_2^2 = \left((1-\tau_y)\lambda\frac{(1-\tau_n)\omega_z+\tau_m\omega_y}{\omega_z+\omega_I(1-\tau_u)\tau_m}\right)^2\sigma_2^2$$

which is indeed constant across universities wince σ_2^2 is an aggregate constant.

A.7. Details on the Positioning Game

In this appendix we give a formal explanation of the positioning game as well as a characterization of the equilibrium. Recall the general environment. There is a continuum of colleges $j \in [0,1]$. At each generation $t \in \mathbb{N}$, they play a positioning game. The games played at any two generations $t > t'$ are independent of each other.

At a given generation $t \in \mathbb{N}$, and before playing the positioning game, each college is given a real number $o \in [0,1]$. The positioning game is sequential and o is the order in which colleges play. Without loss of generality, since all colleges are identical, one can relabel colleges $j = o$ so that their label is also their order.[35] Colleges play sequentially in descending order: j plays before j' if and only if $j > j'$. Each college plays once.

All colleges have the same set of actions: the line of qualities $q \in \mathbb{R}_+$. The history of the sequential game up to college j's turn is a (injective) function $\mathscr{H}_j^+ : (j,1] \to \mathbb{R}_+$ that describes the colleges' actions up to j's turn. A strategy for college j is a choice of quality $q \in \mathbb{R}_+$ whenever it is its turn; abusing notation we denote it $q_j(\mathscr{H}_j^+)$. Denoting $\mathscr{H}_j : [j,1] \to \mathbb{R}_+$ the history including college j's action, one has, for all $k > j$, $\mathscr{H}_j(k) = \mathscr{H}_j^+(k)$ and $\mathscr{H}_j(j) = q_j(\mathscr{H}_j^+)$. $\mathscr{H}_0$ denotes a terminal history.

We now introduce the notion of the set of available students at quality q at history $\mathscr{H}_j$. Denote $S(q) \subset \mathscr{I}$ the subset of students who demand quality q and $card(S(q))$ its cardinal, similarly denote $S(q, \mathscr{H}_j)$ the subset of students demanding quality q who are not in a college yet after history $\mathscr{H}_j$ (we call it the set of available students).

The cardinality of the set of available students at each quality to colleges that play later $j' \leq j$ is a function of the positions of colleges that have already played, $j' > j$, because when college j chooses quality q it takes a subset of these students, $S(q, \mathscr{H}_j) \subset S(q, \mathscr{H}_j^+)$. More specifically, we assume that college j picks a subset of students of cardinality $\aleph_1$ (its assumed size). We further assume that at any history $\mathscr{H}_j^+$, if $card(S(q), \mathscr{H}_j^+) \leq \aleph_1$ and j chooses q, then college j takes all the students at quality q and $card(S(q, \mathscr{H}_{j'})) = 0$ for all $j' \leq j$.[36] If $card(S(q, \mathscr{H}_j^+)) > \aleph_1$,

---

[35] This assumption of an order across colleges captures in a very direct way the notion that colleges do not start on an equal foot in the competition for prestige. In the real world, there are slow-moving state variables that gives an advantage to some colleges in this race, such as their reputation, their faculty member, their stock of publications, their endowment. Our assumption should be seen as a reduced-form expression of this ex ante hierarchy of advantages created by these state variables that this paper abstracts from.

[36] This is indeed a restriction, and not a tautology. It would be possible for a countable number of colleges to offer the same quality q and still respect the size constraint since a countable set of

we assume that college j picks a subset of students of cardinality $\aleph_1$ which implies $card(S(q), \mathscr{H}_j) > \aleph_1$.[37]

Recall that the objective of the college is to deliver the highest quality possible. If they faced no constraint, they would all choose to deliver the highest quality. All colleges would like to be the top college, but there is room for only one. This notion is captured by the size constraint: colleges can't be too small. Specifically, if at history $\mathscr{H}_j^+$ the set of available students at q is lower than the cardinality of the continuum, $card(S(q, \mathscr{H}_j^+)) < \aleph_1$, the payoff of college j if it chooses q is 0, and we say that college j is not operating.[38] If $card(S(q, \mathscr{H}_j^+)) \geq \aleph_1$, and college j chooses q, then its payoff is simply q and we say that it is operating.

This induces a preference relationship over the set of possible terminal histories. Consider any two terminal histories $\mathscr{H}_0, \mathscr{H}_0'$ in which college j is operating. College j prefers $\mathscr{H}_0$ to $\mathscr{H}_0'$, $\mathscr{H}_0 \succsim \mathscr{H}_0'$ if and only if $\mathscr{H}_0(j) = q_j \geq q_j' = \mathscr{H}_0'(j)$ with strict preference for strictly higher quality. A college always prefers a terminal history in which it is operating over one in which it is not.

Denote $q^*_{<j} = \{q^*_k(\mathscr{H}_k^+)\}_{k\in[0,j)}$ the strategy profile of colleges playing (strictly) after j and $\mathscr{H}_0\left(\mathscr{H}_j^+, q_j(\mathscr{H}_j^+), q_{<j}\right)$ the terminal history that follows history $\mathscr{H}_j^+$ and induced by the strategies of college j, $q_j(\mathscr{H}_j^+)$ and of the colleges playing afterwards $q_{<j}$. A subgame perfect Nash equilibrium of this game is a strategy profile $\{q^*_j(\mathscr{H}_j^+)\}_{j\in[0,1]}$ such that for all j, given the strategies of the colleges playing next $q^*_{<j}$

$$\mathscr{H}_0\left(\mathscr{H}_j^+, q^*_j(\mathscr{H}_j^+), q^*_{<j}\right) \succsim \mathscr{H}_0\left(\mathscr{H}_j^+, q, q^*_{<j}\right)$$

for all $q \in \mathbb{R}_+$.

Detail on the Index Set of Households, $\mathscr{I}$. To be consistent with the notion that there is a continuum of colleges and a continuum of students within each college, it has to be the case that the cardinality of the set of students be strictly higher than the cardinality of the set of colleges, i.e. $card(\mathscr{I}) > card([0,1]) = \aleph_1$. It seems natural to consider the smallest such cardinal. Using the axiom of choice, such a cardinal is

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set of cardinal $\aleph_1$ is still of cardinal $\aleph_1$. It is however an inconsequential restriction which allows to associate one college with one quality since in equilibrium it is true that $card(S(q)) = \aleph_1$.

[37] Although this case might arise in some other version of the model, it doesn't happen in any equilibria analyzed in this paper.

[38] The size constraint is what makes the game strategic: the positioning decisions of higher-ranked colleges influence the payoffs of lower-ranked colleges.

$\aleph_2$. To fix ideas, this corresponds for example to the index set $\mathscr{I} = [0,1]^{[0,1]}$.

Assumption 1. *The cardinal of the set of households is the same as the continuum of continua*

$$card(\mathscr{I}) = \aleph_2$$

Equilibrium Characterization. The following lemma says that the quality delivered by each college follows the same order as the order in which colleges play the game.

Lemma 2. *Assume the distribution of students over quality is continuous over $\mathbb{R}_+$. Then in equilibrium,*

$$q_j > q_{j'} \iff j > j'.$$

Proof. Since the distribution is continuous over $\mathbb{R}_+$ and there are a cardinal $\aleph_2$ of students, there must be a cardinal $\aleph_1$ of students demanding a given quality q, i.e. $card(S(q)) = \aleph_1$ for all $q \in \mathbb{R}_+$. (Otherwise there would be a mass point at some q, contradicting the assumption of a continuous distribution). Hence, by the assumption made earlier, whenever college j chooses a location q that is unoccupied $card(S(q, \mathscr{H}_j^+)) = \aleph_1$, it takes all of its students and no students is left for a college playing later, $card(S(q, \mathscr{H}_{j'})) = 0$ for all $j' \leq j$. This implies that if there exists $\underline{q}$ such that the history up to j is bounded on the left by $\underline{q}$: $\mathscr{H}_j^+((j,1]) = (\underline{q}, +\infty)$, then a college j's optimal location is $\underline{q}$: choosing strictly above $\underline{q}$ would mean not operating by the previous argument, and choosing exactly $\underline{q}$ rather than a strictly lower quality is strictly preferred. This shows that in any equilibrium in which the distribution for quality demanded is continuous over $\mathbb{R}_+$, for any $j > j'$, one has $q_{j'} < q_j$.

□

A.8. Existence and Uniqueness of Equilibrium Path

Proposition A.4.

- *If* $\lim_{\Sigma_h \to \infty} \tilde{\rho} < 1$, *there exists at least one locally stable steady state.*
- *For ω_y small enough, there exists a unique globally stable steady state and a unique equilibrium path,*

where $\lim_{\Sigma_h^2 \to \infty} \tilde{\rho} = \alpha_z + \alpha_z \alpha_q (\omega_z + \tau_m (1-\tau_u) \omega_I) + \alpha_q [\omega_I (1-\tau_u)(1-\tau_n) - \omega_y](1-\tau_y)\lambda$. *A high ω_y can lead to multiple equilibria by making inequality Σ_h grow too fast in* *some parts of the state-space (violating the contraction mapping property of* (59)*):* ν *is increasing in* Σ_h*, hence* ε_l *for* $l = I, z, y$*,* ρ *and* X_2 *are all increasing in* Σ_h*.*

The set of equations defining an equilibrium path in proposition A.3 is block-recursive. In particular, the law of motion of Σ_h, is independent and the path of all other variables are pinned-down by the path of Σ_h. It is therefore necessary and sufficient to focus on the existence and uniqueness of the path of Σ_h. We first define new notations:

$$\begin{aligned}
\Sigma_h'^2 &= f(\Sigma_h^2) \\
&= \left[\alpha_z^2 + \left(\frac{A}{1-\nu\omega_y}\right)^2 + \frac{2\alpha_z A}{1-\nu\omega_y}\right]\Sigma_h^2 + \sigma_y^2 + \left[\alpha_z^2 + \frac{B^2}{(1-\nu\omega_y)^2} + \frac{2B\alpha_z}{1-\nu\omega_y}\right]\sigma_z^2
\end{aligned}$$

$$\begin{aligned}
\text{with } A &= \alpha_z\alpha_q(\omega_z + \tau_m(1-\tau_u)\omega_I) + \alpha_q\left(\omega_I(1-\tau_u)(1-\tau_n) - \omega_y\right)(1-\tau_y)\lambda \\
B &= \alpha_z\alpha_q\left(\omega_z + \tau_m(1-\tau_m)\omega_I\right) \qquad \nu = \frac{C}{E\Sigma_h^{-2} + (E+\omega_y)C} \\
C &= \left(\frac{\omega_z}{\omega_I + \omega_z}\right)^{-2}\sigma_z^{-2} \qquad E = \left(\omega_I(1-\tau_u)(1-\tau_n) - \omega_y\right) + \frac{\omega_z}{(1-\tau_y)\lambda}
\end{aligned}$$

$f(\cdot)$ is differentiable for $\Sigma_h^2 \in (0, \infty)$ and

$$\lim_{\Sigma_h^2 \to 0} f(\Sigma_h^2) = \sigma_y^2 + \left[\alpha_z^2 + B^2 + 2B\alpha_z\right]\sigma_z^2 > 0.$$

The derivative $f'(\cdot)$ is

$$\begin{aligned}
&\left[\alpha_z^2 + \left(\frac{A}{1-\nu\omega_y}\right)^2 + \frac{2\alpha_z A}{1-\nu\omega_y}\right] \\
&+ \left[\left[\left(\frac{A}{1-\nu\omega_y}\right)^2 + \frac{\alpha_z A}{1-\nu\omega_y}\right]\Sigma_h^2 + \left[\frac{B^2}{(1-\nu\omega_y)^2} + \frac{B\alpha_z}{1-\nu\omega_y}\right]\sigma_z^2\right]\frac{2\omega_y}{1-\nu\omega_y}\frac{\partial\nu}{\partial\Sigma_h^2}
\end{aligned}$$

with $\dfrac{\partial\nu}{\partial\Sigma_h^2} = \dfrac{CE}{\left(E + \Sigma_h^2(E+\omega_y)C\right)^2}$

$$\begin{aligned}
\text{Hence} \quad \lim_{\Sigma_h^2 \to \infty}\frac{\partial f}{\partial\Sigma_h^2} &= \left[\alpha_z^2 + \left(\frac{A}{1 - \frac{\omega_y}{E+\omega_y}}\right)^2 + \frac{2\alpha_z A}{1 - \frac{\omega_y}{E+\omega_y}}\right] \\
&= \left[\alpha_z + \alpha_z\alpha_q(\omega_z + \tau_m(1-\tau_m)\omega_I) + \alpha_q[\omega_I(1-\tau_u)(1-\tau_n) - \omega_y](1-\tau_y)\lambda\right]^2
\end{aligned}$$

Therefore if

$$[\alpha_z + \alpha_z\alpha_q(\omega_z + \tau_m(1-\tau_m)\omega_I) + \alpha_q[\omega_I(1-\tau_u)(1-\tau_n) - \omega_y](1-\tau_y)\lambda]^2 < 1,$$

the equation $\Sigma_h^2 = f(\Sigma_h^2)$ has at least one solution since f is continuous and $\lim f(0) > 0$. Moreover, it has to be that an odd number of these solutions are characterized by $f'(\Sigma_h) < 1$, which guarantees local stability of the equilibrium path around these solutions.

Let's now show that the equilibrium path is unique for ω_y small enough. A first-order approximation of f in the neighborhood of $\omega_y = 0$ is

$$\begin{aligned} f(\Sigma_h^2) &\simeq \left[\alpha_z^2 + A^2 + 2\alpha_z A\right]\Sigma_h^2 + \sigma_y^2 + \left[\alpha_z^2 + B^2 + 2B\alpha_z\right]\sigma_z^2 \\ &\quad + \left[\left[A^2 + \alpha_z A\right]\Sigma_h^2 + \left[B^2 + \alpha_z B\right]\sigma_z^2\right] 2\nu\omega_y, \\ f'(\Sigma_h^2) &\simeq \left[\alpha_z^2 + A^2 + 2\alpha_z A\right] + \underbrace{G(\Sigma_h^2)}_{F(\Sigma_h^2)}\omega_y, \end{aligned}$$

where

$$\begin{aligned} G(\Sigma_h^2) &\equiv 2\left(H(\Sigma_h^2)\frac{E}{E + EC\Sigma_h^2} + \left[A^2 + \alpha_z A\right]\Sigma_h^2\right)\frac{C}{E + EC\Sigma_h^2}, \\ H(\Sigma_h^2) &\equiv \left[A^2 + \alpha_z A\right]\Sigma_h^2 + \left[B^2 + \alpha_z B\right]\sigma_z^2. \end{aligned}$$

with $\nu = C/(E\Sigma_h^{-2} + EC)$. Since we have assumed that $[\alpha_z^2 + A^2 + 2\alpha_z A] < 1$ and $F(\Sigma_h^2)$ is bounded for $\Sigma_h^2 \in (0, \infty)$, there exists an ω_y small enough such that for all Σ_h^2, $\frac{\partial f(\Sigma_h^2)}{\partial \Sigma_h^2} < 1$. This is sufficient for the existence and uniqueness of a globally stable steady state.

A.9. Rise in the Returns to Human Capital

We prove a more general version of Proposition 4.1 that nests the baseline laissez-faire environment of Section 2 and the policy-augmented environment of Section 5. The general statement holds along the equilibrium path of the policy-augmented economy under the sufficient condition

$$\omega_I(1-\tau_n)(1-\tau_u) > \omega_y. \tag{60}$$

In the baseline of Section 2 all policy instruments are zero ($\tau_n = \tau_u = \omega_y = 0$), so (60) reduces to $\omega_I > 0$, which is satisfied by assumption. At the calibrated policy vector reported in Tables 1–2, the left-hand side of (60) is $0.73 \times (1 - 0.11) \times (1 - 0.35) = 0.42 > 0 = \omega_y$, so (60) holds at the parameters used in the quantitative analysis as well.

The total derivative of the IGE with respect to λ is given by

$$\left[\frac{\partial \nu}{\partial \lambda} + \frac{\partial \nu}{\partial \Sigma_h^2}\frac{\partial \Sigma_h^2}{\partial \lambda}\right]\left[\alpha_z\alpha_q\left(\frac{\partial \vartheta_z}{\partial \nu} + \tau_m\frac{\partial \vartheta_I}{\partial \nu}\right) + \alpha_q\left(\frac{\partial \vartheta_I}{\partial \nu}(1-\tau_n) - \frac{\partial \vartheta_y}{\partial \nu}\right)(1-\tau_y)\lambda\right]$$
$$+\,\alpha_q(\vartheta_I(1-\tau_n) - \vartheta_y)(1-\tau_y)$$

We then compute the derivatives:

$$\frac{\partial \varepsilon_l}{\partial \nu} = \varepsilon_l\vartheta_y > 0 \qquad \frac{\partial \nu}{\partial \Sigma_h^2} = \frac{CE}{\left(E + (E+\omega_y)C\Sigma_h^2\right)^2} > 0$$

with C and E have been defined in the proof of existence and uniqueness.

$$\frac{\partial \nu}{\partial \lambda} = \frac{2C\left(\frac{\omega_z}{\omega_z+\omega_I}\right)\frac{1}{\omega_I}\left[E\Sigma_h^{-2} + \omega_y C\right] + C\frac{\omega_z}{(1-\tau_y)\lambda^2}}{\left(E\Sigma_h^{-2} + (E+\omega_y)C\right)^2} > 0$$
$$\frac{\partial X_2}{\partial \lambda} = \sigma_z^2\alpha_z\left(1 + \alpha_q(\vartheta_z + \tau_m\vartheta_I)\right)\alpha_z\alpha_q\vartheta_y(\vartheta_z + \tau_m\vartheta_I)\vartheta_I\frac{\partial \nu}{\partial \lambda} > 0$$
$$\frac{\partial \Sigma_h^2}{\partial \lambda} = \frac{\frac{\partial X_2}{\partial \lambda} + \Sigma_h^2 2\frac{\partial \tilde{\rho}}{\partial \lambda}\tilde{\rho}}{1 - (\tilde{\rho})^2 - \Sigma_h^2 2\frac{\partial \tilde{\rho}}{\partial \Sigma_h^2}\tilde{\rho} - \frac{\partial X_2}{\partial \Sigma_h^2}} > 0$$

where $\frac{\partial \tilde{\rho}}{\partial \lambda}$ has to be understood as the partial derivative of $\tilde{\rho}$ w.r.t. λ keeping Σ_h^2 constant. The last line stems from the fact that the steady state is locally stable - which requires that $1 - (\tilde{\rho})^2 - \Sigma_h^2 2\frac{\partial \tilde{\rho}}{\partial \Sigma_h^2}\tilde{\rho} - \frac{\partial X_2}{\partial \Sigma_h^2} = \frac{\partial (\Sigma_h')^2}{\partial (\Sigma_h)^2} > 0$. Hence, putting everything together yields

$$\frac{\partial \tilde{\rho}}{\partial \lambda} = \left[\underbrace{\frac{\partial \nu}{\partial \lambda} + \frac{\partial \nu}{\partial \Sigma_h^2}\frac{\partial \Sigma_h^2}{\partial \lambda}}_{>0}\right]\vartheta_y[\alpha_z\alpha_q\left(\vartheta_z + \tau_m\vartheta_I\right)$$
$$+\,\alpha_q\left(\vartheta_I(1-\tau_n) - \vartheta_y\right)(1-\tau_y)\lambda] + \alpha_q(\vartheta_I(1-\tau_n) - \vartheta_y)(1-\tau_y) > 0.$$

This proves not only that the steady-state IGE is increasing in λ but that the variance of human capital in the economy is as well. Given that the variance of market income is given by $\lambda^2\Sigma_h^2$ it is immediate that it increases too. Turning to the private spending on higher education, given by s, it is immediate to see from the expressions (13) and (15) that it is increasing in the future path of λ. Let's now turn to the ratio of within college variance of (log) parental income over economy-wide variance of (log) income: Letting $B \equiv \left(\frac{\varepsilon_{qz}\alpha_z}{\vartheta_I+\varepsilon_{qz}\alpha_z}\right)^2$,

$$\frac{V(\ln y|q)}{V(\ln y)} = \frac{B\,\sigma_z^2}{\Sigma_h^2 + B\,\sigma_z^2},$$

$$\Rightarrow \frac{\partial}{\partial\lambda}\left[\frac{V(\ln y|q)}{V(\ln y)}\right] = \frac{\sigma_z^2\,\frac{\partial B}{\partial\lambda}\,\Sigma_h^2 - B\,\sigma_z^2\,\frac{\partial\Sigma_h^2}{\partial\lambda}}{\left[\Sigma_h^2 + B\,\sigma_z^2\right]^2} < 0.$$

The sign follows from $\frac{\partial\Sigma_h^2}{\partial\lambda} > 0$ and $\frac{\partial B}{\partial\lambda} < 0$.

Finally the variance of (log) college quality is given by $(\varepsilon_{qz}\alpha_z)^2\sigma_z^2 + (\vartheta_I + \varepsilon_{qz}\alpha_z)^2\Sigma_h^2$. It is immediate that it increases with λ since $\vartheta_I, \varepsilon_{qz}\alpha_z, \Sigma_h^2$ increase with λ.

Monotonic Transition Path. From the law of motion of Σ_h^2, in the first period the initial increase in λ raises $\tilde{\rho}$ and triggers the initial increase in the dispersion of human capital. Since $X_2(\Sigma_h)$ and $\tilde{\rho}(\Sigma_h)$ are both increasing in Σ_h it further increases Σ_h^2 at the following period and so on... This establishes that Σ_h^2 is strictly increasing over the transition path. This also establishes the monotonic increase in $\tilde{\rho}$ and all ω's.

Turning to the private spending on higher education, given by s, it is easy to see that it is increasing in the future path of λ, $\tilde{\rho}$ and ϑ_I. Since these three variables are increasing over the transition path, s also increases. The variance of log college quality is also increasing because $\vartheta_I, \varepsilon_{qz}\alpha_z, \Sigma_h^2$ are increasing over the transition path. The ratio of within college variance of (log) parental income over economy-wide variance of (log) income will decrease monotonically over the transition path because of the initial increase in λ, this is the first term in the derivative $\sigma_z^2\frac{\partial B}{\partial\lambda}\Sigma_h^2$, and then decreases further as Σ_h increases, this is the second term $B\sigma_z^2\frac{\partial\Sigma_h^2}{\partial\lambda}$.

A.10. The College Problem in the Quantitative Version

In order to keep the college problem tractable despite the loss of closed-form expressions for the distribution of students within the college and equilibrium tuition, we assume that the problem of the college is still given by (48), even if it is not possible to derive (48) from the primitive problem (11) since the within-college distribution of students isn't joint log-normal anymore.

The alternative way to microfound (48) is to assume that there is a loss in the efficiency with which resources are used when the inequality of tuition fees among students rises, i.e. that σ_u^2 is given by (47), a measure of the dispersion of tuition within the college, instead of the within-college heterogeneity in students. One can interpret it as a rise in human resources and administrative costs or as an increase in the sentiment of unfairness among students when tuition fees become more heterogeneous among students. The first-order conditions for this problem are the same and the equilibrium tuition schedule is identical.

B. From a Full Life-Cycle to a Two-Period Household Problem

We now show that the two-period household problem formulation is a reduced-form formulation of a full life-cycle problem. We allow for taxes and transfers as in section 5. We start by giving the timing of a lifetime in detail. A period corresponds to four years. Each individual lives for five periods as a child, then attends college for one period, and finally works as an adult for ten periods, which we index by $a \in \{1, ..10\}$. Across these periods households can borrow and save at exogenous interest rate r to smooth consumption during their lifetime. The household's time preference is denoted by δ. For simplicity, we assume that $r = 1/\delta - 1$.

Each adult has a child when they are in their fifth period of adult life, $a = 5$, and sends them to college before retiring at $a = 10$. Parents retire after their children graduate from college, so that parents and children do not overlap in the labor market. The household's time preference is denoted by δ.

The household problem can be formulated recursively as

$$U(h, z) = \max_{c_a, q} \left\{ \sum_{a=1}^{10} \delta^{a-1} \left(\ln \left(c_a \right) - \ell_a^{\eta} \right) + \beta \mathbb{E} \left[U(h', z') \right] \right\}$$

where δ and β are the time and intergenerational discount factor, respectively.

Denoting $c_{t,a}, \ell_{t,a}$ the consumption and the labor supply of an individual whose age is a at time t, the household's life-time income and budget constraint is

$$\begin{aligned}\sum_{a=1}^{10}\delta^{a-1}(1+a_{ct+a-1})c_{t+a-1,a}+\delta^{10-1}e_{t+10-1}(q,z,y)&\\ \coloneqq (1-a_y)T_y(h^{\lambda})^{1-\tau_y}\sum_{a=1}^{10}\delta^{a-1}\ell_{t+a-1,a}^{1-\tau_y}.&\end{aligned}$$

where $(1-a_y)T_y\left(h^{\lambda}\ell_{t+a-1,a}\right)^{1-\tau_y}$ is the after tax-and-transfers labor earnings of an individual of age a in time t and we used the assumption that $r=1/\delta-1$.

We then solve for the optimal allocation of consumption and labor across periods of an adult's lifetime. From the assumption that $r=1/\delta-1$, after-tax consumption is constant within one's lifetime:

$$(1+a_{c,t+a-1})c_{t+a-1,a}=\delta^{a-1}\Pi_{\tau=1}^{a-1}(1+r_{t+\tau})(1+a_{ct})c_{t1}=(1+a_{ct})c_{t1}$$

Similarly labor supply is constant over time:

$$\ell_{t+a-1,a}=\ell_{t1}.$$

We define the lifetime after-tax income y_t $\tilde{w}_t$ as

$$\begin{aligned}y_t&=(1-a_y)T_y\left(h^{\lambda}\ell\right)^{1-\tau_y}\\ \text{with}\quad \tilde{\delta}_t&=\delta^{-9}\sum_{a=1}^{10}\delta^{a-1}\end{aligned}$$

Hence the problem of the household can be written more simply as

$$\begin{aligned}U(h,z)&=\max_{c,q}\left\{(\ln(c)-\ell^{\eta})\sum_{a=1}^{10}\delta^{a-1}+\beta\mathbb{E}\left[U(h',z')\right]\right\}\\ \text{s.t.}\quad y&=\delta^{-9}(1+a_{ct})c\sum_{a=1}^{10}\delta^{a-1}+e_{t+10}(q,z,y)\end{aligned}$$

This is isomorphic to the problem in sections 2 and 5.

C. Profit Maximization

In this Appendix we show that at first order the equilibrium tuition schedule implied by profit-maximizing colleges with arithmetic peer-effects and free-entry is the same as the one used in the baseline model.

$$\pi(q) = \max_{\tilde{z},I,\phi} \int e(q,z,y)\phi(z,y)d(z,y) - \int I\phi(z,y)d(z,y) \tag{61}$$

$$q = I^{\omega_I}\tilde{z}^{\omega_z} \tag{62}$$

$$\tilde{z} = E_{\phi(.)}[z] \tag{63}$$

$$\int \phi(z,y)d(z,y) = 1 \tag{64}$$

After substituting $\tilde{z}$ by its expression in the quality function, the Lagrangian is given by

$$\begin{aligned}\mathscr{L} = &\int e(q,z,y)\phi(z,y)d(z,y) - \int I\phi(z,y)d(z,y) \\ &+ \lambda_1\left(\ln I - \frac{1}{\omega_I}\left(\ln q - \omega_z \ln\left(\int z\phi(z,y)d(z,y)\right)\right)\right) + \lambda_2\left(\int \phi(z,y)d(z,y) - 1\right)\end{aligned}$$

The first-order conditions are give by

$$\begin{aligned}-1 + \frac{\lambda_1}{I} &= 0 \quad (I) \\ e(z,y) - I + \lambda_1\left(\frac{\omega_z}{\omega_I}\frac{z}{\tilde{z}}\right) + \lambda_2 &= 0 \quad (\phi(z,y))\end{aligned}$$

The first equation gives $\lambda_1 = I$. Multiplying the second equation by $\phi(z,y)$ and summing over (z,y), using the second constraint $\int \phi(z,y)d(z,y) = 1$, and using the free entry condition $\int e(q,z,y)\phi(z,y)d(z,y) = \int I_j\phi(z,y)d(z,y)$ we get

$$\lambda_2 = -I\frac{\omega_z}{\omega_I}$$

Putting everything together gives

$$e(z,y) = I\left(1 + \frac{\omega_z}{\omega_I}\frac{z}{\tilde{z}} - \frac{\omega_z}{\omega_I}\right) \tag{65}$$

Dividing by I on both sides and taking the log gives

$$\ln\left(\frac{e(z,y)}{I}\right) = \ln\left(1 + \frac{\omega_z}{\omega_I}\frac{z}{\tilde{z}} - \frac{\omega_z}{\omega_I}\right)$$

We now approximate this around $z \simeq \tilde{z}$:

$$\ln\left(\frac{e(z,y)}{I}\right) = \frac{\omega_z}{\omega_I}\left(\frac{z}{\tilde{z}} - 1\right)$$

Around the first point, we can further approximate the above by

$$\ln\left(\frac{e(z,y)}{I}\right) = \frac{\omega_z}{\omega_I}\ln\left(\frac{z}{\tilde{z}}\right).$$

This delivers the same tuition schedule as in the quality-maximizing problem. If the tuition schedule is the same, the sorting is the same. Since the budget constraint in the quality-maximizing case is equivalent to the no-profit condition in the profit-maximizing case, the allocation of spending is the same. Only the peer-effects differ since the technologies are slightly different (arithmetic vs geometric averages). This ends the proof.