Tax on Inflation Policy at the Zero Lower Bound*

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Abstract

This paper studies Tax on Inflation Policy (TIP) at the zero lower bound in a New Keynesian model. By shaping firms' price changes, TIP corrects a pecuniary externality in price setting, mitigates deflation, raises output, and implements the constrained-efficient allocation. The optimal TIP depends on the trap's source: a subsidy to price increases in a fundamental trap, and a tax in a self-fulfilling trap. In the latter, a high tax can rule out deflationary equilibria. A simple rule responding aggressively to deflation with a negative intercept is robust and delivers sizable gains in a medium-scale model calibrated to Japan.

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1 Introduction

Once a Japan-specific issue, liquidity traps have become a global challenge after the Global Financial Crisis. During these episodes, output remained persistently below potential and inflation fell short of its medium-run target. Although the post-pandemic surge in inflation lifted interest rates above the zero lower bound (ZLB), a return to a low-interest-rate environment is likely (IMF, 2023). Since 2025, for instance, the Swiss National Bank has cut its policy rate back to zero to fight deflation. Similarly, inflation in China has remained exceptionally low and aggregate demand persistently weak since 2023. Designing effective stabilization policies at the ZLB therefore remains an urgent and globally relevant challenge. At the ZLB, conventional monetary policy becomes constrained, leaving other instruments such as forward guidance and fiscal interventions to play a central role.

In this paper, we evaluate the effectiveness and robustness of a *Tax on Inflation Policy* (TIP)—a fiscal instrument that taxes or subsidizes firms' price increases—as a tool to improve welfare in a liquidity trap.[1] By embedding TIP in a standard New Keynesian model with a ZLB and multiple sources of liquidity traps, we show how TIP can mitigate the deflationary spirals characteristic of liquidity traps by re-aligning the firms' private and social valuations of pricing decisions. We derive the optimal TIP in each type of liquidity trap and then design a policy rule that is robust across all types. Using a medium-scale model calibrated to Japan, we quantify a robust TIP rule that would have lifted the economy out of its liquidity trap.

We begin with a small-scale New Keynesian model with TIP and three potential sources of traps: a fundamental liquidity trap (FLT) driven by a decline in the neutral rate of interest, a self-fulfilling liquidity trap (SFLT), and a trap driven by shifts in the Phillips curve (PCLT). The model admits closed-form solutions for both decentralized and optimal allocations, allowing us to characterize analytically the role of TIP across different environments.

In a fundamental liquidity trap, a tax on price decreases—or equivalently a subsidy to price increases—addresses the pecuniary externality, mitigates deflation and narrows the output gap. In *laissez-faire*, firms cut prices to attract scarce demand, generating deflation that raises the real interest rate and further depresses demand.[2]

[1] Prior work has shown that TIP effectively regulates inflation outside the liquidity trap (Capelle and Liu, forthcoming).

[2] In practice, this mechanism may be amplified by an increase in real wages and debt deflation, due

Firms do not internalize the pecuniary externality of their private decisions to decrease prices. By giving incentives to firms to increase prices, a tax on price decreases aligns the private with the social benefit of price increases in a liquidity trap, which is to relax the ZLB constraint. TIP can thus implement the constrained-efficient allocation, characterized by positive inflation and a smaller output gap. While TIP cannot restore the first best in this type of liquidity trap, it can substantially improve welfare.

We then compare the welfare gains implied by TIP with those of forward guidance and of government spending, two commonly proposed policies in this type of liquidity trap. Like TIP, these two other policies are second best, as forward guidance relies on overheating the economy after the liquidity trap ends and government spending crowds out private consumption. Numerical simulations of a calibrated small-scale model suggest that, in the empirically-relevant part of the parameter space, TIP performs significantly better than forward guidance. It performs slightly worse than government spending, but welfare is of the same order of magnitude.

In a self-fulfilling liquidity trap, a moderate positive tax on price increases can address the pecuniary externality and a large tax can rule out the deflationary equilibrium. In an SFLT there are two distinct inefficiencies. Like in an FLT, firms do not internalize how their pricing decisions affect aggregate deflation, real interest rates, and demand. Second, the economy may coordinate on the deflationary equilibrium even though a first-best equilibrium also exists. TIP can address both margins. A moderate positive TIP implements the second-best allocation conditional on remaining at the ZLB. A sufficiently large positive TIP goes further and rules out the deflationary equilibrium, leaving the first-best allocation as the unique stationary equilibrium. Importantly, improving welfare in an SFLT requires a tax on price *increases*, not a subsidy to price increases as in an FLT.

We then consider a liquidity trap caused by a shift in the Phillips curve. We find that in this case, TIP can perfectly restore the first-best allocation with full employment and inflation at target. This result is consistent with Capelle and Liu (forthcoming) which shows that TIP can perfectly stabilize macroeconomic fluctuations in the face of cost-push shocks. In this case, the fundamental source of inefficiency—the cost-push shock—is the wedge between the private and social valuations of price increases. A tax on price decreases, or a subsidy to price increases, can

to sticky wages and financial frictions, which would amplify deflation. This paper abstracts from these additional effects, see Eggertsson and Krugman (2012) for an analysis incorporating debt deflation.

perfectly address this inefficiency and re-align private and social benefits.

Given the uncertainty surrounding the underlying drivers of liquidity traps, we analyze policy designs that are robust to these different sources. Like other policies analyzed in the literature and used in practice—forward guidance and government spending—the simple constant TIP studied above is not robust. We thus look for an inflation-targeting rule for TIP and characterize the rule that maximizes welfare. This policy choice is made subject to uncertainty about the model's parameters and shocks.

We show that an inflation-targeting rule that is sufficiently responsive to deflation and with a negative intercept, corresponding to a long-run subsidy to price increases, is a robust optimal policy. We derive this result first analytically under a restricted informational structure. Intuitively, the strong response to deflation eliminates the SFLT, while the negative intercept provides stimulus in the FLT and PCLT. We then show numerically that this finding holds for more general informational structures, following three approaches: (i) the policymaker has a prior on the model's parameters and maximizes expected social utility, (ii) the policymaker is averse to ambiguity, has a reference model in mind and maximizes multiplier preferences (Hansen and Sargent, 2001) and (iii) the policymaker has multiple priors and maximizes the worst possible outcome (Gilboa and Schmeidler, 1989).

These results generalize the "paradox of flexibility" according to which increasing price stickiness at the ZLB can improve welfare (Eggertsson and Krugman, 2012; Billi and Galí, 2020). Indeed, an inflation-targeting TIP increases the degree of aggregate price stickiness. This paradox was derived in the context of the FLT. Our results suggest that if the increase in price stickiness is strong enough it improves welfare in all types of liquidity traps. However, small increases in price stickiness worsen the allocation in a self-fulfilling equilibrium. An important policy implication is that the strength of the response of TIP to deflation needs to be appropriately calibrated.

We then assess the effectiveness of TIP in a medium-scale DSGE model *à la* Smets and Wouters (2007) calibrated to Japan. Japan provides an ideal laboratory for this exercise, having experienced prolonged deflation and a policy rate constrained by the ZLB since the early 2000s. Following Hirose (2020), the model includes a variety of important features and realistic frictions. We consider two different calibrations of the model, corresponding to an FLT and an SFLT, following the approach by Cuba-Borda and Singh (2024). We also introduce multiple temporary shocks driving macroeconomic fluctuations.

Using the calibrated medium-scale model, we construct a simple and robust inflation-targeting rule for TIP that would have lifted Japan out of its deflationary equilibrium during 1998–2012. We find that the rule needs to react aggressively to deflation—corresponding to at least 28 percentage points for each percentage point of additional quarterly deflation—to eliminate the SFLT. In addition, a negative intercept, corresponding to a long-run 5% subsidy to price increases, could have brought back inflation to the 1% annual target in the FLT calibration of the model, at a negligible fiscal cost. Finally, this policy rule would have closed the persistent output gap.

Literature. We show how the traditional deflationary spiral can be interpreted as a pecuniary externality in firms' pricing decisions and show how TIP can directly address it. This externality differs from the aggregate-demand externality emphasized by Blanchard and Kiyotaki (1987) and pecuniary externalities arising from leverage or borrowing constraints (Lorenzoni, 2008; Eggertsson and Krugman, 2012; Korinek and Simsek, 2016; Farhi and Werning, 2016). The pecuniary externality arises even in the representative-agent New Keynesian economy without leverage or heterogeneity: when the ZLB binds, firms ignore that cutting their prices exacerbates aggregate deflation, raises real interest rates, and depresses demand. TIP can address the resulting inefficiency by realigning private and social incentives to adjust prices.

We contribute to the large literature on the monetary and fiscal policies at the ZLB. In an FLT, papers have analyzed the role of tax increases aimed at restricting supply (Eggertsson and Woodford, 2006), tax cuts aimed at stimulating demand (Eggertsson, 2011; Billi and Galí, 2020), temporary government spending (Woodford, 2011; Schmidt, 2013; Nakata, 2016), active fiscal policy (Bianchi and Nicolò, 2021; Billi and Walsh, 2025). In an SFLT, papers have emphasized the role of cuts in marginal labor tax rates that boost confidence (Mertens and Ravn, 2014), non-ricardian fiscal expansion (Benhabib, Schmitt-Grohé, and Uribe, 2001b; Benhabib, Schmitt-Grohé, and Uribe, 2002), ricardian fiscal rules (Schmidt, 2016) or debt issuance (Caramp and Singh, 2023). Benigno and Fornaro (2017) highlight the role of investment subsidies in stagnation traps. Correia et al. (2013) and Farhi et al. (2014) show that well-designed paths of consumption and labor taxes, import and export tariffs can replicate the first-best allocation.

We make several contributions to this rich literature. This paper is the first to consider the stabilization properties of TIP at the ZLB. TIP is conceptually very different from monetary and fiscal policies analyzed in prior literature: it directly shapes firms' pricing decisions, correcting a pecuniary externality at the ZLB, while other policies operate through the interest rates, demand management, and the real side of the economy.

In addition, a novel contribution of our paper is to design a robust rule for TIP across different types of liquidity trap. The lack of robustness of traditional fiscal and monetary policies (forward guidance, interest rate increases, public spending, labor tax cuts) has been highlighted in recent papers (Nakata and Schmidt, 2022; Bilbiie, 2022). Our approach builds on seminal works on model uncertainty, ambiguity and robust policy by Gilboa and Schmeidler (1989), Hansen and Sargent (2001) and Hansen and Sargent (2008). Existing contributions have either studied the robust design of monetary policy away from the liquidity trap or when policymakers are uncertain about the location of the effective lower bound (Tillmann, 2021; Özcan and Traficante, 2024).

Our paper contributes to a literature on TIP. Capelle and Liu (forthcoming) analyze the effectiveness of TIP outside the liquidity trap. This paper builds on an older literature from the 1960s-1970s (Wallich and Weintraub, 1971; Seidman, 1978, 1979). In these decades, TIP was widely discussed in the U.S. and in Western Europe and versions of TIP were even briefly implemented in a few countries. The contribution of this paper is two-fold. This is the first paper focusing on how TIP can be used at the ZLB to address the deflationary spiral whether it comes from a decline in r^* or self-fulfilling expectations. It also takes seriously the issue of robustness and show how to design TIP in a way that is effective even when policymakers are uncertain about the underlying drivers of the ZLB.

Finally, we build on recent studies quantifying the sources and implications of the ZLB in Japan (Aruoba, Cuba-Borda, and Schorfheide, 2018; Hirose, 2020; Cuba-Borda and Singh, 2024; Hirose, Iiboshi, Shintani, and Ueda, 2024). Our quantitative assessment draws on these approaches by studying the effectiveness of TIP in a medium-scale NK model. We find that TIP could have provided stimulus at the ZLB in Japan.

The remainder of this paper is organized as follows. Section 2 introduces a conventional sticky price model augmented with a TIP and a ZLB. Section 3 analyzes the optimal design of TIP in an FLT and Section 4 compares it to forward guidance and government spending. Section 5 analyzes the effectiveness of TIP in an SFLT and PCLT and Section 6 shows that a simple inflation-targeting rule is a robust policy. Section 7 provides a quantitative analysis of TIP at the ZLB in Japan. Section 8 concludes.

2 A New Keynesian Model with TIP and a ZLB

We start with embedding TIP into an otherwise conventional New Keynesian model with a ZLB on the nominal interest rate. We briefly sketch the model, focusing on the implications of TIP. We then derive a three-equation log-linear representation of the macroeconomic dynamics which we use to analyze the positive and normative properties of TIP in the three types of liquidity traps in the following sections.

2.1 Household

There is a representative infinitely-lived household. They maximize their discounted sum of CRRA utility over consumption and labor:

$$\mathcal{U}(B_{t-1}) = \max_{C_t, N_t, B_t} \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} + \beta_t E_t \mathcal{U}(B_t) \right\} \tag{1}$$

where C_t, N_t and B_t denote consumption, labor supply and nominal wealth in period t, $\beta_t > 0$ is the time discount factor, and σ and ψ are the inverse elasticity of intertemporal substitution of consumption and labor respectively.

Importantly, the time discount factor, β_t, is allowed to vary stochastically. In equilibrium, it will be equal to the neutral rate of interest. An increase in the discount factor, corresponding to a decline in the neutral rate of interest, will be one of the three potential causes of the liquidity trap (Eggertsson and Woodford, 2006; Bilbiie, 2019).

The problem of the household is to maximize their lifetime utility subject to the sequence of flow budget constraints $P_t C_t + Q_t B_t = B_{t-1} + W_t N_t + T_t$ and a no-Ponzi condition $\lim_{T\to+\infty} \Pi_{j=0}^{T} Q_j B_T \geq 0$. For all $t = 0, 1...$, the optimality conditions are given by

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\psi} \tag{2}$$

$$Q_t = E_t \left[\beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right]. \tag{3}$$

The first equation determines the optimal labor supply N_t given consumption C_t and the real wage W_t. The second equation is the traditional Euler equation which determines the optimal path of consumption given the nominal returns on bonds $1/Q_t$, the path of inflation, P_{t+1}/P_t, and the discount factor, β_t.

2.2 Final Good Competitive Firms

The final good is produced competitively by a continuum of firms. The production technology combines a continuum of varieties of intermediate goods supplied by monopolistic firms with a CES technology:

$$Y_t = \left(\int_0^1 Y_{ti}^{1-1/\epsilon_t} di \right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{4}$$

where ϵ_t is the elasticity of substitution across varieties. Taking the price of the final good P_t and the prices of inputs $\{P_{ti}\}_i$ as given, final good firms maximize profits $\max_{Y_{ti}} P_t Y_t - \int_0^1 P_{ti} Y_{ti} di$ subject to the technology (4).

2.3 Intermediate Good Monopolistic Firms

There is a continuum of mass one of firms specializing in the production of a single variety of good which they sell to the final good firms. The technology to produce goods displays decreasing marginal returns

$$Y_{ti} = A_t N_{ti}^{1-\alpha} \tag{5}$$

where A_t denotes total factor productivity, which is common across all firms, and $1 - \alpha$ is the elasticity of output to labor. Firms are in monopolistic competition and choose their price, P_{ti}, taking into account the demand schedule of final-good firms. Following Rotemberg (1982), they also face quadratic adjustment costs to price changes $\mathcal{C}_t\left(P_{t-1i}, P_{ti}\right) = \frac{\theta}{2}\left(\frac{P_{ti}}{P_{t-1i}} - 1\right)^2 P_t Y_t$.

The key novelty is that firms pay a tax (or subsidy if negative) proportional to the increase in their price, $\tau_t(P_{ti} - P_{t-1i})$, where $\tau_t \in \mathbb{R}$ is the tax rate set by the government.[3] In addition, the total tax payment scales with the firm's output, Y_{ti}. Hence profits net of taxes are given by

$$\Pi(P_{t-1i}, P_{ti}) = P_{ti}Y_{ti} - W_t N_{ti} - \tau_t(P_{ti} - P_{t-1i})Y_{ti} - \mathcal{C}_t\left(P_{t-1i}, P_{ti}\right). \tag{6}$$

where labor demand N_{it} is given by the technology (5) and output Y_{ti} by the demand schedule from final-goods firms. In recursive form, the problem of the firm is given by

$$V(P_{t-1i}) = \max_{P_{ti}} \Pi(P_{ti}, P_{t-1i}) + E_t\left[Q_t V(P_{ti})\right] \tag{7}$$

After substituting for labor using the production function, and for output using the demand schedule, one can take the first order condition for the optimal price. After imposing that in equilibrium all firms are symmetric, $P_{ti} = P_t$, and denoting the rate of inflation $\pi_t = P_t/P_{t-1} - 1$, the optimality condition is given by

$$\begin{aligned}(\epsilon_t - 1)\left(\mathcal{M}_t MC_t - 1\right) + E_t\left[Q_t \frac{Y_{t+1}}{Y_t}\left(\tau_{t+1} + \theta(\pi_{t+1}+1)\pi_{t+1}\right)\right] \\ = \tau_t\left(1 - \epsilon_t \frac{\pi_t}{1+\pi_t}\right) + \theta\pi_t(\pi_t + 1)\end{aligned} \tag{8}$$

where the real marginal cost is given by $MC_t = \frac{W_t}{P_t(1-\alpha)} \frac{Y_t^{\frac{\alpha}{1-\alpha}}}{A_t^{\frac{1}{1-\alpha}}}$ and the ideal markup in the flexible price equilibrium is given by $\mathcal{M}_t = \frac{\epsilon_t}{\epsilon_t - 1}$.

A negative cost-push shock will be one of the three causes of a liquidity trap we analyze this paper. Although it has not been considered in the literature, we show that it is a theoretical possibility and the quantitative exercise provides some empirical support for it. The cost-push is a reduced-form way to model conflicting aspirations of workers and firms over relative wages and prices, as in Lorenzoni and Werning (2023). It can also be interpreted as a shock to inflation expectations, for

[3] Capelle and Liu (forthcoming) showed that a market for inflation permits can implement the same allocations as a tax on inflation. In that case, τ is the equilibrium price of a permit to increase prices.

instance due to better anchoring of these expectations.

2.4 Central Bank Policy and Zero Lower Bound

The central bank sets the nominal interest rate. Outside of the ZLB the central bank follows an inflation-targeting interest rate rule. However, it is constrained by a ZLB on interest rate. Its policy rate is thus given by

$$i_t = -\log(Q_t) = \max(0, \phi_\pi \pi_t - \log \beta_t) \tag{9}$$

where $-\log \beta_t$ is the equilibrium neutral rate of interest. In addition, monetary policy is active outside the ZLB, in the sense that the policy rate reacts more than one for one to changes in the rate of inflation:

$$\phi_\pi > 1. \tag{10}$$

Benhabib, Schmitt-Grohé, and Uribe (2001b) show that the nonlinearity introduced by the ZLB on nominal interest rates, when combined with a steep Phillips curve, can give rise to multiple rational expectations equilibria, including a deflationary trap alongside the intended target equilibrium. This self-fulfilling deflationary equilibrium is the third and final potential cause of the liquidity trap.

We close the model with a balanced government budget that ensures that the net transfers to firms due to TIP and the wage bill subsidy are funded by lump-sum taxes on households. In addition, the markets for each intermediate goods, the final good, labor and bonds should clear. We give a full definition of the general equilibrium in Appendix A.

2.5 Three-equation Representation

Following the literature and to obtain analytical results, we focus on the linearized version of the model around the efficient equilibrium. We denote $\hat{y}_t = \log(y_t / y_t^e)$ the log-deviation of output from its efficient value, u_t the deviation of markup from its steady-state and $\beta \in (0, 1)$ the steady-state value of the discount factor. We assume that a production subsidy corrects for the steady-state markup distortion so that the flexible price equilibrium is efficient outside the ZLB. Capelle and Liu (forthcoming) show that a steady-state TIP can equivalently eliminate the steady-state markup distortion.

The model boils down to three equations: an Euler equation or IS curve, a Phillips curve and a Taylor rule subject to a ZLB. The following lemma gives their expression.

Lemma 1. *Around a steady-state with zero inflation and a zero TIP, the linearized Euler equation, the Phillips curve and the Taylor rule subject to a ZLB are given by:*

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}\left(i_t - E_t\pi_{t+1} + \log\beta_t\right) \tag{11}$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa\hat{y}_t + \frac{1}{\theta}\left[\beta E_t\tau_{t+1} - \tau_t + u_t\right] \tag{12}$$

$$i_t = \max(0, \phi_\pi\pi_t - \log\beta_t). \tag{13}$$

where $\kappa = \frac{\epsilon-1}{\theta}\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)$.

Capelle and Liu (forthcoming) show that Calvo-type frictions would lead to the same first-order representation.

3 Optimal TIP in a Fundamental Liquidity Trap

We start with analyzing how TIP can improve welfare in a liquidity trap arising from a decline in the neutral rate of interest (r^*). This driver of the ZLB has received significant attention (Eggertsson and Woodford, 2006; Bilbiie, 2019) and is consistent with the secular decrease in the real interest rate documented across many rich countries (Rachel and Summers, 2019). We show that TIP can implement the second-best allocation by addressing the pecuniary externality at the ZLB. In a standard calibration, the optimal TIP is quantitatively reasonable—it entails a negligible fiscal cost—and achieves significant reduction in the output gap and in deflation.

3.1 Liquidity Trap *Laissez-Faire* Equilibrium

We model the decline in the neutral rate of interest as a temporary discount factor shock, $\Delta\beta > 0$.[4] The discount factor can thus take two values $\beta_t \in \{\beta, \beta + \Delta\beta\}$, with $\beta < 1 < \beta + \Delta\beta$ and a ZLB may arise when the discount factor is high $\beta_t = \beta + \Delta\beta$.

[4]The discount factor shock is also a reduced-way of modeling a deleveraging shock as in Eggertsson and Krugman (2012).

The economy starts in the liquidity trap, $\beta_0 = \beta + \Delta\beta$, and the discount factor follows a Markov chain: every period there is a constant probability, p, that the discount factor remains high in the following period. With complementary probability $(1 - p)$, the economy exits the ZLB. Once the economy leaves the liquidity trap, it stays there forever, *i.e.* β is an absorbing state, following Bilbiie (2019) and Bilbiie (2022).

Consistent with the Markov structure of the underlying discount factor shock, we focus on minimal state variable equilibria in which the allocation and prices are only a function the driving variable, β_t. We thus omit the time subscript and use the "L" subscript to refer to the liquidity trap equilibrium. Outside of the liquidity trap ($\beta_t = \beta$), the equilibrium is characterized by $\pi = \hat{y} = 0$ and $i = -\log \beta$. In the ZLB equilibrium ($\beta_t = \beta + \Delta\beta$), the nominal interest rate is at its lower bound $i = 0$ and the inflation rate π_L and the output gap $\hat{y}_L$ are solutions to:[5]

$$\hat{y}_L = p\hat{y}_L + \frac{1}{\sigma}\left(p\pi_L - \log(\beta + \Delta\beta)\right) \tag{14}$$

$$\pi_L = p\beta\pi_L + \kappa\hat{y}_L + \frac{1}{\theta}\left[\beta p\tau - \tau\right]. \tag{15}$$

The following lemma characterizes the size of the negative output gap and of deflation as well as the range of parameters consistent with an FLT.

Lemma 2 (Decentralized equilibrium - FLT)**.** *There exists a unique equilibrium with deflation $\pi_L < 0$ and negative output gap $\hat{y}_L < 0$ when $\beta_t = \beta + \Delta\beta$ if and only if*

1. *The discount factor is sufficiently high,* $\log(\beta + \Delta\beta) > 0$*,*

2. *The Phillips curve is less steep than the Euler equation in the $(\hat{y}_L, \pi_L)$ space:*

$$\frac{1 - p\beta}{\kappa} > \frac{p}{\sigma(1-p)}. \tag{16}$$

[5]Equation (14) makes clear that the persistence of the liquidity trap $p < 1$ gives rise to an upward-sloping Euler equation in the $(\hat{y}_L, \pi_L)$ space. This is an important property to obtain a stationary liquidity trap equilibrium. This property is quite general: it would also arise in heterogeneous agents models in which aggregate savings increase with the real rate of interest (so consumption increases in inflation) or in models with cognitive discounting. Recent papers estimating DSGE models allowing for discounting find significant empirical support for it (Gabaix, 2020; Hirose, Iiboshi, Shintani, and Ueda, 2024).

The decentralized equilibrium is given by

$$\pi_L = -\frac{1}{(1-p\beta)\sigma(1-p)-\kappa p}\left[\kappa \log(\beta+\Delta\beta) + \frac{(1-p\beta)\sigma(1-p)}{\theta}\tau\right] \tag{17}$$

$$\hat{y}_L = \frac{1}{\sigma(1-p)}\left(p\pi_L - \log(\beta+\Delta\beta)\right) \tag{18}$$

Proof. See Appendix B. □

Figure 1 plots the Euler equation (14) and the Phillips curve (15) and shows the laissez-faire equilibrium ($\beta_t = \beta + \Delta\beta$).

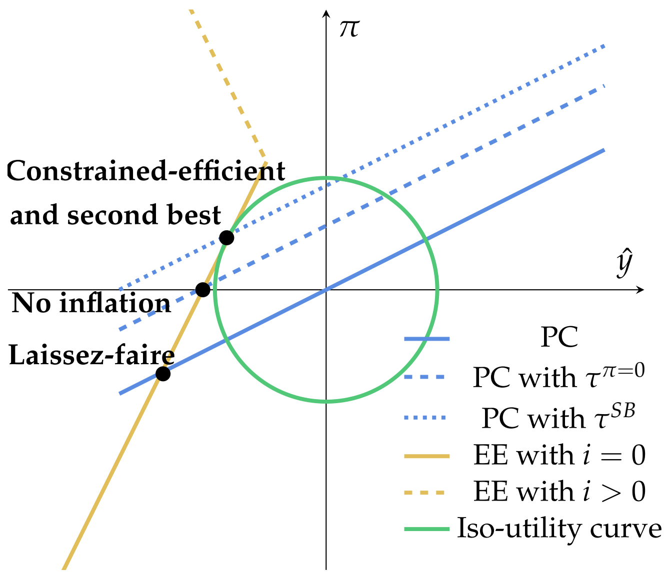


Figure 1: TIP in an FLT

Note: The blue lines represent Phillips curves (PC) under different policies. The green line represents the Euler equation (EE). The iso-utility curve captures preferences over inflation and output.

3.2 Constrained-Efficient Allocation and Pecuniary Externality

We then characterize the constrained-efficient allocation and compare it with the *laissez-faire*. We find that the decentralized equilibrium features a pecuniary externality in the price-setting behaviors of firms which leads to an inefficient deflationary spiral.

We define the constrained-efficient allocation as a path of output, labor, bonds, prices and bond prices $\{C_t, N_t, B_t, Y_t, P_t, Q_t\}_{t=0,1,2..}$ that maximizes the household's utility (1), subject to the technological constraints (4), (5), the resource constraints (45), (46) and the implementation constraint

$$E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \leq 1. \tag{19}$$

Instead of solving for the full non-linear solution of this problem, we reformulate it as a linear-quadratic (LQ) problem, following Rotemberg and Woodford (1999) and Eggertsson and Woodford (2003). The objective of the planner is to minimize the discounted weighted sum of the second-order approximation of welfare losses, given by squared inflation and output gaps:

$$\mathcal{L} = \sum_{t=0}^{\infty} E_0 \beta^t \left[\pi_t^2 + \eta_y \left(\hat{y}_t \right)^2 \right], \tag{20}$$

for $\eta_y = \frac{1}{\theta} \left(\sigma + \frac{\alpha + \psi}{1-\alpha} \right)$, subject to the ZLB constraint (18), which is the linearized version of equation (19).

We restrict the set of allocations the social planner can choose from to those that are a function of the driving variable β_t and that are time consistent. This implies that as soon as the discount factor reverts back to its lower value, β, and the economy exits the liquidity trap, the social planner implements the static first-best allocation, with $\pi = \hat{y} = 0$. It also implies that the social planner doesn't manufacture a boom after the economy exits the liquidity trap. This is an important difference with Eggertsson and Woodford (2003) who solve for the time-varying constrained-efficient allocation with commitment and show how forward guidance can implement it.[6] Our restriction is also consistent with our focus on minimum state variable equilibria in the decentralized equilibrium. In what follows, we omit the time subscript on endogenous variables and use the "CS" superscript for both endogenous variables, π_L^{CS} and $\hat{y}_L^{CS}$.

Importantly, our social planner is not constrained by the optimal price-setting behavior of private agents given by equation (15). She is free to choose the level of inflation as long as it is consistent with the other constraint (18). By abstracting from the behavioral constraint on inflation, our constrained-efficient allocation highlights

[6] Forward guidance type allocations have been extensively analyzed and discussed in prior work, e.g. in Bilbiie (2019). A committed social planner may want to use TIP to manufacture a boom after exiting the ZLB, see Appendix B.2

the pecuniary externality and the inefficiency related to the deflationary spiral. This is another important difference with the problem analyzed in Eggertsson and Woodford (2003). By abstracting from one behavioral constraint, we set a more demanding benchmark for policymakers using TIP in a decentralized economy.

The following proposition characterizes the constrained-efficient allocation. We denote with an "LF" superscript, the variables in the *laissez-faire*.

Proposition 1 (Constrained-Efficient Allocation)**.** *At an interior solution, the constrained-efficient (CS) levels of the output gap and inflation are given by*

$$\hat{y}_L^{LF} < \hat{y}_L^{CS} = -\frac{\sigma(1-p)}{\sigma^2(1-p)^2 + p^2\eta_y}\log(\beta + \Delta\beta) < 0, \tag{21}$$

$$\pi_L^{LF} < 0 < \pi_L^{CS} = \frac{\eta_y p}{\sigma^2(1-p)^2 + p^2\eta_y}\log(\beta + \Delta\beta) > 0. \tag{22}$$

As illustrated in Figure 1, the social planner chooses a positive inflation rate and a negative output gap. By boosting inflation, the social planner relaxes the constraint on the ZLB, reduces the real interest rate and thus increases output. Around the optimal allocation with positive inflation, the social planner trades off lowering inflation—which is costly—and further reducing the output gap by decreasing the real interest rate. At the optimum, the marginal cost of increasing inflation is equal to the marginal gains of reducing the output gap.

Proposition 1 also shows that there is scope for improving upon the *laissez-faire*. This is due to the presence of a pecuniary externality at the ZLB. Although the ZLB is the *causa sine qua non* of the inefficiency in equilibrium, it is compounded by a deflationary spiral stemming from the behavior of firms competing to attract scarce demand when output is below potential. Firms fail to take into account the fact that by decreasing their individual price, they contribute to aggregate deflation which in turn increases the real rate of interest and depresses output, which aggravates the imbalance between aggregate demand and output potential.

3.3 Ramsey Allocation: Welfare-Maximizing TIP

To correct this pecuniary externality giving rise to excessive deflation, policymakers can use TIP to directly affect the price-setting behavior of firms. We show that TIP can implement the second-best allocation and address this pecuniary externality.

The Ramsey optimal problem consists in choosing a TIP rate τ^{SB} and the associated allocation to maximize the household utility (1), subject to the constraint that the allocation is a decentralized equilibrium of the economy. Consistent with our approach in the decentralized economy and constrained-efficient allocation, we restrict attention to Markov policies for TIP and solve the LQ Ramsey problem. The set of constraints is thus the Phillips curve (17) and the IS curve (18). That policymakers are subject to the firms' optimal pricing decision is the key difference with the constrained-efficient allocation.

By giving firms incentives to increase their prices, we find that TIP can implement the constrained-efficient allocation. To see this, consider the graphic representation of the social planner's problem. The circle in Figure 1 represents the iso-welfare locus such that $\pi^2 + \eta_y \hat{y}^2 = \bar{\ell}$, for a given level of welfare loss $\bar{\ell}$. By changing TIP, the social planner is able to shift the Phillips curve. The point at which the Phillips curve is tangent to the indifference locus corresponds the second-best allocation. At the optimum, the social planner chooses a level of TIP that implements an optimal balance between closing the output gap and the costs of higher inflation. Proposition 2 formalizes the result that the second-best allocation implemented by TIP coincides with the constrained-efficient allocation.

Proposition 2 (Implementation of Constrained-Efficient Allocation with TIP)**.** *In the Ramsey optimal second-best allocation (SB), the output gap and inflation rate coincide with their constrained-efficient levels*

$$\pi_L^{SB} = \pi_L^{CS} \quad \textit{and} \quad \hat{y}_L^{SB} = \hat{y}_L^{CS} \tag{23}$$

and the optimal TIP is given by

$$\tau^{SB} = -\theta \frac{\log(\beta + \Delta\beta)}{(1 - p\beta)\sigma(1-p)} \left[\kappa + \eta_y p \frac{(1-p\beta)\sigma(1-p) - \kappa p}{(1-p)^2\sigma^2 + \eta_y p^2} \right]. \tag{24}$$

The important intuition is that TIP gives the right signal to firms about the social cost of their price changes. This makes them internalize the pecuniary externality of their decentralized behavior which can thus break the deflationary spiral at the aggregate level. By addressing the deflationary spiral, TIP can then raise output by lowering the real rate of interest.

3.4 Remarks

First-best allocation. Although TIP corrects the amplification mechanism and addresses the perverse deflationary incentives that firms face when setting their price, it cannot restore the first-best allocation where $\pi_L = 0$ and $\hat{y}_L = 0$. TIP is not able to do so because it cannot address the root cause of the problem: bringing the nominal interest rate down to the negative value of the natural rate of interest.

A necessary condition for any policy to implement the first best is that it increases the relative price of consumption in the future relative to the present. This shift in the Euler equation can be implemented with an appropriate path of increasing consumption taxes and labor subsidies (Correia, Farhi, Nicolini, and Teles, 2013). By contrast, TIP does not affect the relative price of consumption across periods. Instead it affects the marginal cost of changing prices, which shifts the Phillips curve.

Alternative objective: keeping inflation at target $(\eta_y = 0)$**.** One concern with the second-best allocation, which requires to sustain inflation above target, is that it wouldn't be politically feasible given the salience of inflation and because it conflicts with the mandate of inflation-targeting central banks. An alternative objective—perhaps more realistic—is to assume that policymakers prioritize price stability over output stabilization. We model this objective with lexicographic preferences, $\eta_y = 0$.

When preferences are lexicographic, the optimal TIP brings inflation back to target—zero in this simple model $\pi_L = 0$—as displayed in Figure 1. Proposition 3 gives the level of TIP that implements this allocation. We denote $x^{\pi=0}$, the value of variable x in the equilibrium with TIP such that there is no deflation.

Proposition 3. *The TIP that sets inflation to 0, $\pi_L = 0$, at the ZLB is given by*

$$\tau^{SB} < \tau^{\pi=0} = -\frac{\kappa\theta}{1-p\beta}\frac{\log(\beta+\Delta\beta)}{\sigma(1-p)} < 0. \tag{25}$$

This level of TIP implies a reduction in the output gap:

$$\hat{y}^{LF} < \hat{y}^{\pi=0} = -\frac{\log(\beta+\Delta\beta)}{\sigma(1-p)} < \hat{y}^{SB} < 0. \tag{26}$$

The optimal level of TIP is less negative than the second-best level of TIP, because inflation needs not be as high as in the second best. However the output gap remains larger than in the second best. This highlights again the trade-off policymakers face when using TIP in a liquidity trap driven by a decline in the neutral rate of interest.

Recurring liquidity trap and commitment. The analysis above focuses on a time-consistent planner and on an absorbing good state. In Appendix B.2, we relax both assumptions and solve for the more general case in which the liquidity trap can recur and the planner may, or may not, commit. The main insights remain unchanged: TIP can still implement the constrained-efficient allocation by shifting the Phillips curve and correcting the pecuniary externality in firms' pricing decisions; in an SFLT, a sufficiently aggressive TIP still rules out the deflationary allocation. This analysis delivers two intuitive insights. First, a committed planner may promise a good-state boom to improve expectations while the economy is at the ZLB. Second, by promising to eliminate the SFLT should expectations become pessimistic, a committed planner can eliminate the SFLT from the equilibrium path.

Determinacy. Outside of the ZLB and without state switching, determinacy in the baseline New Keynesian model requires that monetary policy implement the Taylor principle according to which the policy rate reacts strongly to inflation. With state switching and one state displaying a ZLB, in which monetary policy does not react to inflation, the Taylor principle may no longer be necessary or sufficient to ensure stability (Davig and Leeper, 2007; Farmer, Waggoner, and Zha, 2009).

In our setting, given that the normal state is absorbing, the Taylor principle ensures determinacy outside of the liquidity trap. But at the ZLB, there is a multiplicity of forward-stable solutions to Equations (11) and (12). In the analysis above we selected the minimal state variable equilibrium, which is only a function of the underlying fundamentals of the economy. Importantly, all equilibria converge to the minimal state variable equilibrium. These two reasons support selecting this equilibrium (McCallum, 1999).

A natural question is whether TIP could help ensure uniqueness in this setting. The following proposition establishes that a TIP rule that responds sufficiently aggressively to the output gap can ensure the uniqueness of the minimum state equilibrium. This result is reminiscent of the Taylor principle for monetary policy.

Proposition 4. *Assume that at the ZLB, TIP responds linearly to inflation and the output gap:* $\tau_L = \tau_0 + \varphi_\pi \pi_L + \varphi_y \hat{y}_L$. *Assume that outside of the ZLB, TIP is zero and the*

Taylor principle applies. The equilibrium path is globally unique if and only if $\frac{\varphi_y}{\theta} p > \frac{\kappa}{1-p\beta} + \left(1 + \frac{\varphi_\pi}{\theta}\right) \sigma(1-p)$

The proof is in Appendix C. TIP is thus another way to pin down a unique equilibrium path at the ZLB besides fiscal policy (Cochrane, 2017).

3.5 Numerical Illustration

We now calibrate the small-scale New Keynesian model to get a sense of the magnitudes. We find that the optimal levels of TIP, given by equation (24) and (25) respectively, are quantitatively reasonable in the sense that they entail negligible fiscal costs and achieve significant reductions in the output gap and in deflation.

Baseline calibration. In our baseline calibration, all parameters are within the range of standard values in the literature. A period is a quarter. The list of parameters is in Table 1 and consists of the elasticity of output to labor $(1-\alpha)$, the discount factor β, the elasticity of intertemporal substitution σ, the Frisch elasticity ψ and the elasticity of substitution ϵ.

Parameters	Description	Value
α	One minus the elasticity of output to labor	0.25
β	Time discount factor	0.99
σ	Elasticity of intertemporal substitution	1
ψ	Inverse Frish elasticity of labor	5
ϵ	Elasticity of substitution across varieties	6
θ	Adjustment cost	372.8
$\beta + \Delta\beta$	Discount factor shock	1.001
p	Persistence of liquidity trap	.7

Table 1: Model parameters

We calibrate the Rotemberg parameter to $\theta = 372.8$ following Capelle and Liu (forthcoming), who target the slope of the Phillips curve in Galí (2015). Following Bilbiie (2019), we set the discount factor at the ZLB to $\beta + \Delta\beta = 1.001$, which amounts to a neutral rate of interest of -.4% per annum, and the persistence probability to $p = .7$.[7]

[7]This value is slightly below that used in Bilbiie (2019). Higher values of p violate condition (31).

Results and sensitivity. In the *laissez-faire*, this parametrization generates an output gap in the liquidity trap of -1.8% and an inflation gap of -.6%. In the second-best allocation, the output gap is equal to -.3% and inflation is above target by .01pp. The level of TIP that implements this allocation, τ^{SB}, is equal to -44%. This subsidy implies a small fiscal cost for the government of .005% of firms' sales. It turns out that, quantitatively, the allocation implemented by a policymaker seeking to bring inflation back to target ($\eta_y = 0$) is almost identical to the second-best one.

Consistent with equations (24) and (25), the optimal level of TIP ($\tau^{SB}, \tau^{\pi=0}$) needs to be more negative as the discount factor shock increases, *i.e.* as the neutral rate of interest declines (Figure 2). The degree of price stickiness has a negligible impact on the optimal of TIP, which is the result of two offsetting effects. On the one hand, stickier prices attenuate the negative effects of the decline in the neutral rate of interest, but on the other it makes a given decrease in TIP less effective at boosting inflation. Finally, a higher persistence parameter and a smaller IES worsen the recession and the deflation, and they thus require a more aggressive decrease in TIP.

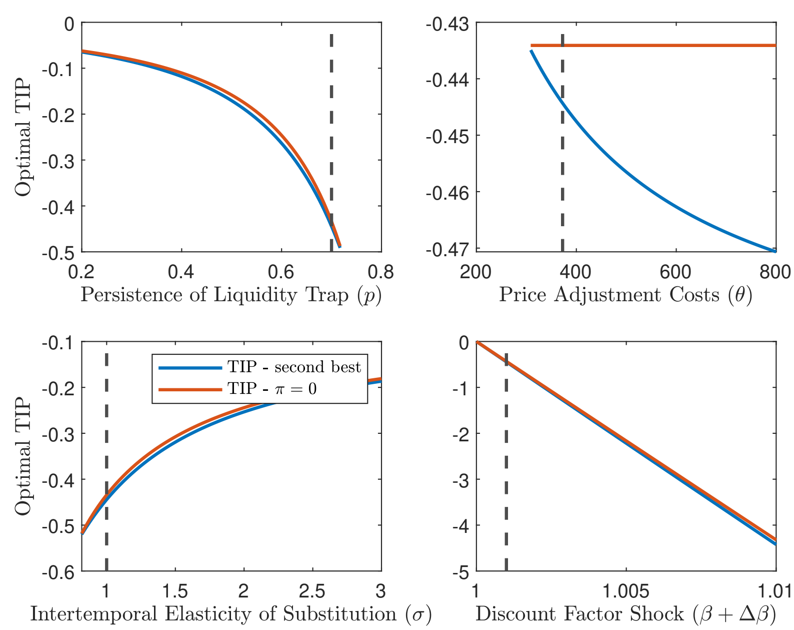


Figure 2: Optimal TIP ($\tau^{SB}, \tau^{\pi=0}$) in an FLT

4 TIP and Other Policies in an FLT

Several instruments have been proposed to improve welfare at the ZLB. Among them, forward guidance and government spending have received particular attention. In this section, we compare the welfare gains from TIP with those from forward guidance and government spending using a simple small-scale New Keynesian model. The analysis, which is intended as a proof of concept, shows that TIP performs significantly better than forward guidance and slightly worse than government spending.

4.1 TIP and Forward Guidance

Forward guidance has been extensively studied in previous literature, including in Eggertsson and Woodford (2003) and Bilbiie (2019). We closely follow the approach proposed in the latter paper in our modeling of forward guidance and assume that when the economy exits the liquidity trap, the central bank commits to keep its interest rate at 0 for some time. As argued in this paper, this second-best policy is well approximated by a time-invariant policy consisting of keeping the interest rate at 0 with some probability q every period.

There are thus three states: a liquidity trap state "L" ($\beta_t = \beta + \Delta\beta$ and $i_t = 0$), a forward guidance state "FG" ($\beta_t = \beta$ and $i_t = 0$) and a normal state, "N" ($\beta_t = \beta$ and $i_t = -\log\beta > 0$). The economy remains in the "L" state with probability p every period. Once it is in the "FG" state, it remains there with probability q every period. Once it exits this second state, it stays in the "N" forever, *i.e.* the normal state is absorbing.

In Appendix D.1 we derive the equilibrium conditions in the liquidity trap and in the forward guidance state. We also derive the expression for the policymakers' objective in these two states. The key trade-off faced by policymakers choosing the persistence of forward guidance q is between boosting the economy in the liquidity trap and overheating the economy after exiting it. The optimal choice of q balances these two forces.

Both TIP and forward guidance are second-best policies. Whether TIP offers better stabilization gains than forward guidance is a complex matter that depends on many parameters. For this reason, we now compare numerically the welfare losses implied by the second-best forward guidance and the second-best TIP. The calibration is the same as in the previous section.

Results. To compare systematically the welfare losses (20) of forward guidance to those implied by TIP, we vary four important parameters, one at a time: the persistence of the liquidity trap p, the slope of the Phillips curve κ, the intertemporal elasticity of substitution σ and the discount factor shock $\beta + \Delta\beta$. Figure 3 shows the results. The dashed black vertical line corresponds to the baseline value of the parameters.

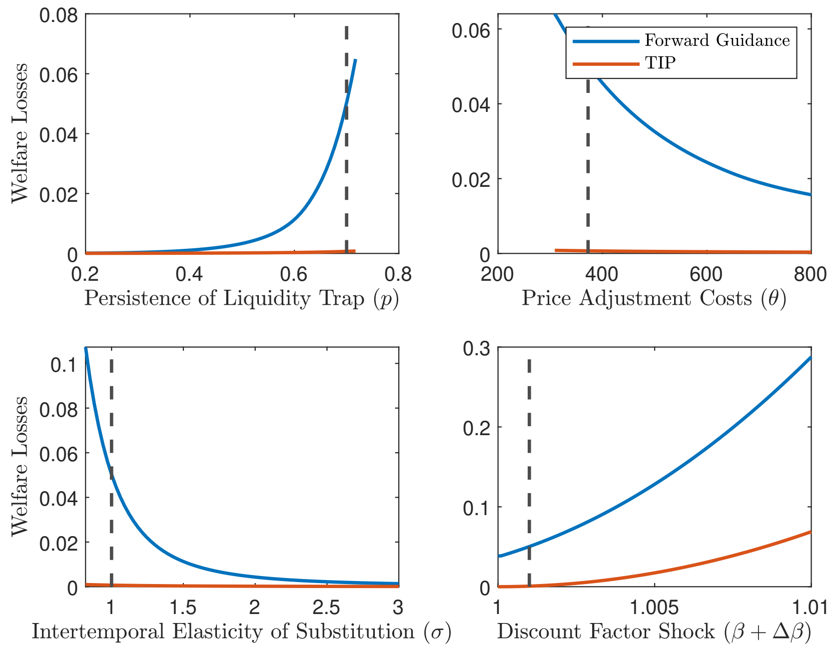


Figure 3: Welfare Losses under Optimal Forward Guidance and TIP

Welfare losses are smaller under TIP than under forward guidance in most of the parameter space. This is because forward guidance needs to generate a boom in the future. This boom, by distorting the future allocation, entails welfare costs. TIP on the contrary does not work through an intertemporal trade-off, but through an intratemporal trade-off: it boosts inflation today—above its first-best level—to decrease the real rate and increase output. It is worth noting that, although we abstract from this possibility by construction, TIP could also be used to manufacture a boom after the economy exits the liquidity trap.

As Eggertsson, Mehrotra, and Robbins (2019) argue, forward guidance becomes less and less effective when the persistence of the liquidity trap increases. This is true as well in our setting as can be seen in the top left panel of Figure 3. It turns out that TIP remains relatively more effective than forward guidance as the persistence of the liquidity trap increases.[8]

4.2 TIP and Government Spending

Government spending is another policy tool that has attracted a lot of attention (Woodford, 2011; Schmidt, 2013; Nakata, 2016; Bilbiie, Monacelli, and Perotti, 2019). We next compare the welfare gains associated with government spending and TIP.

We follow the approach proposed in these papers and assume that utility is separable in government spending with intertemporal elasticity of substitution σ_G:

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} + \frac{G_t^{1-\sigma_G}}{1-\sigma_G} \tag{27}$$

Outside of the liquidity trap, the level of government spending is chosen to equalize the marginal utility of private consumption and government consumption. In the liquidity trap, the optimal choice of government spending balances the gains from a smaller output gap, since government spending boosts output and employment, with its costs. The costs stem from the distortion of the intratemporal allocation across sectors, since government spending increases output, but crowds out private consumption. This distortion shows up as a wedge in the household optimal intratemporal condition between consumption and employment which shifts the Phillips curve.[9]

Whether TIP offers higher or lower welfare gains than government spending depends on many parameters. We thus turn to numerical simulations of the welfare losses implied by both policies for different parameters. We do this comparison

[8] These comparisons abstract from additional challenges associated with forward guidance. First, its effectiveness relies on the central bank's ability to credibly commit to keeping interest rates low and overheating the economy after exiting the ZLB, which may be time-inconsistent and difficult to implement in practice. Second, the model implies strong responses of private agents to future interest rates, a feature known as the forward guidance puzzle. This arises in part because the model omits mechanisms that dampen these effects in reality, such as liquidity constraints and bounded rationality (McKay, Nakamura, and Steinsson, 2016; Gabaix, 2020). Accounting for these realistic features may limit the effectiveness of forward guidance.

[9] In Appendix D.2 we derive the equilibrium conditions in the liquidity trap. We also derive the expression for the second-order approximation of the welfare loss function, which is also the policymakers' objective.

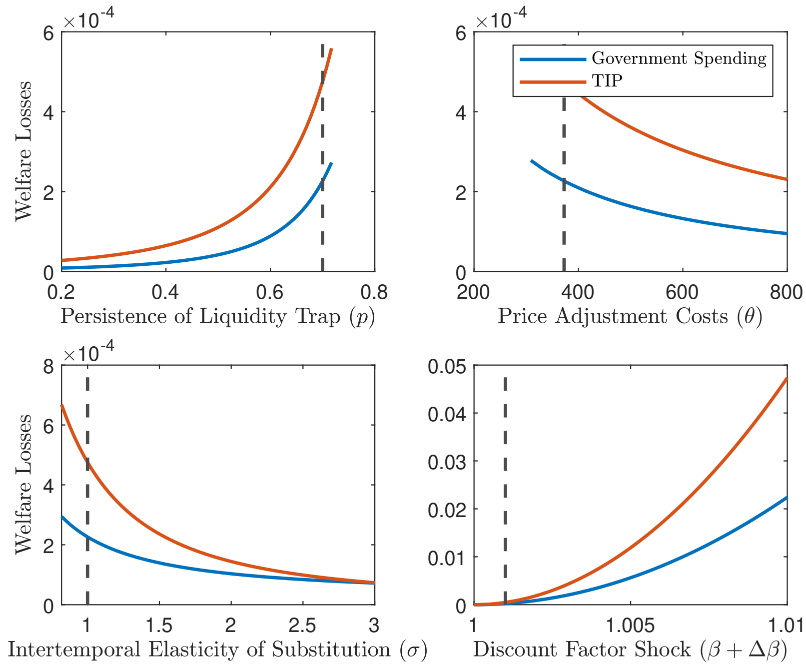


Figure 4: Welfare Losses under Optimal Government Spending and TIP

along the same four important parameters. Consistent with the literature and the relative size of government consumption in advanced economies, we set $\sigma_G = 1.5$ and $w_c = .2$.

Figure 4 shows the welfare losses given by (20) under TIP and under government spending. The welfare losses are higher under TIP but of the same order of magnitude in most of the parameter space. However, in the baseline calibration, the optimal level of government spending implies a fiscal cost that is 60 times higher than that of the optimal TIP. This higher fiscal cost matters in practice due to distortionary taxation and debt sustainability issues.

5 TIP in Self-Fulfilling and Phillips-Curve-Driven Traps

The previous two sections focused on the decline in the neutral rate of interest as driver of the liquidity trap. We now look at two other potential sources of a liquidity trap: self-fulfilling expectations and shifts in the Phillips curve. In an SFLT, TIP has two roles: a moderate TIP implements the second-best allocation conditional on remaining at the ZLB, while a sufficiently aggressive TIP eliminates the SFLT and coordinates expectations on the unique first best. In a PCLT, TIP can restore the first-best allocation.

5.1 A Self-fulfilling Liquidity Trap

An important stream of the literature has shown that liquidity traps could be one of multiple equilibria when monetary policy follows an active policy and the Phillips curve is sufficiently steep (Benhabib, Schmitt-Grohé, and Uribe, 2001b,a; Benhabib, Schmitt-Grohé, and Uribe, 2002; Mertens and Ravn, 2014). In this situation, self-fulfilling pessimistic expectations can lead an economy to the ZLB. Aruoba, Cuba-Borda, and Schorfheide (2018) and Cuba-Borda and Singh (2024) find support for the self-fulfilling hypothesis in Japan.

Model and *laissez-faire*. To keep the same framework as in the previous sections, and following Bilbiie (2019) and Bilbiie (2022), we assume that a sunspot can take two values $x_t \in \{\text{L,FB}\}$ where "L" stands for liquidity trap and "FB" for first best. The economy is initially in the liquidity trap equilibrium, $x_0 = \text{L}$, and the sunspot follows a Markov chain. With probability p, the sunspot stays at L in the following period and with complementary probability, $(1-p)$, it is equal to "FB" and the economy exits the liquidity trap. Like before, the "FB" state is absorbing. In contrast to Section 3, the discount factor is fixed at its value below one, $\beta < 1$ and $\Delta\beta = 0$.

The liquidity trap is one of two possible stationary equilibria.[10] The equilibrium outside the ZLB is characterized by $\pi = \hat{y} = 0$ and $i = -\log\beta$. By contrast, in the liquidity trap equilibrium, the nominal interest rate is at 0, $i = 0$ and the inflation rate and the output gap are solutions to the system of equations given by the Phillips curve (15) and the Euler equation $\hat{y}_L = p\hat{y}_L + \frac{1}{\sigma}(p\pi_L - \log\beta)$ with $\beta < 1$.

The following lemma gives the range of parameters consistent with the existence of multiple stationary equilibria with one displaying negative output gap and deflation in the absence of TIP, $\tau = 0$.

Lemma 3 (Decentralized allocation - Self-fulfilling Expectations)**.** *There exists a ZLB equilibrium with deflation and negative output gap if*

[10] The multiplicity of stationary equilibria is conceptually distinct from the multiplicity of equilibrium paths leading to the same stationary equilibrium. We discussed the latter at the end of Section 3.1. The current section is concerned with the former.

1. *The Taylor coefficient is large enough,* $\phi_\pi > \frac{\kappa p-(1-p\beta)\sigma(1-p)}{\kappa}$*,*

2. *The Phillips curve is steeper than the Euler equation*

$$\frac{1-p\beta}{\kappa} < \frac{p}{\sigma(1-p)}. \tag{28}$$

The equilibrium deflation rate and output gap are given by equations (17) *and* (18) *with* $\Delta\beta = 0$*.*

In particular, when p is high enough, there always exists a deflationary equilibrium, *e.g.* Benhabib, Schmitt-Grohé, and Uribe (2001b) looks at the case $p = 1$. Figure 5 illustrates the multiplicity of stationary equilibria when the Phillips curve is steeper than the Euler equation.

Sources of inefficiency. The self-fulfilling liquidity trap involves two distinct inefficiencies. The first inefficiency arises conditional on the economy remaining in the liquidity trap. Like in the FLT, there is a pecuniary externality in the pricing behavior of firms who ignore that their private decisions to decrease their individual prices exacerbate aggregate deflation. The second is a coordination failure. Because the economy admits both a first-best equilibrium, with $\pi = \hat{y} = 0$ and $i = -\log\beta > 0$, and a deflationary ZLB equilibrium, agents may coordinate on the inefficient liquidity-trap equilibrium. There are therefore two questions for policymakers: how can TIP improve welfare at the SFLT and can it help coordinate agents on the first-best equilibrium?

Second-best TIP. Even when the economy stays at the ZLB, policymakers can set a positive TIP to shift the Phillips curve downward and move the liquidity-trap allocation along the ZLB Euler equation toward a point with higher inflation and higher output, as shown in Figure 5. Graphically, the constrained-efficient allocation corresponds to the tangency between the Euler equation and the iso-welfare curve. At this point, the policymakers balances lower deflation with a positive output gap. The mathematical expression for the constrained-efficient allocation and the level of TIP that implements it are the same as in the case of the FLT.

Proposition 5 (Moderate TIP and Second Best in a SFLT)**.** *At an interior solution, the constrained-efficient deflation rate and output gap are given by equations* (21)–(22) *with* $\Delta\beta = 0$*. The level of TIP that implements that allocation is given by Proposition 2.*

In contrast with Section 3, however, the second-best level of TIP is a positive tax on price increases. Somewhat surprisingly it is optimal to give firms incentives to decrease prices further at the firm level so that prices increase at the aggregate level in general equilibrium. This is another counter-intuitive comparative static result in an SFLT. Indeed, it has been shown that raising the inflation target and an increase in government spending worsens the allocation (Nakata and Schmidt, 2022) or that increases in the nominal interest rate could allow the economy to escape the liquidity trap (Schmitt-Grohé and Uribe, 2014).

Ruling out the SFLT. A sufficiently large positive TIP can rule out the existence of the liquidity trap equilibrium. If TIP is set high enough, the resulting increase in inflation lifts the nominal interest rate away from the zero lower bound. Graphically this corresponds to the dotted line on Figure 5. In that case, the deflationary stationary equilibrium disappears and the remaining equilibrium is the first-best allocation with zero inflation and no output gap. The following proposition gives the lower bound on the level of TIP, $\underline{\tau}^{SF}$, that rules out the existence of the liquidity trap.

Proposition 6 (Aggressive TIP to Rule Out the SFLT)**.** *There exists a minimum level of TIP* $\underline{\tau}^{SF}$ *such that if* $\tau^{SF} > \underline{\tau}^{SF}$*, the first-best allocation is the unique equilibrium of the economy:*

$$\underline{\tau}^{SF} = -\frac{\theta \log \beta}{(1-p\beta)(1-p)\sigma}\left[\kappa + \frac{(1-p\beta)\sigma(1-p) - \kappa p}{\phi_\pi}\right]. \tag{29}$$

Numerical illustration. To get a sense of the magnitudes, we compute the optimal level of TIP given by Propositions 5 and 6. To be consistent with the SFLT, we make two modifications to the parametrization of Section 3: the discount factor is set to $\beta = .999$, corresponding to a .4% annual neutral rate of interest, and the price adjustment cost parameter is recalibrated to $\theta = 248$, corresponding to a steeper Phillips curve, to match the same inflation rate as in the FLT.

We find that the minimum value of TIP that eliminates the self-fulfilling liquidity trap and the optimal second-best TIP are both about 40% (see Appendix Figure 10).

It increases in the persistence of the trap p, in the price adjustment cost θ and in the neutral rate of interest $-\ln(\beta)$. The fiscal costs of the second-best policy are negligible, around .01% of GDP. These negligible fiscal costs suggest that TIP does not suffer from credibility issues, which are more central for government spending and bond issuance (Caramp and Singh, 2023).[11]

Discussion. The optimal TIPs derived in Propositions 5 and 6 have two issues. First, they have the opposite sign of the optimal policy in an FLT, which raises robustness issues if policymakers are unsure about the source of the trap. Second, the elimination of the SFLT results from the violation of equilibrium conditions. We will show in Section 6 that a more intuitive inflation-targeting rule for TIP addresses both of these issues: it is robust and it ensures uniqueness without violating equilibrium conditions.

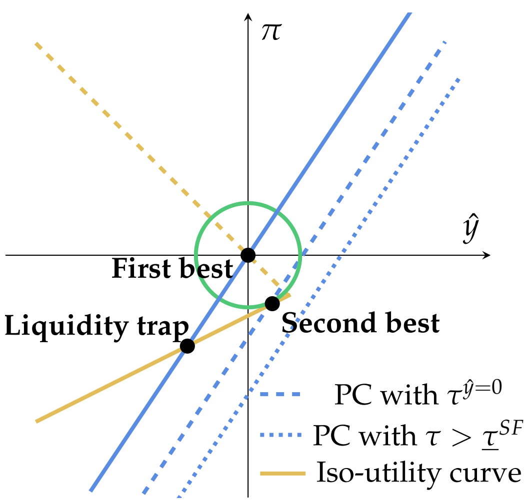


Figure 5: Self-fulfilling liquidity trap

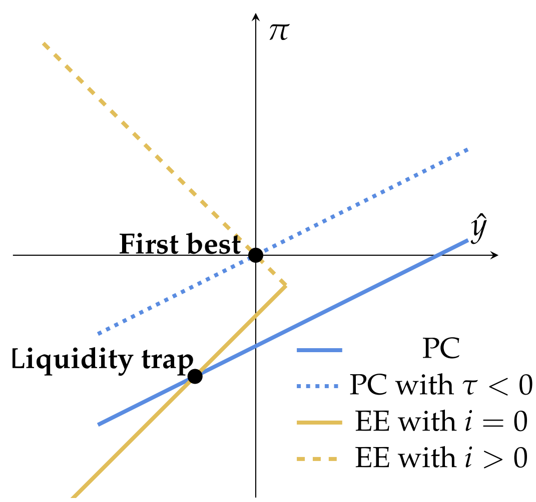


Figure 6: Phillips-curve driven trap

The solid blue line represents the Phillips curve in the laissez-faire equilibrium, $\tau = 0$. The dashed blue line represents the Phillips curve when TIP is used. The green line corresponds to the Euler equation.

5.2 Shift in the Phillips Curve

We now consider a third possible explanation for an economy to fall into a liquidity trap: a persistent negative cost-push shock that shifts the Phillips curve. This

[11]When TIP is aggressive enough to rule out the SFLT, there is no fiscal transfer *ex post* because deflation is 0.

explanation is an interesting theoretical possibility. Although it hasn't received as much attention in the literature as the previous two explanations, it relates to well-understood ideas that pro-competition and supply-side policies can have detrimental effects in an FLT (Eggertsson, 2010; Eggertsson, Ferrero, and Raffo, 2014). We show that in this type of liquidity trap, TIP can implement the first-best allocation with no inflation and no output gap.

Model and *laissez-faire*. To model a shift in the Phillips curve, we augment the model with a cost-push shock, u_t, which can take two values $u_t \in \{\underline{u}, 0\}$ with $\underline{u} < 0$. The negative cost-push shock $\underline{u}$ is the driver of a liquidity trap.[12] The cost-push shock starts at the low value, $u_0 = \underline{u}$ and follows a Markov chain. Every period there is a constant probability p that the cost-push shock remains negative, $u = \underline{u}$, in the following period. Once the shock reverts back to zero, it stays there forever.

Given this stochastic structure, the equilibrium outside the ZLB is characterized by $\pi = \hat{y} = \tau = u = 0$ and $i = -\log \beta$. By definition of the ZLB equilibrium $i = 0$ and the Phillips curve is given by

$$\pi_L = p\beta\pi_L + \kappa\hat{y}_L + \frac{1}{\theta}\left[\beta p \tau_L - \tau_L + \underline{u}\right]. \tag{30}$$

The following lemma gives the equilibrium expressions of inflation and the output gap. It also shows that for a negative cost-push shock to lead to a liquidity trap, it must be that the Phillips curve is less steep than the Euler equation and that the cost-push shock is sufficiently negative.

Lemma 4 (Decentralized Equilibrium - Shifts in the Phillips Curve)**.** *There exists a unique ZLB equilibrium with deflation and negative output gap with $\underline{u} < 0$ if and only if*

1. *The negative cost-push shock is large enough, $\underline{u} < \frac{\theta \log \beta}{\sigma(1-p)} \left(\kappa(1+p) - (1-p\beta)\sigma(1-p)\right)$*
2. *The Phillips curve is less steep than the Euler equation*

$$\frac{1-p\beta}{\kappa} > \frac{p}{\sigma(1-p)}. \tag{31}$$

[12] A negative cost-push shock can be interpreted as a decline in the bargaining power of workers, a decline in the firms markup or a negative shock to inflation expectations due, for instance, to better anchoring of expectations.

In the decentralized equilibrium, the output gap is given by Equation (18) *with* $\Delta\beta = 0$ *and deflation by*

$$\pi_L = -\frac{1}{(1-p\beta)\sigma(1-p) - \kappa p}\left[\kappa \log \beta + \frac{\sigma(1-p)}{\theta}\left((1-p\beta)\tau_L - \underline{u}\right)\right] \tag{32}$$

Figure 6 shows how a liquidity trap can arise due to a downward shift in the Phillips curve.

Constrained-efficient and first-best allocations. Both the constrained-efficient and the first-best allocations feature zero inflation and no output gap, $\pi_L = \hat{y}_L = 0$. The key source of inefficiency in the decentralized economy is the cost-push shock. This shock opens a wedge between the private and the social returns to decreasing prices: firms' private valuations of price decreases exceed their social counterparts.

This externality in the pricing decisions of firms is different from the pecuniary externality in the FLT analyzed in Section 3. In this earlier section, the pecuniary externality was amplifying a fundamental inefficiency that was due to the ZLB in the context of a negative neutral rate of interest. The constrained-efficient social planner could address this amplification mechanism, but could not eliminate the fundamental source of inefficiency. In the case of a shift in the Phillips curve, the source of inefficiency is the wedge introduced by the cost-push shock itself. But the social planner is not subject to the cost-push shock, since it is not constrained by the price-setting behaviors of firms. As a result, the constrained-efficient social planner can implement the first-best allocation.

Optimal TIP. It turns out that a very simple negative TIP can restore the first-best allocation at the liquidity trap. The following proposition gives the expression of the optimal level of TIP.

Proposition 7 (Optimal TIP in a PCLT)**.** *TIP can implement the first-best allocation with* $\pi_L = \hat{y}_L = 0$*, it is given by*

$$\tau_L = \frac{\underline{u}}{1-p\beta} < 0. \tag{33}$$

Given that the cost-push shock is negative, $\underline{u}$, the optimal TIP should also be negative. A negative TIP, by giving incentives to increase prices, closes the wedge between the private and the social returns to decreasing prices and therefore corrects excessive aggregate deflation. Figure 6 shows how TIP can shift the Phillips curve upward, which increases inflation and output.

Numerical illustration. To get a sense of the magnitude we compute the optimal level of TIP in the small-scale model. The calibration follows the baseline FLT parametrization with $\beta = .999$, the original price adjustment cost parameter $\theta = 372.8$ and the cost-push shock $\underline{u}$ is recalibrated so that the *laissez-faire* inflation rate matches the same inflation rate as in the FLT. We find that the value of TIP that restores the first-best with zero inflation is -44% (see Appendix Figure 11 for a sensitivity analysis to key parameters). There is no fiscal transfer *ex post* because deflation is zero.

6 A Robust Inflation-Targeting Rule

The previous sections showed that the optimal TIP depends on the sources of the liquidity traps. In the FLT and PCLT, the optimal policy is to set a negative TIP while in an SFLT, TIP should be positive and sufficiently high. This is problematic when policymakers are uncertain about the true underlying states of the economy.[13] In this section, we show that an inflation-targeting TIP rule that responds aggressively to inflation and with a negative intercept is a robust policy to increase welfare in a liquidity trap.

6.1 Environment: an Uncertain Policymaker with a TIP rule

An inflation-targeting rule for TIP. In the spirit of designing a robust policy rule, we allow the policymaker to choose among the set of inflation-targeting rules for TIP. This targeting rule parallels the Taylor rule for monetary policy and Capelle and Liu (forthcoming) showed it has appealing stabilization properties outside the ZLB. This rule has an intercept τ_0 and a term that is linear in inflation $\varphi_\pi \pi$:

$$\tau = \tau_0 + \varphi_\pi \pi \tag{34}$$

where φ_π parametrizes the strength of the reaction of TIP to inflation.

[13] Other policies such as forward guidance, government spending and change in inflation targets also lack robustness (Bilbiie, 2022; Nakata and Schmidt, 2022).

Policymaker's information set. To model the policymaker's (P) uncertainty about the source of the liquidity trap, we assume that she is uncertain about the elasticity of intertemporal substitution σ, the degree of price stickiness θ, the cost-push shock u, and the discount factor shock $\Delta\beta$. This means that she is uncertain about the slopes (σ, θ) and the intercepts $(\Delta\beta, u)$ of the IS curve and the Phillips curve. A state s of the economy is thus described by a vector of these parameters, $(\sigma_s, \theta_s, \Delta\beta_s, u_s)$. The remaining parameters are known with certainty.

P does not know which state of the world has realized. However she observes the *laissez-faire* (LF) allocation, $(\pi^{LF}, \hat{y}^{LF})$. This imposes restrictions on the set of possible states, $(\sigma_s, \theta_s, \Delta\beta_s, u_s)$, since P understands the data-generating process is given by

$$\log(\beta + \Delta\beta_s) - p\pi^{LF} + \hat{y}^{LF}\sigma_s(1-p) = 0 \tag{35}$$

$$u_s - \theta_s\left[\pi^{LF}(1-\beta p) - \kappa_s\hat{y}^{LF}\right] = 0. \tag{36}$$

where κ_s depends on θ_s and σ_s.

Finally, P understands that the allocation resulting from implementing the rule is given by:

$$\log(\beta + \Delta\beta_s) - p\pi_L + \hat{y}_L\sigma_s(1-p) = 0 \tag{37}$$

$$u_s - \theta_s\left[\pi_L(1-\beta p) - \kappa_s\hat{y}_L\right] = (1-\beta p)(\tau_0 + \varphi_\pi\pi). \tag{38}$$

Problem of the policymaker. P chooses the intercept, $\tau_0 \in [\underline{\tau_0}, \overline{\tau_0}]$ and the slope of the rule $\varphi_\pi \in [0, \overline{\varphi_\pi}]$ to minimize expected losses (20) under a distribution, g, and subject to ambiguity aversion

$$W(\tau_0^*, \varphi_\pi^*) = \max_{\tau_0, \varphi_\pi}\min_{g\in G}\left\{\int -\mathcal{L}(\theta, \sigma, \tau_0, \varphi_\pi)g(d\theta, d\sigma) + c(g)\right\} \text{ s.t. } (35) - (38). \tag{39}$$

where $c(g)$ is an ambiguity index and a convex function on the simplex. As shown by Maccheroni, Marinacci, and Rustichini (2006), this general representation encompasses the three specific approaches to ambiguity we simulate numerically: expected social welfare with no aversion to ambiguity, multiplier preferences as in Hansen and Sargent (2008) and multiple priors preferences as in Gilboa and Schmeidler (1989).

As evident from the objective (39), a robust policy can be interpreted as the outcome of a maxmin problem, *i.e.* the outcome of a two-stage game between the

Planner (P) and a malign Agent (A). In the first stage, P chooses the rule parameters (τ_0, φ_π) to maximize welfare anticipating the response of A. In the second stage of the game, A chooses a density over the potential states of the economy $(\Delta\beta, u, \sigma, \theta)$ to minimize welfare, given the instruments (τ_0, φ_π) set by P.

A and P make these choices subject to the constraints (35)-(38). The fact that P observes the *laissez-faire* allocation $(\pi^{LF}, \hat{y}^{LF})$ imposes two restrictions to the set of possible parameters A can choose from. More specifically, by combining equations (35) and (37) on the one hand and (36) and (38) on the other, we can eliminate $\log(\beta + \Delta\beta)$ and u and reduce the number of free parameters to two, (θ, σ). This is why the expectation is taken over the prior distribution over θ and σ. We denote G the set of probability distributions consistent with these restrictions and g the elements of this set. Finally, the policymaker chooses the optimal *ex ante* rule and commits to applying it as long as the economy is in the L state.[14]

6.2 Preliminary Results: Comparative Static

Before turning to simulations, we derive analytical results to build intuition. Consider for a moment that there are only three possible states corresponding to the liquidity traps analyzed previously, $s = r^*, SF, PC$. In the FLT, $\Delta\beta_{r^*} > 0$, $u_{r^*} = 0$ and $(\sigma_{r^*}, \theta_{r^*})$ are such that the Phillips curve is less steep than the IS curve, as in Lemma (2); in the SF-trap $\Delta\beta_{SF} = u_{SF} = 0$ and the slopes are consistent with Lemma (3); and in the PC-trap, $u_{PC} < 0$, $\Delta\beta_{PC} = 0$ and $(\sigma_{PC}, \theta_{PC})$ are such that the Phillips curve is less steep than the IS curve as in Lemma (4).

A first important result is that a TIP rule that responds aggressively to inflation improves welfare in all three states. Lemma 5 formalizes how the allocation and welfare change with the value of φ_π.

Lemma 5 (Comparative statics - φ_π)**.**

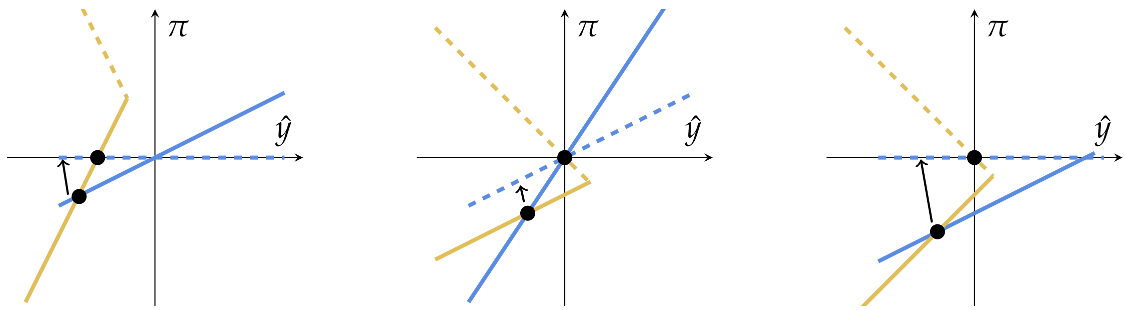
1. *In an FLT, the output gap, inflation and welfare increase monotonically with φ_π. In the limit $\varphi_\pi \to +\infty$, inflation converges to 0 and the output gap to $\hat{y}_L = -\frac{1}{\sigma(1-p)} \log(\beta + \Delta\beta)$.*

2. *In an SF liquidity trap, the first-best equilibrium $\pi = \hat{y} = 0$ is unique if and only if $\varphi_\pi \geq \varphi_\pi^{SF}$ with $\varphi_\pi^{SF} = \theta\left(\frac{\kappa p}{(1-p\beta)\sigma(1-p)} - 1\right) > 0$.*

[14] Introducing period-by-period re-optimization would introduce additional experimentation motives and learning behaviors that are interesting but beyond the scope of this paper.

3. *In a PC liquidity trap, inflation increases monotonically to 0 with φ_π. The output gap increases with φ_π for $\varphi_\pi < \varphi_\pi^{PC}$ (defined in Appendix) then decreases for $\varphi_\pi > \varphi_\pi^{PC}$ and converges monotonically to 0 as $\varphi_\pi \to +\infty$.*

Proof. Proofs are in Appendix F. □



(a) Decline in r^* (b) Self-fulfilling (c) Shift in Phillips curve

Figure 7: Effects of Increasing the Response of TIP to Inflation, φ_π.

The solid blue line represents the Phillips curve in the laissez-faire equilibrium, $\varphi_\pi = 0$. The dashed blue line represents the Phillips curve for a stronger response of TIP to inflation, $\varphi_\pi > 0$. In Panel (a) and (c), $\varphi_\pi \to +\infty$. In panel (b), φ_π is such that the Phillips curve and the Euler equation are parallel and the ZLB equilibrium does not exist.

To understand Lemma 5, it is important to see that increasing the strength of the response of TIP to inflation, φ_π, decreases the sensitivity of inflation to output and flattens the Phillips curve. Using the inflation-targeting rule for TIP gives the following Phillips curve:

$$\pi_L = \frac{\kappa \hat{y}_L + (\underline{u} - (1 - p\beta)\tau_0)/\theta}{(1 - p\beta)\left(1 + \frac{\varphi_\pi}{\theta}\right)}. \tag{40}$$

It is clear from equation (40) that increasing φ_π, decreases the sensitivity of inflation to the output gap and to the cost-push shock.

Figure 7 illustrates how the flattening the Phillips curve improves the allocation in all three types of traps. In a r^*-driven liquidity trap, a clockwise rotation of the Phillips curve around the point where it crosses the x-axis in Panel (a) implies a lower inflation and a lower output gap. In the expectation-driven liquidity trap, a large enough rotation of the Phillips curve in Panel (b) eliminates the intersection of the Euler equation and the Phillips curve at the liquidity trap point, leaving only the first-best equilibrium. In the Phillips-curve-driven trap in Panel (c), a rotation of the Phillips curve leads to a reduction in deflation. The effect on the output gap is however non-monotonic: at first the output gap increases as the real rate decreases with the rise of inflation, but then once the economy exits the ZLB, further increases in inflation leads to an increase in the real rate as the central bank reacts strongly to inflation.

Drawing on the previous lemma, Proposition 8 shows that it is weakly welfare maximizing to choose the strongest possible response $\varphi_\pi = \overline{\varphi_\pi}$ provided $\overline{\varphi_\pi} > \max(\varphi_\pi^{SF}, \varphi_\pi^{PC})$. When that condition is met, a negative intercept $\tau_0 < 0$ further improves welfare in the FLT and PCLT.

Proposition 8. *Assume $\varphi_\pi \in [0, \overline{\varphi_\pi})$ with $\overline{\varphi_\pi} > \max(\varphi_\pi^{SF}, \varphi_\pi^{PC})$.*

- *In all three types of liquidity traps, setting $\varphi_\pi = \overline{\varphi_\pi}$ is (weakly) welfare-maximizing.*
- *There exist $\tau_0 < 0$ such that welfare is strictly higher with τ_0 than in the* laissez-faire, *in both an FLT and PCLT.*

The second result—that there exist negative intercepts, $\underline{\tau}_0 < 0$, such that a TIP rule can increase welfare in an FLT and PCLT—is similar to Propositions 3 and 7. It can be best understood graphically. Recall that a negative intercept entails an upward shift of the Phillips curve—in blue in Panel (a) and (c) of Figure 7. It is easy to see on the Figure how an upward shift of the Phillips curve in both Panel (a) and (c) can result in lower deflation and lower output gap.

6.3 Numerical Simulations

Drawing on the analytical results of Proposition 8, we now show numerically that a TIP rule that responds aggressively to inflation and with a negative intercept is a robust policy to increase welfare in a liquidity trap. We derive this finding following three separate and complementary approaches based on the robust control literature.

A policymaker maximizing expected welfare with a prior. In the first approach, P simply maximizes expected social welfare subject to her subjective priors g^* about the distribution of (θ, σ):

$$W(\tau_0^*, \varphi_\pi^*) = \max_{\tau_0, \varphi_\pi} \int -\mathcal{L}(\theta, \sigma, \tau_0, \varphi_\pi) g^*(d\theta, d\sigma). \tag{41}$$

It is the special case of Equation (39) with no ambiguity $G = \{g^*\}$ and $c(.) = 0$. We calibrate the prior distribution and the choice sets. We assume θ and σ are log-normally distributed with mean (standard deviation) 372 (80) and 2 (1), respectively. For parameters that are common knowledge and for the *laissez-faire* allocation — $\pi_L^{LF} = -.6$, $\hat{y}_L^{LF} = -1.8$ — we follow the calibration in Section 3. The set of possible policy parameters encompasses the range of reasonable values, $[\underline{\tau_0}, \overline{\tau_0}] = [-1, 1]$, and $\overline{\varphi_\pi} = 1500$. Finally, allocations should be consistent with a liquidity trap, so we exclude those in which inflation is 1 percentage point above target. In sensitivity analysis, we find that our results are robust to the specific value of the upper bound.

A policymaker with multiplier preferences. Our second approach follows Hansen and Sargent (2008) and Woodford (2010). P now faces ambiguity and has a reference prior in mind, g^*. She fears that the data is generated by an unknown perturbation of her approximating model. P has multiplier preferences and seek to maximize

$$W(\tau_0^*, \varphi_\pi^*) = \max_{\tau_0, \varphi_\pi} \min_{g \in G} \int -\mathcal{L}(\theta, \sigma, \tau_0, \varphi_\pi) g(d\theta, d\sigma) + \nu \int \ln\left(\frac{dg}{dg^*}\right) dg. \qquad (42)$$

This is the special case of Equation (39) with $c(g) = \nu \int \ln\left(\frac{dg}{dg^*}\right) dg$, which is the relative entropy of the prior g with respect to a reference prior g^*. The calibration is the same as in the first approach. In particular, the reference prior g^* follows the same joint log-normal distribution.

A max-min policymaker. Our third approach is a pure maxmin problem following the seminal work by Gilboa and Schmeidler (1989). P's objective is given by

$$W(\tau_0^*, \varphi_\pi^*) = \max_{\tau_0, \varphi_\pi} \min_{g \in G} \int -\mathcal{L}(\theta, \sigma, \tau_0, \varphi_\pi) g(d\theta, d\sigma). \qquad (43)$$

We restrict the support of the distributions in G to be bounded, *i.e.* $\sigma \in [\underline{\sigma}, \bar{\sigma}], \theta \in [\underline{\theta}, \bar{\theta}]$, otherwise A would choose infinite values, which is not realistic. These sets are calibrated to encompass the range of reasonable values $[\underline{\sigma}, \bar{\sigma}] = [.5, 10]$ and $[\underline{\theta}, \bar{\theta}] = [50, 700]$.

Results. All three approaches lead to the same conclusions: the optimal rule features a strong positive reaction of TIP to inflation, $\varphi_\pi = \bar{\varphi}_\pi$, combined with a negative intercept $\tau_0 = \underline{\tau}_0$. These results are consistent with Lemma 5 and Proposition 8. To illustrate these results, we plot welfare, inflation and the output gap as a function of (τ_0, φ_π) in all three approaches in Appendix F. In the case of the maxmin policymaker, we also report the optimal policy rules for σ and θ by A. Whereas the first and third approaches generate sharp non-monotonicities for low values of φ_π, under multiplier preferences, the welfare surface is considerably smoother over the entire parameter space.

6.4 The Paradox of Flexibility Revisited

The finding that an inflation-targeting TIP can reduce deflation and the output gap generalizes the paradox of flexibility according to which reducing price flexibility at the ZLB can improve welfare (Eggertsson and Krugman, 2012; Billi and Galí, 2020). An inflation-targeting TIP indeed increases the degree of aggregate price stickiness. While this paradox was derived in the context of a liquidity trap caused by a decline in the neutral rate of interest, we show that it holds across all types of liquidity traps provided the increase in aggregate price stickiness is strong enough. By contrast, small increases in price stickiness worsen the allocation in a self-fulfilling equilibrium. An important policy implication is that the strength of the response of TIP to inflation needs to be appropriately calibrated.[15]

7 A Quantitative Assessment for Japan

We previously showed that TIP can enhance welfare at the ZLB in a small-scale New Keynesian model. This section uses a medium-scale DSGE model calibrated to Japan. Japan provides an ideal laboratory, having experienced prolonged deflation and a policy rate constrained by the ZLB since the early 2000s, as shown in Figure 9. We find that a simple inflation-targeting rule for TIP could have brought inflation back to target, with a negligible fiscal cost. Importantly, this policy rule is robust to the underlying source of the trap.

[15] A concern with the aggressive response of TIP to inflation is that it increases the firms' cost of changing prices and thereby distorts relative prices. This concern, however, is misplaced. As Capelle and Liu (forthcoming) show, an inflation-targeting TIP increases the aggregate stickiness without distorting prices. This is because TIP operates as a linear tax, which doesn't impede the adjustment of relative prices, unlike price controls, which act as convex taxes and distort them.

7.1 A Quantitative Model

Model structure. We extend the small-scale New Keynesian model considered in the previous sections to include a variety of important features and frictions. More specifically, we allow for bonds in the utility, habits in consumption, investment in physical capital subject to adjustment costs, sticky nominal wages and indexation of prices and wages, variable capital utilization and fixed costs in production, following Hirose (2020) and Cuba-Borda and Singh (2024) (CBS, henceforth). We also allow for multiple temporary shocks driving fluctuations of the economy around its steady state: a price markup, wage markup, risk premium, investment-specific technology, and monetary policy shocks. The model features a sunspot variable that indexes inflation expectation errors to address the local indeterminacy around the expectation-driven ZLB steady state, following closely CBS and Bianchi and Nicolò (2021).[16] The full model is described in Appendix G.

We depart from CBS in three ways. First, we introduce a discount factor to expectations, p. It captures cognitive discounting, which has been shown to be a realistic feature at the ZLB (Gabaix, 2020; Hirose et al., 2024).[17] It can also be interpreted as the persistence parameter of the liquidity trap state as in Section 3-5. Second, we assume that both prices and wages are partially indexed to lagged inflation π_{t-1}, while CBS assume that they are fully indexed to a weighted average of lagged and steady-state inflation. These two features ensure that the IS and Phillips curves remain upward sloping in the long run. Third, and most importantly, we introduce a TIP which follows an inflation-targeting rule

$$\tau_t = \rho\tau_{t-1} + \tau_0 + \varphi_\pi(\pi_t - \pi^*). \tag{44}$$

This rule is a more realistic version of the simple rule used in Section 2 for two reasons. First, policymakers prefer smoother paths of policies to more volatile ones, as evident from the smooth paths of central bank interest rates. This motivates the introduction of the backward-looking term, $\rho\tau_{t-1}$. Second we allow for a strictly

[16] There are two types of indeterminacy in the self-fulfilling expectations version of the model. First there are multiple steady-state equilibria, a feature we have analyzed in Section 5. Second, there is local indeterminacy around the liquidity trap steady state, which we address by introducing a sunspot shock on inflation expectation errors.

[17] We apply p to only households and final-good producers but not to capital goods producers in order to stay close to the small-scale model while minimizing deviations from the original medium-scale model.

positive inflation target, π^*, consistent with the stated objective of the Bank of Japan.

We consider two versions of the economy corresponding to the FLT and SFLT. These two economies differ in the values of the utility of bonds and the price stickiness parameters, as in CBS, as well as the expectation discount parameter. We consider only these two explanations of the liquidity trap, and abstract from the PCLT, as these two have been the focus of the quantitative literature.

Table 2: Calibrated Parameters

Parameters	Description	Value (self-fulfilling / r^* decline)
i. Externally calibrated parameters		
β	Time discount factor	0.942
σ	Elasticity of intertemporal substitution	1
ψ	Inverse Frish elasticity of labor	2.27
ϵ_p	Elasticity of substitution across varieties	6
ϵ_w	Elasticity of substitution across workers	6
α	Capital share	0.37
δ_k	Capital depreciation rate	0.015
h	Consumption habit	0.358
ω	Labor disutility	0.588
$A''(1)$	Capital utilization elasticity	2.246
ψ_I	Investment adjustment cost	5.16
ι_p	Price indexation	0.225
ι_w	Wage indexation	0.295
g	Autonomous spending	1.333
G_z	TFP trend growth	0.26
ii. Internally calibrated parameters		
δ	Marginal utility of bonds	0.0398 / 0.0764
ψ_p	Price adjustment cost	314.2 / 312.2
ψ_w	Wage adjustment cost	583.2 / 595.0
p	Persistence probability/cognitive discounting	0.998 / 0.943
iii. Counterfactual TIP rule		
π^*	Inflation target	1%
ρ	Policy inertia	0.9
τ_0	Intercept	-0.51%
φ_π	Response to deviation of inflation from target	50

Calibration. The upper panel of Table 2 reports all externally calibrated parameters, which are taken directly from CBS. Below, we describe how we choose the four internally calibrated parameters, $\delta, \psi_p, \psi_w, p$, as well as the shock processes.

As in CBS, we calibrate the marginal utility of bonds, δ, to obtain a steady-state r^* of 0% in the expectation-driven scenario and -1.1% in the FLT. Numerically, this implies setting δ to 0.0398 in the former and 0.0764 in the latter.

We deviate from CBS by assuming a lower degree of price and wage stickiness to match empirical estimates in the literature. Hirose (2020) estimates the slope of the price Phillips curve to be 0.0205 and that of the wage Phillips curve to be 0.00967 for the post-1999 period. Given that Hirose (2020) abstract from indexation, we interpret these estimates as the long-run slopes. Hence in our model with partial indexation, this corresponds to $(1 - \iota_p) \times 0.0205$ for the price and $(1 - \iota_w) \times 0.00967$ for the wage Phillips curve. The price adjustment cost, ψ_p, are set to 314.2 in the expectation-driven scenario and to 312.2 in the FLT to exactly match these slopes. Similarly, we set the wage adjustment cost, ψ_w, to 583.2 and 595.0.

The discount factor p is pinned down by the steady-state level of inflation. Like CBS, we target a steady-state inflation rate of -1.06% per annum, which implies $p = 0.998$ in the expectation-driven scenario and $p = 0.943$ in the FLT.

Finally, in both scenarios we estimate the parameters governing the shock processes assuming no TIP, $\tau_t = 0$. The results are reported in Tables 3 and 4.

7.2 Counterfactual Analyses

We next use the calibrated medium-scale model to construct a TIP rule that would have lifted Japan out of its deflationary equilibrium during the period 1998–2012. We leverage the insights derived in the small-scale model to design a rule that is robust to both interpretations of the trap—FLT and SFLT.

Self-fulfilling liquidity trap. How strong must TIP's response to inflation, φ_π, be to eliminate the self-fulfilling trap? To answer this question, we numerically compute the stationary liquidity trap equilibrium in the self-fulfilling calibration for different values of φ_π. Figure 8 plots the steady-state inflation, output, and fiscal revenues from TIP as a function of φ_π. Inflation is annualized, and output is normalized relative to the output level in the flexible-price steady state. In the third panel, we also compute fiscal revenues from TIP relative to output.

Quantitatively, we find that φ_π needs to exceed 28.0 to eliminate the SFLT when TIP inertia ρ in Equation (44) equals 0.9. To give a sense of magnitude, when

$\varphi_\pi = 28.0$, a one-percentage-point increase in annual inflation, equivalent to 0.25 percentage point per quarter, raises the TIP rate by 7.00 percentage points on impact. Interestingly, as φ_π approaches 28.0, the left-hand limit remains bounded and converges to -3.7% for inflation and -13.2% for the output loss. This is in contrast with the small-scale linear model and is due to the non-linearities of the medium-scale model.

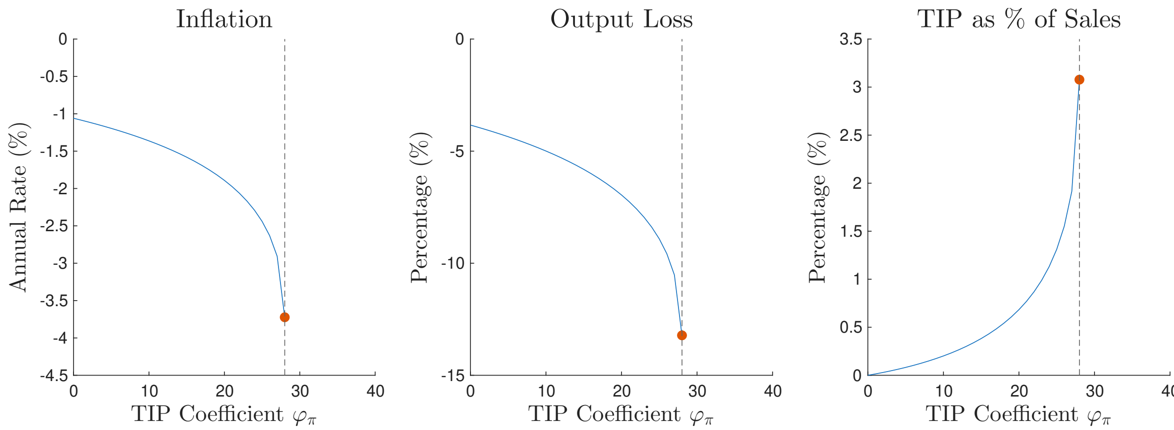


Figure 8: Self-fulfilling Deflationary Equilibrium as a Function of φ_π

Note: Inflation is measured in annualized percentage points. Output loss is defined as the percentage deviation of output from its flexible-price steady-state level. TIP as % of sales is calculated as $100\tau_t\pi_t$.

Liquidity trap caused by a decline of r^*. We then ask what value of the intercept in the TIP rule, τ_0, would bring the steady-state level of inflation back to the 1% target in the FLT? Starting from the calibration for the r^*-driven case, we set $\rho = 0.9$ and $\varphi_\pi = 50$ for the TIP rule while keeping monetary policy at the ZLB.[18] We find that the required value of τ_0 is -0.51%, which implies a steady-state TIP rate of -5.1%. The fact that the intercept is negative is consistent with the analysis above that an upward shift of the Phillips curve can boost inflation and output at the ZLB.

Next, we trace out the counterfactual path of the Japanese economy if the TIP rule we just calibrated had been used. To do this, we first estimate the empirical shock series using the log-linearized version of the model and a Kalman filter and then recompute the equilibrium path with the TIP rule.

[18]Given an r^* of -1.1% and steady-state inflation of 1%, the steady-state nominal rate is -0.1%, which remains below the zero lower bound, consistent with our assumption.

The top panel of Figure 9 shows that deflation would have been mostly eliminated under TIP, with inflation positive and close to the 1% target on average. The rule raises inflation by about 2.1 percentage points. Mirroring its strong effect on inflation, TIP raises the output level by as much as 4.5 percentage points. Given that TIP responds to variations of inflation around the target through φ_{π}, it also dampens inflation fluctuations.

In most periods, the TIP rate is negative and not very large, fluctuating between -30% and 20% as shown in the lower panels of Figure 9. The only exceptions are Q4 2008 and Q1 2009, when the TIP rate rises to 40%. This spike reflects the one-time increase in inflation in Q4 2008, together with the inertia embedded in the TIP rule. Although double-digit TIP rates may appear large, they imply only modest effective fiscal costs (or modest fiscal revenues when the TIP rate is positive) as measured by the ratio of fiscal expenses to GDP. The fiscal impact fluctuates between -0.1% and 0.1% of GDP, except in Q4 2008. The effective transfers are small because the tax base—the change in prices—is not only small to begin with but also contracts as the TIP rate increases.

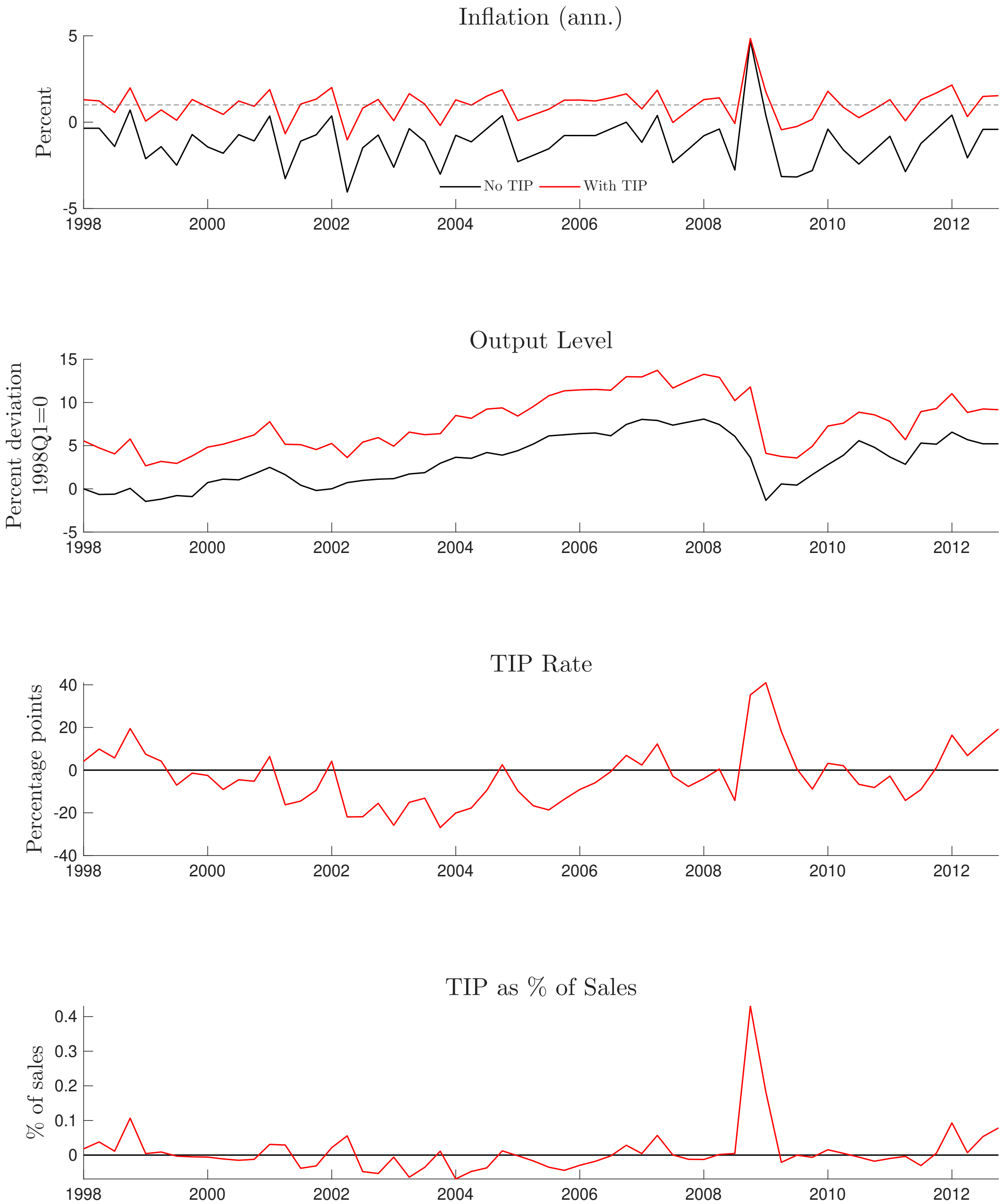


Figure 9: Counterfactual Results

Note: Inflation and output data are from Cuba-Borda and Singh (2024). Inflation is measured in the annualized log growth rate, multiplied by 100. The output level is the log deviation from its true value in 1998Q1, multiplied by 100. Both the TIP rate and TIP as % of sales are in percentage points. TIP as % of sales is calculated as $100\tau_t\pi_t$.

8 Conclusion

This paper has assessed the effectiveness of a Tax on Inflation Policy (TIP)—a fiscal instrument that alters firms' incentives to adjust prices—as a tool to lift the economy out of the ZLB and improve welfare. In an FLT, by encouraging firms to raise prices, a negative TIP can implement the constrained-efficient allocation by mitigating the deflationary spiral. It can thus increase inflation and boost output. In conventional calibrations, TIP performs better than forward guidance.

In a liquidity trap caused by coordination failures and self-fulfilling expectations, TIP has two roles. At the ZLB, a moderate positive TIP implements the second-best allocation by correcting the pecuniary externality in firms' pricing decisions. A sufficiently aggressive positive TIP goes further and eliminates the deflationary equilibrium, restoring the first best as the unique stationary equilibrium. In a liquidity trap caused by a shift in the Phillips curve, a negative TIP can restore the first best. These results imply that the optimal constant TIP is not robust across sources of liquidity traps. We therefore design an inflation-targeting TIP rule that responds strongly to deflation and has a negative intercept. This rule eliminates the SFLT while providing stimulus in the FLT and PCLT. Simulations of a medium-scale model calibrated to Japan suggest significant stabilization gains from TIP.

The results derived in this paper have implications for other policy instruments at the ZLB. They apply most directly to those instruments that affect the Phillips curve, such as labor subsidies and, to some extent, to government spending.

Our paper also opens avenues for future research. A few implementation issues deserve a more in-depth quantitative inquiry. Tax avoidance, for example, warrants more attention. The main risk is that firms relabel old products as seemingly new ones, or reduce their quality. Quantifying the effects of alternative designs of TIP in a framework with endogenous product creation, information asymmetries about product quality and costly monitoring by the tax administration is an important next step. In addition, delving into the political economy of TIP is another important avenue for future research. What are the risks that TIP be used for objectives other than macroeconomic stabilization? Could it lead to less independent monetary policy in countries with weak institutional frameworks?

References

B. Aruoba, P. Cuba-Borda, and F. Schorfheide. Macroeconomic dynamics near the zlb: A tale of two countries. *The Review of Economic Studies*, 85(1):87–118, 01 2018. ISSN 0034-6527. . URL .

J. Benhabib, S. Schmitt-Grohé, and M. Uribe. The perils of taylor rules. *Journal of Economic Theory*, 96(1):40–69, 2001a. ISSN 0022-0531. . URL .

J. Benhabib, S. Schmitt-Grohé, and M. Uribe. Monetary policy and multiple equilibria. *American Economic Review*, 91(1):167–186, March 2001b. .

J. Benhabib, S. Schmitt-Grohé, and M. Uribe. Avoiding liquidity traps. *Journal of Political Economy*, 110(3):535–563, 2002. .

G. Benigno and L. Fornaro. Stagnation traps. *The Review of Economic Studies*, 85(3): 1425–1470, 11 2017. ISSN 0034-6527. .

F. Bianchi and G. Nicolò. A generalized approach to indeterminacy in linear rational expectations models. *Quantitative Economics*, 12(3):843–868, 2021.

F. O. Bilbiie. Optimal forward guidance. *American Economic Journal: Macroeconomics*, 11(4):310–45, October 2019. .

F. O. Bilbiie. Neo-fisherian policies and liquidity traps. *American Economic Journal: Macroeconomics*, 14(4):378–403, October 2022. . URL .

F. O. Bilbiie, T. Monacelli, and R. Perotti. Is government spending at the zero lower bound desirable? *American Economic Journal: Macroeconomics*, 11(3):147–73, July 2019. .

R. M. Billi and J. Galí. Gains from wage flexibility and the zero lower bound. *Oxford Bulletin of Economics and Statistics*, 82(6):1239–1261, 2020. .

R. M. Billi and C. E. Walsh. Inflation, fiscal rules and cognitive discounting. 2025.

O. J. Blanchard and N. Kiyotaki. Monopolistic competition and the effects of aggregate demand. *American Economic Review*, 77(4):647–666, 1987.

D. Capelle and Y. Liu. Optimal Taxation of Inflation. *American Economic Journal: Macroeconomics*, forthcoming.

N. Caramp and S. R. Singh. Bond premium cyclicality and liquidity traps. *The Review of Economic Studies*, 90(6):2822–2879, None 2023. . URL .

J. H. Cochrane. The new-keynesian liquidity trap. *Journal of Monetary Economics*, 92: 47–63, 2017. ISSN 0304-3932. .

I. Correia, E. Farhi, J. P. Nicolini, and P. Teles. Unconventional Fiscal Policy at the Zero Bound. *American Economic Review*, 103(4):1172–1211, June 2013. .

P. Cuba-Borda and S. R. Singh. Understanding persistent zlb: Theory and assessment. *American Economic Journal: Macroeconomics*, 16(3):389–416, July 2024. .

T. Davig and E. M. Leeper. Generalizing the taylor principle. *American Economic Review*, 97(3):607–635, June 2007. .

G. Eggertsson, A. Ferrero, and A. Raffo. Can structural reforms help europe? *Journal of Monetary Economics*, 61(C):2–22, None 2014. .

G. B. Eggertsson. The paradox of toil. Staff Reports 433, Federal Reserve Bank of New York, 2010.

G. B. Eggertsson. What Fiscal Policy Is Effective at Zero Interest Rates? In *NBER Macroeconomics Annual 2010, volume 25*, NBER Chapters, pages 59–112. National Bureau of Economic Research, Inc, June 2011.

G. B. Eggertsson and P. Krugman. Debt, deleveraging, and the liquidity trap: A fisher-minsky-koo approach*. *The Quarterly Journal of Economics*, 127(3):1469–1513, 07 2012. ISSN 0033-5533. .

G. B. Eggertsson and M. Woodford. The Zero Bound on Interest Rates and Optimal Monetary Policy. *Brookings Papers on Economic Activity*, 34(1):139–235, 2003.

G. B. Eggertsson and M. Woodford. Optimal Monetary and Fiscal Policy in a Liquidity Trap. In *NBER International Seminar on Macroeconomics 2004*, NBER Chapters, pages 75–144. National Bureau of Economic Research, Inc, June 2006.

G. B. Eggertsson, N. R. Mehrotra, and J. A. Robbins. A model of secular stagnation: Theory and quantitative evaluation. *American Economic Journal: Macroeconomics*, 11 (1):1–48, January 2019. .

E. Farhi and I. Werning. A theory of macroprudential policies in the presence of nominal rigidities. *Quarterly Journal of Economics*, 131(1):223–261, 2016.

E. Farhi, G. Gopinath, and O. Itskhoki. Fiscal Devaluations. *Review of Economic Studies*, 81(2):725–760, 2014.

R. E. Farmer, D. F. Waggoner, and T. Zha. Understanding markov-switching rational expectations models. *Journal of Economic Theory*, 144(5):1849–1867, September 2009. .

X. Gabaix. A Behavioral New Keynesian Model. *American Economic Review*, 110(8): 2271–2327, August 2020. .

J. Galí. *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications Second edition*. Number 10495 in Economics Books. Princeton University Press, 2015.

I. Gilboa and D. Schmeidler. Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics*, 18(2):141–153, 1989. ISSN 0304-4068. .

L. Hansen and T. J. Sargent. Robust control and model uncertainty. *American Economic Review*, 91(2):60–66, May 2001. .

L. P. Hansen and T. J. Sargent. *Robustness*. Princeton University Press, 2008.

Y. Hirose. An Estimated Dsge Model With A Deflation Steady State. *Macroeconomic Dynamics*, 24(5):1151–1185, July 2020.

Y. Hirose, H. Iiboshi, M. Shintani, and K. Ueda. Estimating a behavioral new keynesian model with the zero lower bound. *Journal of Money, Credit and Banking*, 56(8): 2185–2197, 2024. . URL .

IMF. *Chapter 2 The Natural Rate of Interest: Drivers and Implications for Policy*, page CH002. International Monetary Fund, USA, 2023. ISBN 9798400224119. .

A. Korinek and A. Simsek. Liquidity trap and excessive leverage. *American Economic Review*, 106(3):699–738, 2016.

G. Lorenzoni. Inefficient credit booms. *The Review of Economic Studies*, 75(3):809–833, None 2008. . URL .

G. Lorenzoni and I. Werning. Wage-price spirals. *Brookings Papers on Economic Activity*, 54(2 (Fall)):317–393, None 2023. . URL .

F. Maccheroni, M. Marinacci, and A. Rustichini. Ambiguity aversion, robustness, and the variational representation of preferences. *Econometrica*, 74(6):1447–1498, November 2006. .

B. McCallum. Role of the minimal state variable criterion in rational expectations models. *International Tax and Public Finance*, 6(4):621–639, November 1999. .

A. McKay, E. Nakamura, and J. Steinsson. The power of forward guidance revisited. *American Economic Review*, 106(10):3133–58, October 2016. .

K. R. S. M. Mertens and M. O. Ravn. Fiscal Policy in an Expectations-Driven Liquidity Trap. *The Review of Economic Studies*, 81(4 (289)):1637–1667, 2014. ISSN 00346527, 1467937X.

T. Nakata. Optimal fiscal and monetary policy with occasionally binding zero bound constraints. *Journal of Economic Dynamics and Control*, 73:220–240, 2016. ISSN 0165-1889. .

T. Nakata and S. Schmidt. Expectations-driven liquidity traps: Implications for monetary and fiscal policy. *American Economic Journal: Macroeconomics*, 14(4): 68–103, October 2022. . URL .

L. Rachel and L. H. Summers. On Secular Stagnation in the Industrialized World. *Brookings Papers on Economic Activity*, 50(1 (Spring):1–76, 2019.

J. J. Rotemberg. Sticky Prices in the United States. *Journal of Political Economy*, 90(6): 1187–1211, 1982. ISSN 00223808, 1537534X.

J. J. Rotemberg and M. Woodford. Interest Rate Rules in an Estimated Sticky Price Model. In *Monetary Policy Rules*, NBER Chapters, pages 57–126. National Bureau of Economic Research, Inc, March 1999.

S. Schmidt. Optimal monetary and fiscal policy with a zero bound on nominal interest rates. *Journal of Money, Credit and Banking*, 45(7):1335–1350, 2013. .

S. Schmidt. Lack of confidence, the zero lower bound, and the virtue of fiscal rules. *Journal of Economic Dynamics and Control*, 70(C):36–53, None 2016. . URL .

S. Schmitt-Grohé and M. Uribe. Liquidity Traps: an Interest-rate-based Exit Strategy. *Manchester School*, 82(S1):1–14, September 2014.

L. S. Seidman. Tax-Based Incomes Policies. *Brookings Papers on Economic Activity*, 9 (2):301–361, 1978.

L. S. Seidman. The Role of a Tax-Based Incomes Policy. *American Economic Review*, 69 (2):202–206, May 1979.

F. Smets and R. Wouters. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3):586–606, June 2007.

P. Tillmann. Robust monetary policy under uncertainty about the lower bound. *The B.E. Journal of Macroeconomics*, 21(1):309–321, 2021. .

H. C. Wallich and S. Weintraub. A Tax-Based Incomes Policy. *Journal of Economic Issues*, 5(2):1–19, June 1971. .

M. Woodford. Robustly optimal monetary policy with near-rational expectations. *American Economic Review*, 100(1):274–303, March 2010. .

M. Woodford. Simple Analytics of the Government Expenditure Multiplier. *American Economic Journal: Macroeconomics*, 3(1):1–35, January 2011. .

G. Özcan and G. Traficante. Optimal robust monetary and fiscal policy under uncertainty on the lower bound. *Journal of Macroeconomics*, 81:103605, 2024. ISSN 0164-0704. .

Appendix

A Market Clearing and Equilibrium Definition

Market Clearing. In equilibrium, the markets for each intermediate good Y_{ti} and for the final good should clear

$$Y_t = C_t + \int_0^1 \frac{\theta}{2} \left(\frac{P_{ti}}{P_{t-1i}} - 1 \right)^2 Y_t di \tag{45}$$

The sum of labor hired in all firms should be equal to the supply of labor by the household:

$$N_t = \int_0^1 N_{ti} di \tag{46}$$

With no government debt, holdings of bonds by the household are zero: $B_t = 0$. Finally, transfers received by the household include profits and tax receipts:

$$T_t = \int_0^1 \Pi_{it} + \tau_t (P_{ti} - P_{t-1i}) Y_{ti} di. \tag{47}$$

Equilibrium definition. An equilibrium is a path of output, labor, bonds, wages, price level, bond prices and TIP $\{C_t, N_t, B_t, \{Y_{ti}, N_{ti}, P_{ti}\}_i, W_t, P_t, Q_t, \tau_t\}_{t=0,1,2..}$, such that

- Taking TIP as given, intermediate firms maximize their discounted sum of profits (7) subject to the definition of profits (6), the demand schedule from final goods firms, and the technology (5).
- Taking prices as given, final good firms maximize their profits subject to the technology (4).
- Taking prices and transfers (47) as given, the household maximizes their utility (1) subject to their budget constraint, no-Ponzi condition and demand for labor.
- The markets for final good (45), intermediate goods, labor (46) and bonds clear.

B Optimal Tax on Inflation Policy

We fist provide the proofs for the simpler case, shown in the draft, in which the good state is absorbing, so $q = 1$ and the planner is time consistent, so that after the economy reaches the good state, she implements the first-best allocation and does not promise a future boom. Next we provide more general proofs when the trap may be recurring, i.e. with $q \leq 1$, and we contrast the time-consistent allocation with the committed allocation.

B.1 Non-recurring liquidity trap and time-consistent planner

B.1.1 Proof of proposition 1 : Constrained-efficient allocation

The Lagrangian is given by

$$\mathcal{L} = \frac{(\pi_L)^2 + \eta_y(\hat{y}_L)^2}{1-\beta p} + \mu \left[\hat{y}_L - \frac{1}{\sigma(1-p)}\left(p\pi_L - \log(\beta+\Delta\beta)\right)\right]$$

The FOCs with respect to $\pi, \hat{y}$ are given by

$$\frac{2\pi}{1-\beta p} - \mu\frac{p}{\sigma(1-p)} = 0$$
$$\frac{\eta_y 2\hat{y}}{1-\beta p} + \mu = 0$$

which gives

$$\pi = -\eta_y\hat{y}\frac{p}{\sigma(1-p)}.$$

Replacing this into the constraint gives

$$\hat{y}^{SB} = -\frac{\sigma(1-p)}{\sigma^2(1-p)^2 + p^2\eta_y}\log(\beta+\Delta\beta).$$

Since this is negative, it implies, together with the optimality condition of the policymakers, that inflation should be positive $\pi = -\eta_y\hat{y}\frac{p}{\sigma(1-p)} > 0$.

B.1.2 Proof of proposition 2: Ramsey allocation

The Lagrangian for the Ramsey optimal allocation problem is given by

$$\mathcal{L} = \frac{\pi^2 + \eta_y \hat{y}^2}{1-\beta p} + \mu \left[\hat{y} - \frac{1}{\sigma(1-p)} \left(p\pi - \log(\beta + \Delta\beta) \right) \right] + \lambda \left[\pi + \frac{1}{(1-p\beta)\sigma(1-p) - \kappa p} \left[\kappa \log(\beta + \Delta\beta) + \frac{(1-p\beta)\sigma(1-p)}{\theta} \tau \right] \right]$$

The F.O.C. with respect to $\pi, \hat{y}, \tau$ are given by

$$2\pi + \lambda - \mu \frac{p}{\sigma(1-p)} = 0$$
$$\eta_y 2\hat{y} + \mu = 0$$
$$\lambda = 0$$

The solution to this problem is thus exactly the same as the constrained-efficient allocation. We can then solve for the level of TIP that implements this allocation:

$$\eta_y \frac{\sigma(1-p)}{\sigma^2(1-p)^2 + p^2\eta_y} \log(\beta + \Delta\beta) \frac{p}{\sigma(1-p)} = -\frac{1}{(1-p\beta)\sigma(1-p) - \kappa p} \left[\kappa \log(\beta + \Delta\beta) + \frac{(1-p\beta)\sigma(1-p)}{\theta} \tau \right]$$

$$-\frac{(1-p\beta)\sigma(1-p)}{\theta\left((1-p\beta)\sigma(1-p) - \kappa p\right)} \tau = \log(\beta + \Delta\beta) \left[\eta_y \frac{\sigma(1-p)}{\sigma^2(1-p)^2 + p^2\eta_y} \frac{p}{\sigma(1-p)} + \frac{\kappa}{(1-p\beta)\sigma(1-p) - \kappa p} \right]$$

This gives the following second-best TIP rate

$$\tau = -\log\left(\beta + \Delta\beta\right) \theta \frac{\left[\eta_y \frac{p((1-p\beta)\sigma(1-p) - \kappa p)}{\sigma^2(1-p)^2 + p^2\eta_y} + \kappa \right]}{(1-p\beta)\sigma(1-p)} \tag{48}$$

B.1.3 Proof of proposition 6

To rule out the liquidity trap equilibrium, TIP has to be such that inflation in that equilibrium is above the level below which the nominal interest rate is zero. This implies the following condition

$$\pi_L = \frac{1}{\kappa p - (1-p\beta)\sigma(1-p)} \left[\kappa \log\beta + \frac{(1-p\beta)\sigma(1-p)}{\theta}\tau\right] > \frac{\log\beta}{\phi_\pi}$$
$$\iff \kappa\log\beta + \frac{(1-p\beta)\sigma(1-p)}{\theta}\tau > \frac{(\kappa p - (1-p\beta)\sigma(1-p))\log\beta}{\phi_\pi}$$

Rearranging gives directly the results.

B.2 Recurring liquidity traps

This appendix extends the analysis of Section 3 to allow for recurrent liquidity traps. The goal is twofold. First, we characterize the constrained-efficient allocation when the good state is no longer absorbing. Second, we clarify how an aggressive TIP can rule out a recurrent self-fulfilling liquidity trap.

B.2.1 Environment

There are two states, $s \in \{L, G\}$. In the fundamental liquidity trap, L denotes the liquidity-trap state, with $\beta_L = \beta + \Delta\beta$, and G denotes the good state, with $\beta_G = \beta$. The transition probabilities are

$$\Pr(L'|L) = p, \qquad \Pr(G'|L) = 1-p, \qquad \Pr(G'|G) = q, \qquad \Pr(L'|G) = 1-q. \quad (49)$$

The absorbing-good-state case studied in the main text corresponds to $q = 1$. For any variable x,

$$E_L x' = p x_L + (1-p) x_G, \quad (50)$$
$$E_G x' = (1-q) x_L + q x_G. \quad (51)$$

In state L, the ZLB binds, $i_L = 0$, so the Euler equation is

$$\hat{y}_L = p\hat{y}_L + (1-p)\hat{y}_G + \frac{1}{\sigma}\left[p\pi_L + (1-p)\pi_G - \log(\beta+\Delta\beta)\right]. \quad (52)$$

In state G, the ZLB is slack. We allow the central bank to choose the intercept of the Taylor rule so that it targets the neutral rate of interest rate in the good state of the world,

$$i_G = -\log\beta + \mu_G + \phi_\pi \pi_G. \tag{53}$$

Combining (53) with the good-state Euler equation, the intercept required to implement any target allocation $(\hat{y}_L, \pi_L, \hat{y}_G, \pi_G)$ is

$$\mu_G = (1-q)\pi_L + (q-\phi_\pi)\pi_G + \sigma(1-q)(\hat{y}_L - \hat{y}_G). \tag{54}$$

Thus the good-state Taylor-rule intercept implements the Euler-equation restriction, while TIP implements the Phillips curve.

The Phillips curves in the two states are

$$\pi_L = \beta\left[p\pi_L + (1-p)\pi_G\right] + \kappa\hat{y}_L + \frac{1}{\theta}\left[\beta\left(p\tau_L + (1-p)\tau_G\right) - \tau_L\right], \tag{55}$$

$$\pi_G = \beta\left[(1-q)\pi_L + q\pi_G\right] + \kappa\hat{y}_G + \frac{1}{\theta}\left[\beta\left((1-q)\tau_L + q\tau_G\right) - \tau_G\right]. \tag{56}$$

We define

$$\Delta_q \equiv (1-\beta p)(1-\beta q) - \beta^2(1-p)(1-q). \tag{57}$$

B.2.2 Committed constrained-efficient allocation

We first consider a planner who can commit to a state-contingent allocation $(\hat{y}_L, \pi_L, \hat{y}_G, \pi_G)$. Starting from state L, the expected discounted weights on losses in the two states are

$$\omega_L \equiv \frac{1-\beta q}{\Delta_q}, \qquad \omega_G \equiv \frac{\beta(1-p)}{\Delta_q}. \tag{58}$$

These weights are the discounted occupation times implied by the two-state Markov chain.

The committed constrained-efficient allocation solves

$$\min_{\hat{y}_L, \pi_L, \hat{y}_G, \pi_G} \quad \omega_L\left(\pi_L^2 + \eta_y \hat{y}_L^2\right) + \omega_G\left(\pi_G^2 + \eta_y \hat{y}_G^2\right) \tag{59}$$

$$\text{s.t.} \quad (1-p)\hat{y}_L - (1-p)\hat{y}_G - \frac{p}{\sigma}\pi_L - \frac{1-p}{\sigma}\pi_G + \frac{\log(\beta+\Delta\beta)}{\sigma} = 0. \tag{60}$$

The good-state Euler equation is not an additional restriction on this allocation because the intercept μ_G in (53) can be chosen to implement the desired value of $\hat{y}_G$.

Proposition 9 (Committed constrained-efficient allocation)**.** *Let*

$$\mathcal{T}_q^C \equiv \frac{\sigma(1-p)^2}{\omega_L \eta_y} + \frac{\sigma(1-p)^2}{\omega_G \eta_y} + \frac{p^2}{\sigma \omega_L} + \frac{(1-p)^2}{\sigma \omega_G}. \tag{61}$$

The committed constrained-efficient allocation is

$$\hat{y}_L^C = -\frac{1-p}{\omega_L \eta_y \mathcal{T}_q^C} \log(\beta + \Delta\beta), \qquad \pi_L^C = \frac{p}{\sigma \omega_L \mathcal{T}_q^C} \log(\beta + \Delta\beta), \tag{62}$$

$$\hat{y}_G^C = \frac{1-p}{\omega_G \eta_y \mathcal{T}_q^C} \log(\beta + \Delta\beta), \qquad \pi_G^C = \frac{1-p}{\sigma \omega_G \mathcal{T}_q^C} \log(\beta + \Delta\beta). \tag{63}$$

Proof. Let λ denote the multiplier on (60). The first-order conditions are

$$2\omega_L \eta_y \hat{y}_L + \lambda(1-p) = 0, \tag{64}$$

$$2\omega_G \eta_y \hat{y}_G - \lambda(1-p) = 0, \tag{65}$$

$$2\omega_L \pi_L - \lambda \frac{p}{\sigma} = 0, \tag{66}$$

$$2\omega_G \pi_G - \lambda \frac{1-p}{\sigma} = 0. \tag{67}$$

Therefore,

$$\hat{y}_L = -\frac{\lambda(1-p)}{2\omega_L \eta_y}, \qquad \hat{y}_G = \frac{\lambda(1-p)}{2\omega_G \eta_y}, \tag{68}$$

$$\pi_L = \frac{\lambda p}{2\sigma \omega_L}, \qquad \pi_G = \frac{\lambda(1-p)}{2\sigma \omega_G}. \tag{69}$$

Substituting these expressions into (60) gives

$$\lambda = \frac{2 \log(\beta + \Delta\beta)}{\mathcal{T}_q^C}.$$

Substitution into the first-order conditions gives (62)–(63). □

The committed planner generally chooses positive inflation and a positive output gap in the good state. This is a forward-guidance-like margin: by promising a good-state boom, the planner improves expectations in the liquidity-trap state.

The good-state Taylor-rule intercept implementing the committed allocation is

$$\mu_G^C = (1-q)\pi_L^C + (q-\phi_\pi)\pi_G^C + \sigma(1-q)(\hat{y}_L^C - \hat{y}_G^C). \tag{70}$$

To implement the Phillips curves, define

$$d_L^C \equiv \theta\left[\pi_L^C - \beta\left(p\pi_L^C + (1-p)\pi_G^C\right) - \kappa\hat{y}_L^C\right], \tag{71}$$

$$d_G^C \equiv \theta\left[\pi_G^C - \beta\left((1-q)\pi_L^C + q\pi_G^C\right) - \kappa\hat{y}_G^C\right]. \tag{72}$$

Then the state-contingent TIPs are

$$\tau_L^C = -\frac{(1-\beta q)d_L^C + \beta(1-p)d_G^C}{\Delta_q}, \qquad \tau_G^C = -\frac{\beta(1-q)d_L^C + (1-\beta p)d_G^C}{\Delta_q}. \tag{73}$$

B.2.3 Markov constrained-efficient allocation

We now consider a Markov, time-consistent planner. Since the planner reoptimizes once the good state is reached, and since the ZLB is slack in that state, the good-state allocation is first best:

$$\hat{y}_G^M = 0, \qquad \pi_G^M = 0. \tag{74}$$

Thus the good state cannot be used as a promise to improve expectations in the liquidity-trap state. In state L, the planner solves

$$\min_{\hat{y}_L,\pi_L} \quad \pi_L^2 + \eta_y\hat{y}_L^2 \qquad \text{s.t.} \quad (1-p)\hat{y}_L - \frac{p}{\sigma}\pi_L + \frac{\log(\beta+\Delta\beta)}{\sigma} = 0. \tag{75}$$

Proposition 10 (Markov constrained-efficient allocation)**.** *The Markov constrained-efficient allocation is*

$$\hat{y}_L^M = -\frac{\sigma(1-p)}{\sigma^2(1-p)^2 + \eta_y p^2}\log(\beta+\Delta\beta), \quad \pi_L^M = \frac{\eta_y p}{\sigma^2(1-p)^2 + \eta_y p^2}\log(\beta+\Delta\beta), \tag{76}$$

$$\hat{y}_G^M = 0, \qquad \pi_G^M = 0. \tag{77}$$

Proof. Let λ denote the multiplier on (75). The first-order conditions are

$$2\eta_y \hat{y}_L + \lambda(1-p) = 0, \qquad 2\pi_L - \lambda \frac{p}{\sigma} = 0.$$

Substituting these conditions into (75) gives the stated expressions. □

The good-state Taylor-rule intercept required to implement the Markov allocation is

$$\mu_G^M = (1-q)\pi_L^M + \sigma(1-q)\hat{y}_L^M. \tag{78}$$

The Phillips-curve residuals are

$$d_L^M \equiv \theta\left[(1-\beta p)\pi_L^M - \kappa \hat{y}_L^M\right], \qquad d_G^M \equiv -\theta\beta(1-q)\pi_L^M. \tag{79}$$

Hence

$$\tau_L^M = -\frac{(1-\beta q)d_L^M + \beta(1-p)d_G^M}{\Delta_q}, \quad \tau_G^M = -\frac{\beta(1-q)d_L^M + (1-\beta p)d_G^M}{\Delta_q}. \tag{80}$$

Equivalently, the good-state TIP can be written as

$$\tau_G^M = \frac{\beta(1-q)}{1-\beta q}\left(\theta\pi_L^M + \tau_L^M\right). \tag{81}$$

When $q = 1$, the good state is absorbing, $\mu_G^M = \tau_G^M = 0$, and the liquidity-trap allocation collapses to the constrained-efficient allocation in Proposition 1.

B.2.4 Aggressive TIP in a recurrent self-fulfilling liquidity trap

We finally extend Proposition 6 to a recurrent self-fulfilling liquidity trap. In this subsection $\beta_t = \beta < 1$ in both states, so the first-best allocation is

$$\pi = \hat{y} = 0, \qquad i = -\log\beta > 0.$$

The states L and G should now be interpreted as sunspot labels. A recurrent SFLT is a nontrivial sunspot equilibrium in which the label L is associated with a ZLB allocation. The ZLB boundary is

$$\bar{\pi} \equiv \frac{\log \beta}{\phi_\pi} < 0. \tag{82}$$

A candidate ZLB allocation must have $\pi_L \leq \bar{\pi}$. Aggressive TIP rules out the SFLT by shifting the candidate L-state Phillips curve so that the ZLB solution would require $\pi_L > \bar{\pi}$.

For any good-state continuation $(\hat{y}_G, \pi_G, \tau_G)$, the L-state Euler equation evaluated at $\pi_L = \bar{\pi}$ gives

$$\hat{y}_L^B(\hat{y}_G, \pi_G) = \hat{y}_G + \frac{p\bar{\pi} + (1-p)\pi_G - \log \beta}{\sigma(1-p)}. \tag{83}$$

The corresponding boundary value of τ_L implied by the L-state Phillips curve is

$$\underline{\tau}_L^{SF}(\hat{y}_G, \pi_G, \tau_G) = \frac{\theta}{1-\beta p}\left[\kappa \hat{y}_L^B(\hat{y}_G, \pi_G) + \beta(1-p)\pi_G - (1-\beta p)\bar{\pi}\right] + \frac{\beta(1-p)}{1-\beta p}\tau_G. \tag{84}$$

If $\tau_L > \underline{\tau}_L^{SF}(\hat{y}_G, \pi_G, \tau_G)$, no ZLB equilibrium of this form exists.

Markov continuation. For the Markov planner, the good-state continuation is first best:

$$\hat{y}_G = \pi_G = 0.$$

If agents still contemplate future transitions to the low-confidence sunspot label, the good-state Phillips curve at the boundary requires

$$\tau_G = \frac{\beta(1-q)}{1-\beta q}\left(\theta \bar{\pi} + \tau_L\right). \tag{85}$$

Substituting (85) into (84) gives

$$\underline{\tau}_L^{SF,M} = -\frac{\theta \log \beta}{\Delta_q \, \phi_\pi \, \sigma(1-p)}\left\{(1-\beta q)\left[\kappa(\phi_\pi - p) + (1-\beta p)\sigma(1-p)\right] - \beta^2\sigma(1-p)^2(1-q)\right\}. \tag{86}$$

The associated good-state TIP is

$$\underline{\tau}_G^{SF,M} = \frac{\beta(1-q)}{1-\beta q}\left(\theta\bar{\pi} + \underline{\tau}_L^{SF,M}\right). \tag{87}$$

When $q = 1$, $\underline{\tau}_G^{SF,M} = 0$ and (86) collapses to

$$\underline{\tau}^{SF} = -\frac{\theta \log\beta}{(1-p\beta)(1-p)\sigma}\left[\kappa + \frac{(1-p\beta)\sigma(1-p) - \kappa p}{\phi_\pi}\right], \tag{88}$$

which is Proposition 6.

Committed continuation. Under commitment, the good-state continuation is the committed allocation $(\hat{y}_G^C, \pi_G^C)$. At the ZLB boundary,

$$\hat{y}_L^{B,C} = \hat{y}_G^C + \frac{p\bar{\pi} + (1-p)\pi_G^C - \log\beta}{\sigma(1-p)}. \tag{89}$$

The good-state Phillips curve evaluated at the boundary implies

$$\begin{aligned} d_G^{B,C} &\equiv \theta\left[\pi_G^C - \beta\left((1-q)\bar{\pi} + q\pi_G^C\right) - \kappa\hat{y}_G^C\right] && (90)\\ &= \beta(1-q)\tau_L - (1-\beta q)\tau_G. && (91)\end{aligned}$$

Thus

$$\tau_G = \frac{\beta(1-q)}{1-\beta q}\tau_L - \frac{d_G^{B,C}}{1-\beta q}. \tag{92}$$

Define

$$B_C \equiv \frac{\theta}{1-\beta p}\left[\kappa\hat{y}_L^{B,C} + \beta(1-p)\pi_G^C - (1-\beta p)\bar{\pi}\right]. \tag{93}$$

Substituting (92) into the L-state boundary condition gives

$$\underline{\tau}_L^{SF,C} = \frac{(1-\beta p)(1-\beta q)B_C - \beta(1-p)d_G^{B,C}}{\Delta_q}, \tag{94}$$

with associated good-state TIP

$$\underline{\tau}_G^{SF,C} = \frac{\beta(1-q)}{1-\beta q}\underline{\tau}_L^{SF,C} - \frac{d_G^{B,C}}{1-\beta q}. \tag{95}$$

If $\tau_L > \underline{\tau}_L^{SF,C}$, the recurrent ZLB branch associated with this committed continuation is ruled out.

When $q = 1$, the good state is absorbing. Then

$$\Delta_q = (1-\beta p)(1-\beta),$$

and

$$d_G^{B,C} = \theta\left[(1-\beta)\pi_G^C - \kappa\hat{y}_G^C\right]. \tag{96}$$

The good-state TIP associated with the committed continuation is therefore

$$\underline{\tau}_G^{SF,C}\big|_{q=1} = -\frac{d_G^{B,C}}{1-\beta} \tag{97}$$

$$= -\theta\pi_G^C + \frac{\theta\kappa}{1-\beta}\hat{y}_G^C. \tag{98}$$

The liquidity-trap threshold becomes

$$\underline{\tau}_L^{SF,C}\big|_{q=1} = \underline{\tau}^{SF} + \frac{\theta\kappa}{1-\beta}\hat{y}_G^C + \frac{\theta\kappa}{\sigma(1-\beta p)}\pi_G^C, \tag{99}$$

where $\underline{\tau}^{SF}$ is the baseline threshold in (88). Hence, when the good-state continuation is first best, $\hat{y}_G^C = \pi_G^C = 0$, the committed threshold coincides with the Markov threshold. If the committed planner promises a nonzero good-state continuation, the threshold is shifted by the two continuation terms in (99).

Equilibrium after the SFLT is ruled out. An aggressive TIP does not alter the law of motion of sunspotsand the probability that governs it (p, q) but it alters the set of equilibrium allocations that can be supported by those sunspots. In particular, it eliminates the nontrivial sunspot equilibrium in which the L sunspot maps into a deflationary ZLB allocation. The remaining local equilibrium is sunspot-irrelevant:

$$\pi_L = \pi_G, \qquad \hat{y}_L = \hat{y}_G.$$

With $\mu_s = 0$, $\tau_s = 0$, and $\phi_\pi > 1$, the Euler equation and Phillips curve then imply

$$\pi_L = \pi_G = 0, \qquad \hat{y}_L = \hat{y}_G = 0.$$

Thus the low-confidence sunspot may still be realized, but it cannot generate a change in equilibrium allocations. Equivalently, $1 - q$ remains the probability of a low-confidence sunspot realization, while the probability that this realization translates into a liquidity trap is zero.

C Determinacy

In this Appendix we analyze the conditions ensuring the uniqueness of equilibria. Outside of the ZLB and without state switching, determinacy in the baseline New Keynesian model requires that monetary policy implement the Taylor principle according to which the policy rate reacts strongly to inflation. With state switching and one state displaying a ZLB, in which monetary policy does not react to inflation, the Taylor principle may longer be necessary or sufficient as discussed in prior literature (Davig and Leeper, 2007; Farmer, Waggoner, and Zha, 2009). A natural question is whether TIP could help ensure uniqueness in this setting.

Replacing the Taylor rule into the IS curve, we have that the system is described by the following pair of equations outside of the liquidity trap (N)

$$\hat{y}_{Nt} = E_{Nt}\hat{y}_{t+1} - \frac{1}{\sigma}\left(\phi_{N\pi}\pi_{Nt} - E_{Nt}\pi_{t+1}\right) \tag{100}$$

$$\pi_{Nt} = \beta E_{Nt}\pi_{t+1} + \kappa\hat{y}_{Nt} + \frac{1}{\theta}\left[\beta E_{Nt}\tau_{t+1} - \tau_{Nt}\right] \tag{101}$$

with $\phi_{N\pi} > 1$ and by this system of equations at the liquidity trap (L)

$$\hat{y}_{Lt} = E_{Lt}\hat{y}_{t+1} - \frac{1}{\sigma}\left(\phi_{L\pi}\pi_{Lt} - E_{Lt}\pi_{t+1} + \log(\beta + \Delta\beta)\right) \tag{102}$$

$$\pi_{Lt} = \beta E_{Lt}\pi_{t+1} + \kappa\hat{y}_{Lt} + \frac{1}{\theta}\left[\beta E_{Lt}\tau_{t+1} - \tau_{Lt}\right]. \tag{103}$$

with $\phi_{L\pi} = 0$ and all parameters other the Taylor coefficients are the same in both states.

Proposition 11. *Assume $\tau_{Nt} = 0$ for all t. The equilibrium path is unique if $\frac{\theta\kappa}{1-p\beta} > \varphi_{Ny}$ and $\phi_{N\pi} > 1$.*

The proof of this proposition is as follows. Recall that N is an absorbing state. Therefore the set of solutions to the system (100) and (101) is independent of the parameters in the L state. We can thus analyze the parameters restrictions the same way one would do in the constant parameter baseline new Keynesian model. In absence of TIP in the normal state, $\tau_{Nt} = 0$, it is well-known that the Taylor principle is necessary and sufficient in that case, $\phi_{N\pi} > 1$. We refer the reader to Capelle and Liu (forthcoming) for a generalized Taylor principle with TIP.

Assume the Taylor principle is fulfilled in the N state. Then we have $\hat{y}_{Nt} = \pi_{Nt} = 0$. As a result, we can rewrite the system of equations in the liquidity trap state as

$$\hat{y}_{Lt} = p\hat{y}_{Lt+1} - \frac{1}{\sigma}\left(\phi_{L\pi}\pi_{Lt} - p\pi_{Lt+1} + \log(\beta + \Delta\beta)\right) \tag{104}$$

$$\pi_{Lt} = \beta p\pi_{Lt+1} + \kappa\hat{y}_{Lt} + \frac{1}{\theta}\left[\beta p\tau_{Lt+1} - \tau_{Lt}\right]. \tag{105}$$

Given that N is an absorbing state, the system of equations is block-recursive, and we can again analyze the determinacy of the equilibrium by looking at the roots of (104) and (105). The conditions for stability in systems with Markov switching cannot be analyzed state by state, as emphasized by Farmer, Waggoner, and Zha (2010). We first rewrite this system in matrix form

$$A\begin{pmatrix}\pi_{Lt}\\ x_{Lt}\end{pmatrix} = pB\begin{pmatrix}\pi_{Lt+1}\\ x_{Lt+1}\end{pmatrix} + C$$

$$\text{with} \quad A = \begin{pmatrix}\phi_{L\pi} & \sigma + \phi_{Ly}\\ 1 + \frac{\varphi_{L\pi}}{\theta} & \frac{\varphi_{Ly}}{\theta} - \kappa\end{pmatrix}, \quad B = \begin{pmatrix}1 & \sigma\\ \beta\left(1 + \frac{\varphi_{L\pi}}{\theta}\right) & \frac{\varphi_{Ly}}{\theta}\beta\end{pmatrix}$$

where C contains parameters and shocks and does not matter for stability. We now solve for the eigenvalues of this system. We first invert A

$$A^{-1} = \frac{1}{\phi_{L\pi}\left(\frac{\varphi_{Ly}}{\theta} - \kappa\right) - (\sigma + \phi_{Ly})\left(1 + \frac{\varphi_{L\pi}}{\theta}\right)} \begin{pmatrix} \frac{\varphi_{Ly}}{\theta} - \kappa & -\sigma - \phi_{Ly} \\ -1 - \frac{\varphi_{L\pi}}{\theta} & \phi_{L\pi} \end{pmatrix}$$

and then multiply it by pB:

$$\begin{aligned} A^{-1}pB &= \frac{p}{\phi_{L\pi}\left(\frac{\varphi_{Ly}}{\theta} - \kappa\right) - (\sigma + \phi_{Ly})\left(1 + \frac{\varphi_{L\pi}}{\theta}\right)} \begin{pmatrix} \frac{\varphi_{Ly}}{\theta} - \kappa & -\sigma - \phi_{Ly} \\ -1 - \frac{\varphi_{L\pi}}{\theta} & \phi_{L\pi} \end{pmatrix} \begin{pmatrix} 1 & \sigma \\ \beta\left(1 + \frac{\varphi_{L\pi}}{\theta}\right) & \frac{\varphi_{Ly}}{\theta}\beta \end{pmatrix} \\ &= \Omega \begin{pmatrix} \frac{\varphi_{Ly}}{\theta} - \kappa - (\sigma + \phi_{Ly})\beta\left(1 + \frac{\varphi_{L\pi}}{\theta}\right) & \sigma\left(\frac{\varphi_{Ly}}{\theta} - \kappa\right) - (\sigma + \phi_{Ly})\beta\frac{\varphi_{Ly}}{\theta} \\ \left(1 + \frac{\varphi_{L\pi}}{\theta}\right)[\beta\phi_{L\pi} - 1] & -\sigma\left(1 + \frac{\varphi_{L\pi}}{\theta}\right) + \phi_{L\pi}\frac{\varphi_{Ly}}{\theta}\beta \end{pmatrix} \\ &= -\Omega \begin{pmatrix} -\frac{\varphi_{Ly}}{\theta} + \kappa + (\sigma + \phi_{Ly})\beta\left(1 + \frac{\varphi_{L\pi}}{\theta}\right) & \sigma\left(\kappa - \frac{\varphi_{Ly}}{\theta}\right) + (\sigma + \phi_{Ly})\beta\frac{\varphi_{Ly}}{\theta} \\ \left(1 + \frac{\varphi_{L\pi}}{\theta}\right)[1 - \beta\phi_{L\pi}] & \sigma\left(1 + \frac{\varphi_{L\pi}}{\theta}\right) - \phi_{L\pi}\frac{\varphi_{Ly}}{\theta}\beta \end{pmatrix} \end{aligned}$$

with $\Omega = \frac{p}{\phi_{L\pi}\left(\frac{\varphi_{Ly}}{\theta} - \kappa\right) - (\sigma + \phi_{Ly})\left(1 + \frac{\varphi_{L\pi}}{\theta}\right)}$. We denote this matrix A', and we now compute its trace and determinant.

$$\begin{aligned} TrA' &= -\Omega\left[-\frac{\varphi_{Ly}}{\theta} + \kappa + (\sigma + \phi_{Ly})\beta\left(1 + \frac{\varphi_{L\pi}}{\theta}\right) + \sigma\left(1 + \frac{\varphi_{L\pi}}{\theta}\right) - \phi_{L\pi}\frac{\varphi_{Ly}}{\theta}\beta\right] \\ &= \frac{p}{(\sigma + \phi_{Ly})\left(1 + \frac{\varphi_{L\pi}}{\theta}\right) + \phi_{L\pi}\left(\kappa - \frac{\varphi_{Ly}}{\theta}\right)} \\ &\qquad \left[-\frac{\varphi_{Ly}}{\theta}(1 + \beta\phi_{L\pi}) + \kappa + (\sigma(\beta + 1) + \phi_{Ly}\beta)\left(1 + \frac{\varphi_{L\pi}}{\theta}\right)\right] \end{aligned}$$

$$
\begin{aligned}
detA' = &\left(\frac{p}{(\sigma+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)+\phi_{L\pi}\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)}\right)^2 \\
&\times\left[\left[-\frac{\varphi_{Ly}}{\theta}+\kappa+(\sigma+\phi_{Ly})\beta\left(1+\frac{\varphi_{L\pi}}{\theta}\right)\right]\left[\sigma\left(1+\frac{\varphi_{L\pi}}{\theta}\right)-\phi_{L\pi}\frac{\varphi_{Ly}}{\theta}\beta\right]\right. \\
&\qquad\left.-\left[\sigma\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)+(\sigma+\phi_{Ly})\beta\frac{\varphi_{Ly}}{\theta}\right]\left[\left(1+\frac{\varphi_{L\pi}}{\theta}\right)[1-\beta\phi_{L\pi}]\right]\right. \\
= &\left(\frac{p}{(\sigma+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)+\phi_{L\pi}\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)}\right)^2 \\
&\times\left(\left[-\frac{\varphi_{Ly}}{\theta}+\kappa+(\sigma+\phi_{Ly})\beta\left(1+\frac{\varphi_{L\pi}}{\theta}\right)\right]\left[\sigma\left(1+\frac{\varphi_{L\pi}}{\theta}\right)-\phi_{L\pi}\frac{\varphi_{Ly}}{\theta}\beta\right]\right. \\
&\qquad\left.-\left[\sigma\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)+(\sigma+\phi_{Ly})\beta\frac{\varphi_{Ly}}{\theta}\right]\left[\left(1+\frac{\varphi_{L\pi}}{\theta}\right)[1-\beta\phi_{L\pi}]\right]\right) \\
= &\left(\frac{p}{(\sigma+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)+\phi_{L\pi}\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)}\right)^2 \\
&\times\left(\beta\left[(\sigma+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)+\phi_{L\pi}\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)\right]\left[\sigma\left(1+\frac{\varphi_{L\pi}}{\theta}\right)-\frac{\varphi_{Ly}}{\theta}\right]\right. \\
= &\left(\frac{p}{(\sigma+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)+\phi_{L\pi}\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)}\right)^2 \\
&\times\beta\left(\left[(\sigma+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)+\phi_{L\pi}\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)\right]\left[\sigma\left(1+\frac{\varphi_{L\pi}}{\theta}\right)-\frac{\varphi_{Ly}}{\theta}\right]\right)
\end{aligned}
$$

$$
\begin{aligned}
detA' = &\left(\frac{p}{(\sigma+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)+\phi_{L\pi}\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)}\right)^2 \\
&\times\beta\left[(\sigma+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)+\phi_{L\pi}\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)\right]\left[\sigma\left(1+\frac{\varphi_{L\pi}}{\theta}\right)-\frac{\varphi_{Ly}}{\theta}\right] \\
= &\left(\frac{p^2}{(\sigma+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)+\phi_{L\pi}\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)}\right)\beta\left[\sigma\left(1+\frac{\varphi_{L\pi}}{\theta}\right)-\frac{\varphi_{Ly}}{\theta}\right]
\end{aligned}
$$

Let's assume $\varphi_{Ly}, \varphi_{L\pi}, \phi_{L\pi}, \phi_{Ly} \geq 0$ and restrict our analysis to the case where the determinant is positive (both eigenvalues have the same sign and are non-imaginary), i.e. assume that $\left(\sigma\left(1+\frac{\varphi_{L\pi}}{\theta}\right)-\frac{\varphi_{Ly}}{\theta}\right)\left((\sigma+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)+\phi_{L\pi}\kappa-\phi_{L\pi}\frac{\varphi_{Ly}}{\theta}\right)>0$. These restrictions are consistent with any empirically reasonable parametrization.

This system with two non predetermined variables is determinate if and only if both eigenvalues are within the unit circle (Blanchard and Kahn, 1980). There are then two sufficient and necessary conditions for both eigenvalues to be within the unit circle: $detA' < 1$ and $TrA < 1 + detA'$. The latter condition can be derived from the condition that both eigenvalues are strictly below 1 $(1-\lambda_1)(1-\lambda_2) > 0$ when both are positive, or both strictly above -1 $(-1-\lambda_1)(-1-\lambda_2) > 0$ when both are negative. The condition $detA' < 1$ gives:

$$\frac{\varphi_{Ly}}{\theta}\left(\phi_{L\pi}-p^2\beta\right) < \phi_{L\pi}\kappa + (\sigma(1-p^2\beta)+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)$$

This condition is that φ_{Ly} is small enough:

$$\varphi_{Ly} < \frac{\theta\phi_{L\pi}\kappa + \theta(\sigma(1-p^2\beta)+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)}{\phi_{L\pi}-p^2\beta}$$

The second necessary and sufficient condition is that $TrA' < 1 + detA'$ which gives

$$p\frac{\left[-\frac{\varphi_{Ly}}{\theta}(1+\beta\phi_{L\pi})+\kappa+(\sigma(\beta+1)+\phi_{Ly}\beta)\left(1+\frac{\varphi_{L\pi}}{\theta}\right)\right]}{(\sigma+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)+\phi_{L\pi}\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)} < 1+\frac{p^2\beta\left[\sigma\left(1+\frac{\varphi_{L\pi}}{\theta}\right)-\frac{\varphi_{Ly}}{\theta}\right]}{(\sigma+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)+\phi_{L\pi}\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)}$$

$$-\frac{\varphi_{Ly}}{\theta}p(1+\beta\phi_{L\pi})+p\kappa+p(\sigma(\beta+1)+\phi_{Ly}\beta)\left(1+\frac{\varphi_{L\pi}}{\theta}\right) < (\sigma+\phi_{Ly})\left(1+\frac{\varphi_{L\pi}}{\theta}\right)+\phi_{L\pi}\left(\kappa-\frac{\varphi_{Ly}}{\theta}\right)+p^2\beta\left[\sigma\left(1+\frac{\varphi_{L\pi}}{\theta}\right)-\frac{\varphi_{Ly}}{\theta}\right]$$

$$
\begin{aligned}
-\frac{\varphi_{Ly}}{\theta}p(1+\beta\phi_{L\pi}) + p\kappa + \left(1+\frac{\varphi_{L\pi}}{\theta}\right)\left[p(\sigma(\beta+1)+\phi_{Ly}\beta) - \sigma - \phi_{Ly} - p^2\beta\sigma\right] \\
< \phi_{L\pi}\left(\kappa - \frac{\varphi_{Ly}}{\theta}\right) - p^2\beta\frac{\varphi_{Ly}}{\theta}
\end{aligned}
$$

$$
(\phi_{L\pi}-1)\kappa + \frac{\varphi_{Ly}}{\theta}\left[p(1+\beta\phi_{L\pi}) - p^2\beta - \phi_{L\pi}\right] + \left(1+\frac{\varphi_{L\pi}}{\theta}\right)\left[p(\sigma(\beta+1)+\phi_{Ly}\beta) - \sigma - \phi_{Ly} - p^2\beta\sigma\right] >
$$

Imposing $\phi_{L\pi} = 0 = \phi_{Ly}$, we get

$$
\begin{aligned}
[1-p\beta]\left[\frac{\varphi_{Ly}}{\theta}p - \left(1+\frac{\varphi_{L\pi}}{\theta}\right)\sigma(1-p)\right] &> \kappa \\
\frac{\varphi_{Ly}}{\theta}p &> \frac{\kappa}{1-p\beta} + \left(1+\frac{\varphi_{L\pi}}{\theta}\right)\sigma(1-p)
\end{aligned}
$$

This ends the proof.

D TIP and Other Policies

D.1 Forward Guidance

We follow Bilbiie (2019) in that we assume that when the economy exits the liquidity trap, the interest rate is maintained at 0 with probability q every period. With this, we can define the inflation rate and output gap in the forward guidance state "F".

$$
\begin{aligned}
\hat{y}^F &= \frac{1}{\sigma(1-q)}\left(q\pi^F - \log\beta\right) \\
\pi^F &= \frac{-\log\beta}{(1-q\beta)\sigma(1-q) - \kappa q}
\end{aligned}
$$

We can then solve for the inflation rate and output gap in the liquidity trap state "L" by backward induction.

$$
\begin{aligned}
\pi_L &= \frac{\beta(1-p)q\pi^F + \kappa\hat{y}_L}{1-\beta p} \\
\hat{y}_L &= \frac{(1-p)q\hat{y}^F + \frac{1}{\sigma}\left(p\pi_L + (1-p)q\pi^F - \log(\beta+\Delta\beta)\right)}{1-p}
\end{aligned}
$$

Solving for π_L gives

$$\pi_L = \frac{\beta(1-p)q\pi^F + \kappa\frac{(1-p)q\hat{y}^F + \frac{1}{\sigma}\left(p\pi_L + (1-p)q\pi^F - \log(\beta+\Delta\beta)\right)}{1-p}}{1-\beta p}$$

$$\Rightarrow \pi_L = \frac{\left(\beta(1-p)^2 q + \kappa\frac{(1-p)q}{\sigma}\right)\pi^F + \kappa\left[(1-p)q\hat{y}^F - \frac{1}{\sigma}\log(\beta+\Delta\beta)\right]}{(1-p)(1-\beta p) - \kappa\frac{p}{\sigma}}$$

We can also compute welfare losses conditional on starting in the liquidity trap

$$\begin{aligned}
X^F &= (\pi^F)^2 + \eta_y(\hat{y}^F)^2 \\
X^L &= (\pi_L)^2 + \eta_y(\hat{y}_L)^2 \\
W^F &= E_F \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \eta_y\left(\hat{y}_t^e\right)^2\right] = X^F + \beta q Y^F = \frac{X^F}{1-\beta q} \\
W^L &= E_L \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \eta_y\left(\hat{y}_t^e\right)^2\right] = X^L + \beta\left(pW^L + (1-p)qW^F\right) = \frac{X^L + \beta(1-p)qW^F}{1-\beta p}.
\end{aligned}$$

D.2 Government Spending

The model is given by the following system of linearized equations:

$$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma}\left(i_t - E_t\pi_{t+1} + \log(\beta)\right) \tag{106}$$

$$\hat{w}_t - \hat{p}_t = \sigma\hat{c}_t + \psi\hat{n}_t \tag{107}$$

$$\hat{y}_t = (1-\alpha)\hat{n}_t \tag{108}$$

$$\hat{mc}_t = \hat{w}_t - \hat{p}_t + \frac{\alpha}{1-\alpha}\hat{y}_t - \log(1-\alpha) \tag{109}$$

$$\hat{y}_t = w_c\hat{c}_t + (1-w_c)\hat{g}_t \tag{110}$$

$$\pi_t = \beta E_t\pi_{t+1} + \frac{\epsilon-1}{\theta}\hat{mc}_t + \frac{1}{\theta}\left(\beta E_t\tau_{t+1} - \tau_t\right) + u_t \tag{111}$$

Combining the definition of marginal cost, the goods market clearing condition and the intratemporal condition of the household gives

$$\hat{mc}_t = \hat{y}_t\left(\frac{\sigma}{w_c} + \frac{\psi}{1-\alpha} + \frac{\alpha}{1-\alpha}\right) - \frac{\sigma(1-w_c)}{w_c}\hat{g}_t$$

Using this expression to substitute for the marginal cost in the Phillips curve and using the market clearing condition to substitute for consumption in the Euler equation gives

$$\hat{y}_t - (1-w_c)\hat{g}_t = E_t\left(\hat{y}_{t+1} - (1-w_c)\hat{g}_{t+1}\right) - \frac{w_c}{\sigma}\left(i_t - E_t\pi_{t+1} + \log(\beta)\right) \quad (112)$$

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa}\hat{y}_t - \frac{\epsilon-1}{\theta}\frac{\sigma(1-w_c)}{w_c}\hat{g}_t + \frac{1}{\theta}\left(\beta E_t \tau_{t+1} - \tau_t\right) + u_t \quad (113)$$

with $\tilde{\kappa} = \frac{\epsilon-1}{\theta}\left(\frac{\sigma}{w_c} + \frac{\psi}{1-\alpha} + \frac{\alpha}{1-\alpha}\right)$.

In the liquidity trap without TIP, the Euler equation and the Phillips curve become

$$\hat{y}_L = (1-w_c)\hat{g}^L + \frac{w_c}{(1-p)\sigma}\left(p\pi_L - \log(\beta+\Delta\beta)\right) \quad (114)$$

$$\pi_L = \frac{1}{1-\beta p}\left[\tilde{\kappa}\hat{y}_L - \frac{\epsilon-1}{\theta}\frac{\sigma(1-w_c)}{w_c}\hat{g}^L + u_t\right] \quad (115)$$

Abstracting from cost-push shocks, and substituting y^L into the second equation, we can solve for π_L :

$$\pi_L\left(1-\beta p - \tilde{\kappa}\frac{w_c p}{(1-p)\sigma}\right) = \left[\tilde{\kappa}\left((1-w_c)\hat{g}^L - \frac{w_c}{(1-p)\sigma}\log(\beta+\Delta\beta)\right) - \frac{\epsilon-1}{\theta}\frac{\sigma(1-w_c)}{w_c}\hat{g}^L\right] \quad (116)$$

We thus obtain:

$$\pi_L = -\frac{w_c\tilde{\kappa}}{(1-p)\sigma(1-\beta p) - \tilde{\kappa}p w_c}\log(\beta+\Delta\beta) + \Gamma_g\hat{g}_t \quad (117)$$

$$\Gamma_g = \frac{\tilde{\kappa}(1-w_c) - \frac{\epsilon-1}{\theta}\frac{\sigma(1-w_c)}{w_c}}{\left(1-\beta p - \tilde{\kappa}\frac{w_c p}{(1-p)\sigma}\right)} \quad (118)$$

$$\hat{y}_L = (1-w_c)\hat{g}^L + \frac{w_c}{(1-p)\sigma}\left(p\pi_L - \log(\beta+\Delta\beta)\right). \quad (119)$$

D.3 Second-order approximation of welfare loss

We next show that the welfare loss function of the household depends on the inflation rate, the output gap and the government spending gap.

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 - \frac{1-\sigma}{\theta w_c} \left(\hat{y}_t - (1-w_c)\hat{g}_t \right)^2 + \frac{1+\psi}{\theta(1-\alpha)} \left(\hat{y}_t \right)^2 - \frac{(1-w_c)}{\theta}(1-\sigma_G)\hat{g}_t^2 - \frac{\Phi}{\theta}\hat{y}_t \right]$$

Without government spending and without steady-state distortion this simplifies to the usual welfare loss function

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \frac{1}{\theta} \left(\sigma + \frac{\alpha+\psi}{1-\alpha} \right) \left(\hat{y}_t \right)^2 \right]$$

The log deviation of the representative household is given by

$$\frac{U_t - U}{U_c C} = \left(\hat{c}_t + \frac{1-\sigma}{2} \hat{c}_t^2 \right) + \frac{U_N N}{U_c C} \left(\hat{n}_t + \frac{1+\psi}{2} \hat{n}_t^2 \right) + \frac{U_g G}{U_c C} \left(\hat{g}_t + \frac{1-\sigma_G}{2} \hat{g}_t^2 \right)$$

.

We next use a second order approximation of the goods market clearing condition to get an approximation for the first bracket

$$w_c \hat{c}_t + (1-w_c)\hat{g}_t = \hat{y}_t - \frac{1}{2}\theta \pi_t^2.$$

with $w_c = \frac{C^{SS}}{Y^{SS}}$.

We next use the labor market clearing condition to get an approximation for the second bracket:

$$\hat{n}_t = \frac{1}{1-\alpha} \left(\hat{y}_t - \hat{a}_t \right)$$

We next use the optimality of spending on government and private consumption in steady-state $U_c = U_g$.

Combining these three conditions and keeping only the terms of first and second order, we obtain

$$\frac{U_t - U}{U_c C} = \left(\frac{\hat{y}_t - (1-w_c)\hat{g}_t - \frac{1}{2}\theta\pi_t^2}{w_c} + \frac{1-\sigma}{2}\left(\frac{\hat{y}_t - (1-w_c)\hat{g}_t}{w_c}\right)^2 \right) + \frac{U_N N}{U_c C}\left(\frac{1}{1-\alpha}(\hat{y}_t - \hat{a}_t) + \frac{1+\psi}{2}\left(\frac{1}{1-\alpha}\right)^2 (\hat{y}_t - \hat{a}_t)^2\right) + \frac{G}{C}\left(\hat{g}_t + \frac{1-\sigma_G}{2}\hat{g}_t^2\right)$$

Denote Φ the steady-state distortion implicitly defined by $-\frac{U_N N}{U_C(C+G)(1-\alpha)} = 1 - \Phi$. We obtain:

$$\frac{U_t - U}{U_c C} = \left(\frac{\hat{y}_t - (1-w_c)\hat{g}_t - \frac{1}{2}\theta\pi_t^2}{w_c} + \frac{1-\sigma}{2}\left(\frac{\hat{y}_t - (1-w_c)\hat{g}_t}{w_c}\right)^2 \right) - \frac{1-\Phi}{w_c}\left((\hat{y}_t - \hat{a}_t) + \frac{1+\psi}{2}\left(\frac{1}{1-\alpha}\right)(\hat{y}_t - \hat{a}_t)^2\right) + \frac{1-w_c}{w_c}\left(\hat{g}_t + \frac{1-\sigma_G}{2}\hat{g}_t^2\right)$$

We can simplify the terms in $\hat{g}_t$.

$$\frac{U_t - U}{U_c C} = \left(\frac{\hat{y}_t - \frac{1}{2}\theta\pi_t^2}{w_c} + \frac{1-\sigma}{2}\left(\frac{\hat{y}_t - (1-w_c)\hat{g}_t}{w_c}\right)^2 \right) - \frac{1-\Phi}{w_c}\left((\hat{y}_t - \hat{a}_t) + \frac{1+\psi}{2}\left(\frac{1}{1-\alpha}\right)(\hat{y}_t - \hat{a}_t)^2\right) + \frac{1-w_c}{w_c}\frac{1-\sigma_G}{2}\hat{g}_t^2$$

Abstracting from TFP shocks and under the "small distortion" assumption, we can ignore the product of Φ with a second-order term

$$w_c\frac{U_t - U}{U_c C} = \Phi\hat{y}_t - \frac{1}{2}\theta\pi_t^2 + \frac{1-\sigma}{2w_c}(\hat{y}_t - (1-w_c)\hat{g}_t)^2 - \frac{1+\psi}{2(1-\alpha)}(\hat{y}_t)^2 + (1-w_c)\frac{1-\sigma_G}{2}\hat{g}_t^2 + t.i.p$$

E Self-fulfilling and Phillips-curve-driven ZLB

E.1 Self-fulfilling liquidity trap

Proof. It has to be the case that inflation, given by $-\frac{1}{(1-p\beta)\sigma(1-p)-\kappa p}\left[\kappa \log \beta\right]$, is negative without TIP, which requires $\frac{1-p\beta}{\kappa} < \frac{p}{\sigma(1-p)}$ given that $\log \beta < 0$. It also has to be the case that the nominal interest rate that would be implemented following the rule is indeed negative,

$$\begin{aligned}
\phi_\pi \pi_L - \log \beta &< 0 \\
\pi_L &< \frac{\log \beta}{\phi_\pi} \\
-\frac{1}{(1-p\beta)\sigma(1-p)-\kappa p}\left[\kappa \log \beta\right] &< \frac{\log \beta}{\phi_\pi} \\
\phi_\pi &> \frac{\kappa p-(1-p\beta)\sigma(1-p)}{\kappa}
\end{aligned}$$

where we used the fact that $\log \beta < 0$. □

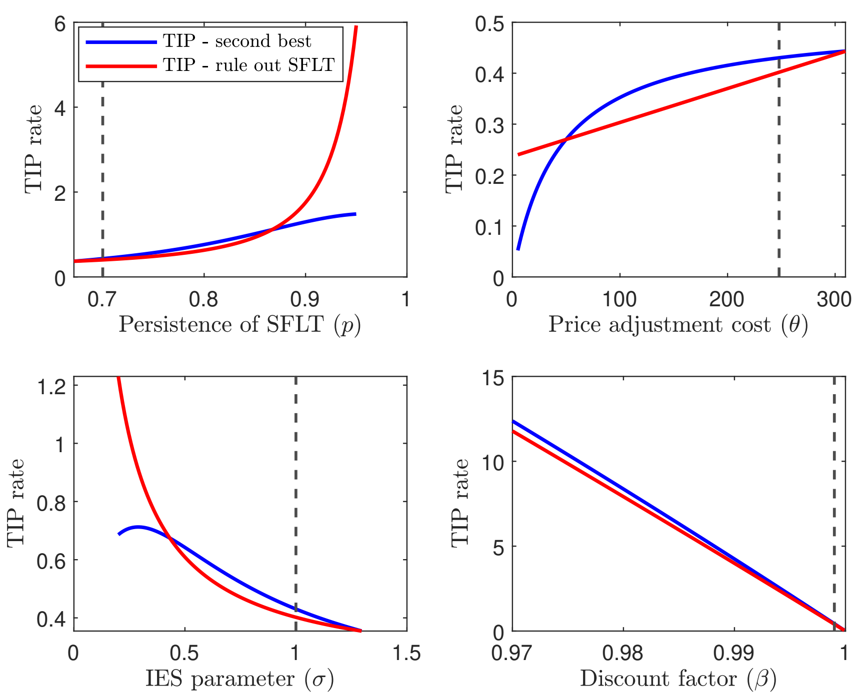


Figure 10: Optimal TIP in the SFLT

E.2 Shift in the Phillips curve

Proof. For the ZLB to bind, it should be that deflation is below the neutral rate

$$\pi_L = -\frac{1}{(1-p\beta)\sigma(1-p)-\kappa p}\left[\kappa \log\beta - \frac{\sigma(1-p)}{\theta}\underline{u}\right] < \log\beta$$

$$\iff \kappa\log\beta - \frac{\sigma(1-p)}{\theta}\underline{u} > ((1-p\beta)\sigma(1-p)-\kappa p)\log\beta \quad \text{and} \quad \frac{1-p\beta}{\kappa} > \frac{p}{\sigma(1-p)}$$

$$\iff \underline{u} < \frac{\theta\log\beta}{\sigma(1-p)}\left(\kappa - ((1-p\beta)\sigma(1-p)-\kappa p)\right) \quad \text{and} \quad \frac{1-p\beta}{\kappa} > \frac{p}{\sigma(1-p)}$$

$$\iff \underline{u} < \frac{\theta\log\beta}{\sigma(1-p)}\left(\kappa(1+p) - (1-p\beta)\sigma(1-p)\right) \quad \text{and} \quad \frac{1-p\beta}{\kappa} > \frac{p}{\sigma(1-p)}$$

□

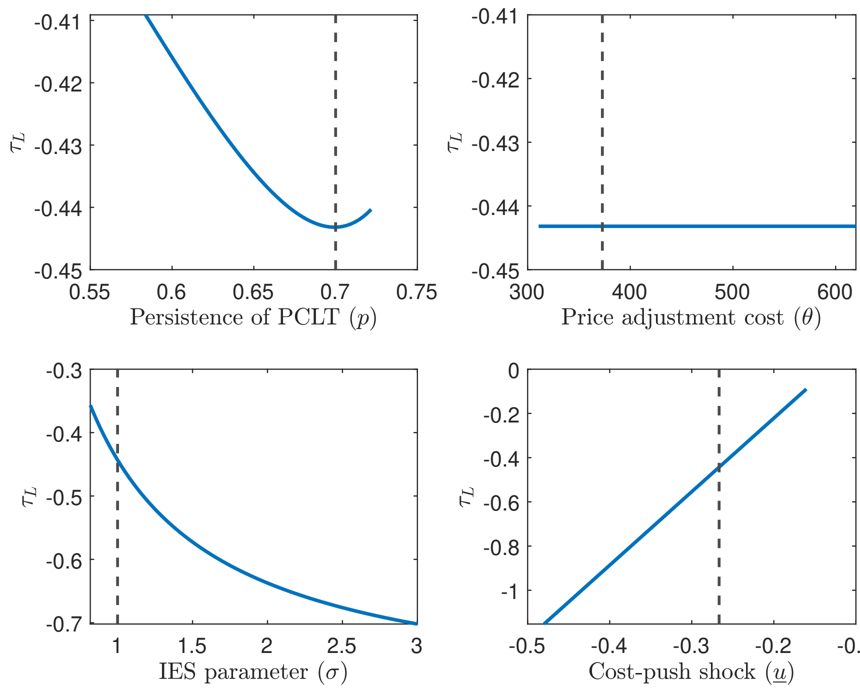


Figure 11: Optimal TIP in the PCLT

F Robust TIP

1. FLT. The proof is direct.

2. SFLT. Recall that for a liquidity trap equilibrium to exist, we need $-\frac{1}{(1-p\beta)\sigma(1-p)-\kappa p}[\kappa \log \beta] < 0$. It thus has to be for a liquidity trap to not exist with an inflation-targeting TIP that $-\frac{1}{(1-p\beta)\sigma(1-p)\left(1+\frac{\varphi_\pi}{\theta}\right)-\kappa p}[\kappa \log \beta] > 0$. This immediately implies $\varphi_\pi > \theta\left(\frac{\kappa p}{(1-p\beta)\sigma(1-p)} - 1\right)$.

3. PCLT.

   Let's first consider the case where $\pi < \frac{\log \beta}{\phi_\pi}$ which corresponds to the case where the economy is at the ZLB with $i = 0$. In that case, inflation and output are given by

$$\pi_L = -\frac{1}{(1-p\beta)\sigma(1-p)\left(1+\frac{\varphi_\pi}{\theta}\right)-\kappa p}\left[\kappa \log(\beta) - \frac{\sigma(1-p)}{\theta}\underline{u}\right] \tag{120}$$

$$\hat{y}_L = \frac{1}{\sigma(1-p)}\left(p\pi_L - \log(\beta)\right) \tag{121}$$

   It is clear that π is monotonically increasing (decreasing in absolute terms) in φ_π and that $\hat{y}$ is monotonically increasing in φ_π.

   Let's then consider the case where $\pi > \frac{\log \beta}{\phi_\pi}$. In that case inflation and output are given by

$$\pi_L = \frac{1}{(1-p\beta)\sigma(1-p)\left(1+\frac{\varphi_\pi}{\theta}\right)+\kappa(\phi_\pi - p)}\frac{\underline{u}\sigma(1-p)}{\theta} \tag{122}$$

$$\hat{y}_L = \frac{1}{\sigma(1-p)}\left(p - \phi_\pi\right)\pi_L \tag{123}$$

   The threshold φ_π^{PC} is solution to

$$\pi = \frac{\log \beta}{\phi_\pi} = -\frac{1}{(1-p\beta)\sigma(1-p)\left(1+\frac{\varphi_\pi^{PC}}{\theta}\right)-\kappa p}\left[\kappa \log(\beta) - \frac{\sigma(1-p)}{\theta}\underline{u}\right]$$

$$\iff \varphi_\pi^{PC} = \theta\left[\frac{\frac{\phi_\pi}{\log \beta}\frac{\sigma(1-p)}{\theta}\underline{u} - \kappa(\phi_\pi - p)}{(1-p\beta)(1-p)\sigma} - 1\right]$$

F.1 Numerical Simulations - Figures

F.1.1 Welfare

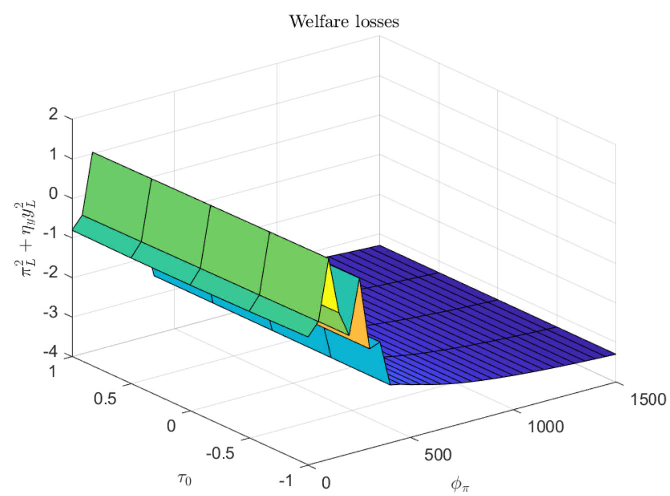


Figure 12: Welfare losses - Policymaker maximizing expected welfare with a prior

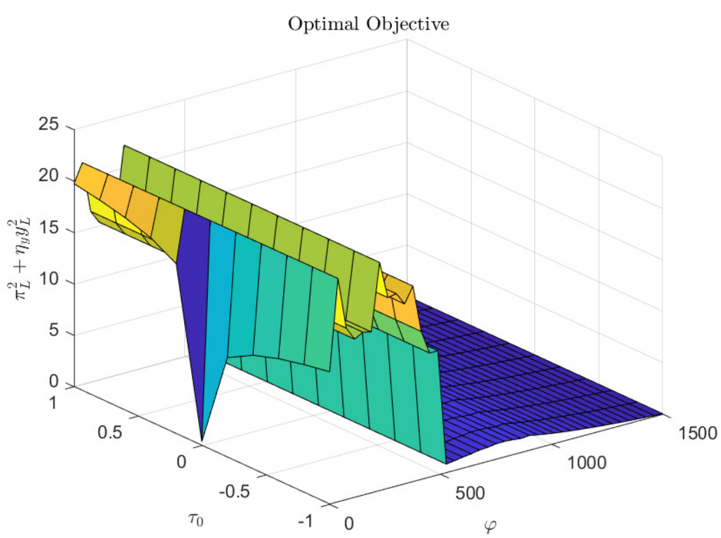


Figure 13: Welfare losses - Max-min policymaker

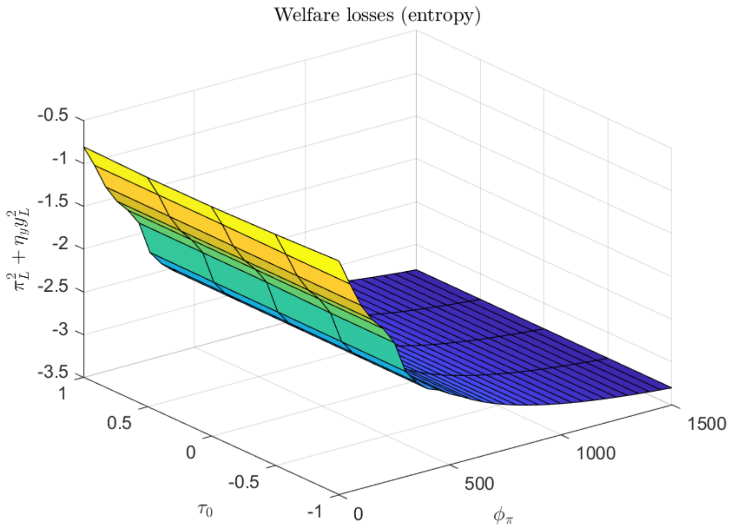


Figure 14: Welfare losses - Policymaker with multiplier preferences

F.1.2 Inflation and output gap

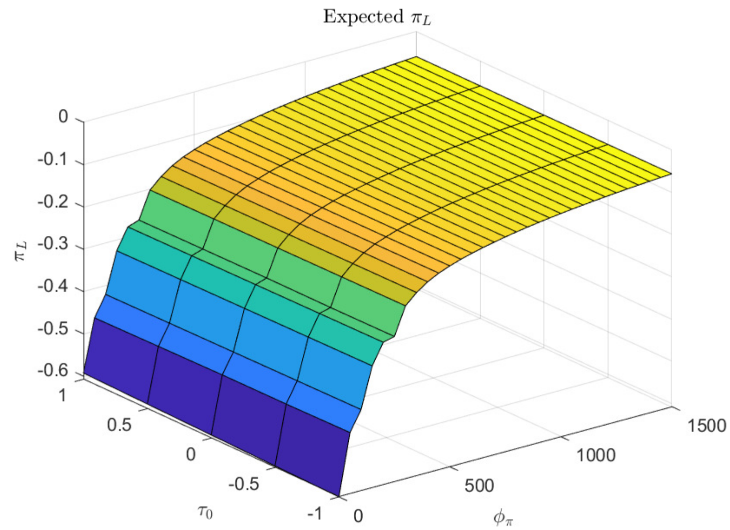


Figure 15: Inflation - Policymaker with prior distribution and multiplier preferences

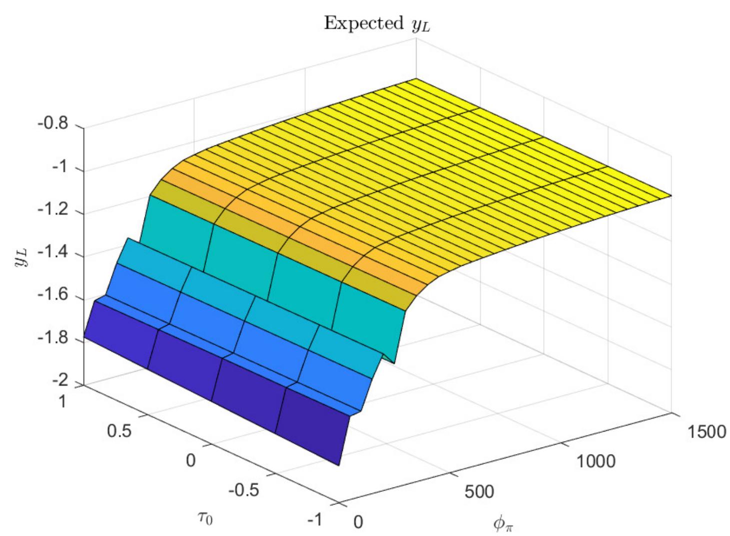


Figure 16: Output Gap - Policymaker with prior distribution and multiplier preferences

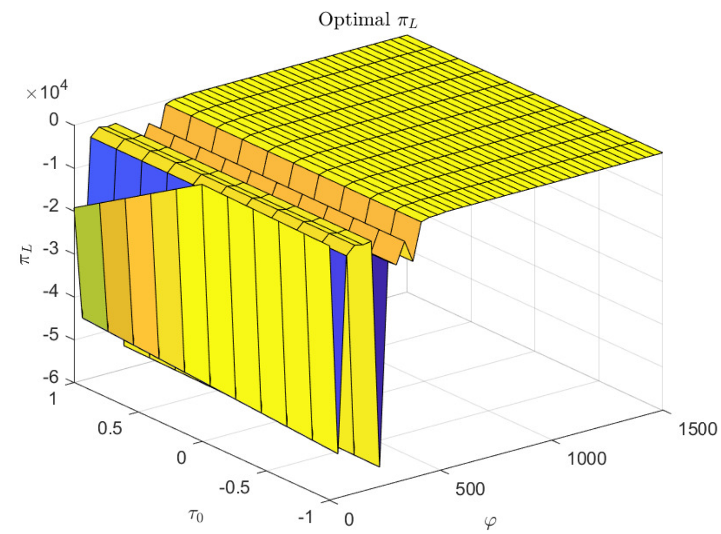


Figure 17: Inflation - Max-min policymaker

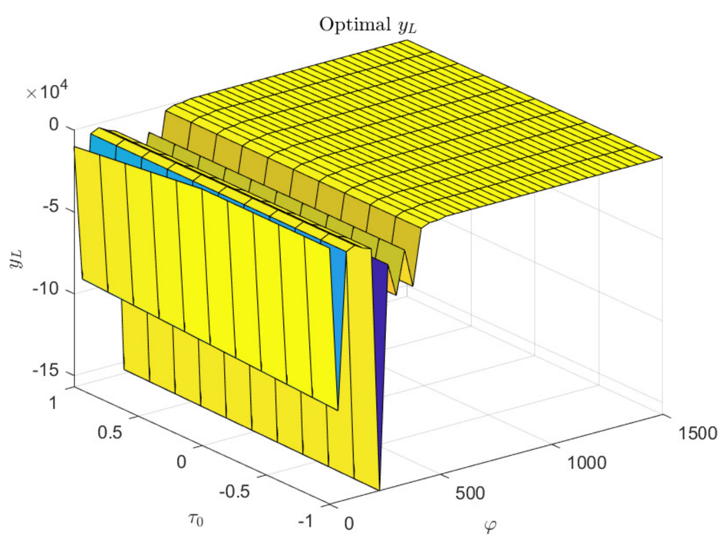


Figure 18: Output gap - Max-min policymaker

F.1.3 Optimal strategy of malign agent (A)

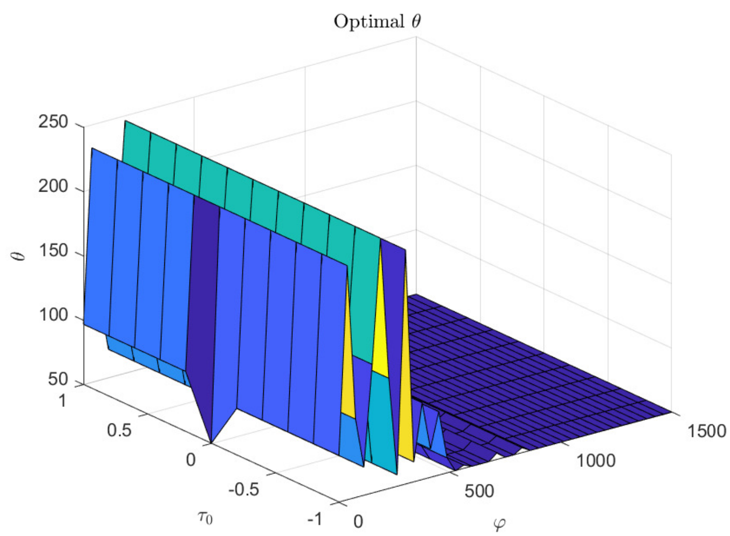


Figure 19: Optimal θ - Max-min policymaker

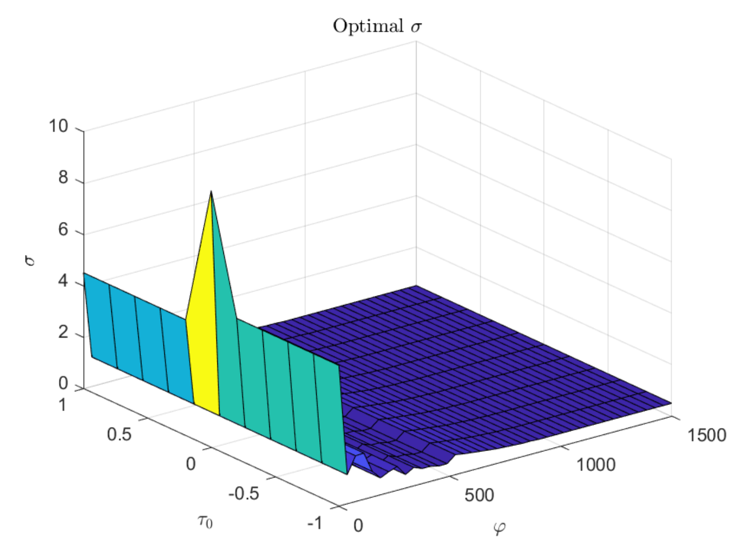


Figure 20: Optimal σ - Max-min policymaker

G Medium-scale Model

G.1 Model

We follow Appendix F.2 of Cuba-Borda and Singh (2024) and describe the final model below. Please refer to CBS for details of the micro-foundation. As in CBS, we de-trend the following variables by productivity: $Y_t = Y_t^*/Z_t$, $C_t = C_t^*/Z_t$, $K_t = K_t^*/Z_t$, $K_t^u = {K^u}_t^*/Z_{t-1}$, $I_t = I_t^*/Z_t$, $W_t = W_t^*/(Z_t P_t)$. Variables with an asterisk denote original levels.

Consumption block. p is the newly introduced discounting factor. We set $p = 1$ when solving for the flexible-price steady state. Otherwise, p is calibrated as in Table 2. Since CBS assume a bond-in-utility specification, households face two first-order conditions related to consumption:

$$\lambda_t = \beta p(1+i_t)E_t\left[\frac{\lambda_{t+1}}{G_{Z,t+1}}\frac{1}{\Pi_{t+1}}\right] + \delta_t, \tag{124}$$

$$\lambda_t = \frac{1}{C_t - \frac{hC_{t-1}}{G_{Z,t}}} - h\beta p E_t \frac{1}{G_{Z,t+1}\left[C_{t+1} - \frac{hC_t}{G_{Z,t+1}}\right]} \tag{125}$$

where λ_t is the marginal effect of de-trended C_t on the household's lifetime utility. δ_t governs the utility gain from bonds. $G_{Z,t} = \frac{Z_t}{Z_{t-1}}$ is the growth rate of TFP Z_t. $\Pi_t = \frac{P_t}{P_{t-1}}$ is the growth rate of price level. h is the degree of external habit formation, and β is the standard discount factor.

Price setting. Intermediate goods producers set prices subject to a quadratic adjustment cost:

$$\frac{\phi_p}{2}\left(\frac{P_{i,t}}{\tilde{\Pi}_{t-1}P_{i,t-1}} - 1\right)^2 P_t Y_t, \quad \text{where} \quad \tilde{\Pi}_{t-1} = \bar{\Pi}^{1-\iota_p}\Pi_{t-1}^{\iota_p}. \tag{126}$$

Parameters ϕ_p and ι_p govern the degree of nominal rigidity and indexation, respectively. $\tilde{\Pi}_{t-1}$ is the cost-free inflation rate implied by indexation. As explained in Section 7, we set $\bar{\Pi} \equiv 1$.

In addition to the Rotemberg adjustment cost, firms also pay the TIP cost, $\tau_t(P_{i,t} - P_{i,t-1})Y_{i,t}$, as specified in (6). Let $v_{p,t}$ denote the inverse demand elasticity, following

CBS. Under symmetry, the price Phillips curve is:

$$
\begin{aligned}
0 = \frac{1}{v_{p,t}} - 1 - \frac{1}{v_{p,t}} MC_t + \phi_p \left(\frac{\Pi_t}{\tilde{\Pi}_{t-1}} - 1 \right) \frac{\Pi_t}{\tilde{\Pi}_{t-1}} \\
- \phi_p \beta p E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\Pi_{t+1}}{\tilde{\Pi}_t} - 1 \right) \frac{\Pi_{t+1}}{\tilde{\Pi}_t} \frac{Y_{t+1}}{Y_t} \\
- \beta p E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{Y_{t+1}}{Y_t} \tau_{t+1} + \tau_t \left(1 - \frac{1}{v_{p,t}} + \frac{1}{v_{p,t}} \frac{1}{\Pi_t} \right).
\end{aligned} \tag{127}
$$

Wage setting. Wages are also subject to indexation and quadratic adjustment costs:

$$
\frac{\phi_w}{2} \left(\frac{W_{i,t}}{\tilde{\Pi}^w_{t-1} W_{i,t-1}} - 1 \right)^2 P_t Y_t, \tag{128}
$$

where

$$
\tilde{\Pi}^w_{t-1} = G_Z \bar{\Pi}^{1-\iota_w} \left(\frac{G_{Z,t}}{G_Z} \Pi_{t-1} \right)^{\iota_w}, \quad \text{and} \quad \Pi^w_t = \frac{W_t}{W_{t-1}} \Pi_t G_{Z,t}. \tag{129}
$$

Under symmetry, and defining $v_{w,t}$ as the inverse labor demand elasticity, the wage Phillips curve is:

$$
\begin{aligned}
v_{w,t} \phi_w \left(\frac{\Pi^w_t}{\tilde{\Pi}^w_{t-1}} - 1 \right) \frac{\Pi^w_t}{\tilde{\Pi}^w_{t-1}} = v_{w,t} \phi_w \beta p E_t \left(\frac{\Pi^w_{t+1}}{\tilde{\Pi}^w_t} - 1 \right) \frac{\Pi^w_{t+1}}{\tilde{\Pi}^w_t} \\
+ \omega L_t^{1+\frac{1}{\eta}} - (1 - v_{w,t}) \lambda_t W_t L_t,
\end{aligned} \tag{130}
$$

where ω governs the disutility of labor.

Investment block. Let K^u_t denote the capital stock at the beginning of period t (deflated by Z_{t-1}) and K_t the capital used in production at the end of period t (deflated by Z_t). With utilization rate u_t,

$$
K_t = u_t \frac{K^u_t}{G_{Z,t}}. \tag{131}
$$

Next, the investment adjustment cost is $S\left(\frac{I_t}{I_{t-1}} \frac{G_{Z,t}}{G_Z} \right)$, where I_t is the de-trended investment. As in CBS, we assume $S(1) = S'(1) = 0$ and $S''(1) = 5.16$. The law of motion for capital is:

$$K_{t+1}^u = \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\frac{G_{Z,t}}{G_Z}\right)\right] I_t + (1-\delta_k)\frac{K_t^u}{G_{Z,t}}. \tag{132}$$

δ_k is the depreciation rate, and μ_t is the investment-specific technology shock ($E_t\mu_t = 1$).

Third, the real utilization cost is $A(u_t)$ per unit of K_t^u. We assume $A(1) = 0$ and $A'' = 2.246$ as in CBS. Note that CBS also assume that utilization is 100% in the steady state, which implies $A'(1) = G_Z\beta^{-1} - (1-\delta_k)$. The required real return on capital r_t^k is

$$r_t^k = A'(u_t). \tag{133}$$

The two FOCs associated with Tobin's Q are:

$$q_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t G_{Z,t+1}}\left(r_{t+1}^k u_{t+1} - A(u_{t+1}) + q_{t+1}(1-\delta_k)\right)\right], \tag{134}$$

$$\begin{aligned} 1 = q_t\mu_t &\left[1 - S\left(\frac{I_t}{I_{t-1}}\frac{G_{Z,t}}{G_Z}\right) - S'\left(\frac{I_t}{I_{t-1}}\frac{G_{Z,t}}{G_Z}\right)\frac{I_t}{I_{t-1}}\frac{G_{Z,t}}{G_Z}\right] \\ &+ \beta E_t\left[\mu_{t+1}\frac{\lambda_{t+1}}{\lambda_t}q_{t+1}\frac{G_{Z,t+1}}{G_Z}\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\frac{G_{Z,t+1}}{G_Z}\right)\right]. \end{aligned} \tag{135}$$

Monetary policy. At the binding ZLB,

$$i_t = 0. \tag{136}$$

A Taylor rule can be applied in the non-ZLB equilibrium, though this is beyond the scope of our analysis.

Production and others. The production function is $Y_t = K_t^\alpha L_t^{1-\alpha}$. Cost minimization implies:

$$r_t^k = \alpha MC_t \frac{Y_t}{K_t}, \tag{137}$$

$$W_t = (1-\alpha) MC_t \frac{Y_t}{L_t}, \tag{138}$$

$$P_t MC_t = \frac{1}{Z_t^{1-\alpha}} \left(\frac{r_t^k}{\alpha}\right)^\alpha \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}. \tag{139}$$

Market clearing requires:

$$\frac{1}{g_t} Y_t = C_t + I_t + A(u_t) \frac{K_t^u}{G_{Z,t}} + \frac{\phi_p}{2} \left(\frac{\Pi_t}{\tilde{\Pi}_{t-1}} - 1\right)^2 Y_t. \tag{140}$$

where $\frac{1}{g_t}$ is the share of non-government spending.

Flexible-price steady state To find the flexible-price steady state (FSS), simply set $p = 1$, $\Pi_t = \frac{\Pi_t^w}{G_Z} = \bar{\Pi} = 1$, and $\frac{1+i_t}{\Pi_{t+1}} = \exp\left(r^*\right) = 1$. Denote the δ compatible with this steady state as δ^{FSS}.

G.2 Self-Fulfilling Deflationary Steady State

We first solve for the self-fulfilling deflationary steady state (DSS). By assumption, r^* does not change, so we keep δ^{FSS}.

The model has three free parameters: p^{DSS}, ϕ_p, and ϕ_w. The parameters ϕ_p and ϕ_w are determined by the short-run slopes of the price and wage NKPCs. The parameter p^{DSS} is calibrated so that steady-state inflation satisfies $\exp \frac{-1.06\%}{4} = 0.99735$.

Consumption block. Given the ZLB ($i_t = 0$) and p^{DSS}, the DSS Euler equation implies:

$$\lambda^{DSS} = \beta p^{DSS} (1+0) \frac{1}{G_Z} \frac{1}{\Pi^{DSS}} \lambda^{DSS} + \delta^{FSS}, \tag{141}$$

This pins down λ^{DSS}. Using the consumption FOC, we then recover C^{DSS}.

Price setting. Indexation implies $\tilde{\Pi}^{DSS} = (\Pi^{DSS})^{\iota_p}$ in the DSS. The steady-state price Phillips curve becomes:

$$0 = (1 - v_p) - MC^{DSS} + v_p \phi_p (1 - \beta p^{DSS}) \left[(\Pi^{DSS})^{1-\iota_p} - 1 \right] (\Pi^{DSS})^{1-\iota_p} \\ + v_p \tau^{DSS} \left(1 - \beta p^{DSS} - \frac{1}{v_p} (1 - (\Pi^{DSS})^{-1}) \right), \tag{142}$$

When $\tau^{DSS} = 0$, the long-run Phillips curve simplifies to:

$$MC^{DSS} = (1 - v_p) - v_p \phi_p (1 - \beta p^{DSS}) \left[(\Pi^{DSS})^{1-\iota_p} - 1 \right] (\Pi^{DSS})^{1-\iota_p}. \tag{143}$$

Since Π^{DSS} is exogenously set to $\exp\left(\frac{-1.06\%}{4}\right)$ (-1.06% annualized), this equation pins down MC^{DSS}.

Investment block. To get K^{DSS}, we combine $r_t^k = \alpha MC_t \frac{Y_t}{K_t}$ and $K_t = u_t \frac{K_t^u}{G_{Z,t}}$:

$$\frac{K^{u\,DSS}}{Y^{DSS}} = \alpha MC^{DSS} \frac{G_Z}{r^{k\,DSS} u^{DSS}}. \tag{144}$$

Together with the FOCs w.r.t. q_t:

$$1 = q^{DSS} \left[1 - S(1) - S'(1) \right] + \beta \left[q^{DSS} S'(1) \right], \tag{145}$$

$$q^{DSS} = 1. \tag{146}$$

and

$$q^{DSS} = 1 = \beta \left[\frac{1}{G_Z} \left(r^{k\,DSS} u^{DSS} - A(u^{DSS}) + q^{DSS} (1 - \delta_k) \right) \right], \tag{147}$$

$$r^{k\,DSS} u^{DSS} = A'(u^{DSS}) u^{DSS} = A(u^{DSS}) + \frac{G_Z}{\beta} - (1 - \delta_k) \tag{148}$$

Easy to verify that $A'(1) = G_Z \beta^{-1} - (1 - \delta_k) \Leftrightarrow u^{DSS} = 1$. Then:

$$\frac{K^{u\,DSS}}{Y^{DSS}} = \alpha MC^{DSS} \frac{G_Z}{r^{k\,DSS} u^{DSS}} = \alpha MC^{DSS} \frac{G_Z}{G_Z \beta^{-1} - (1 - \delta_k)}. \tag{149}$$

Investment then satisfies:

$$\frac{I^{DSS}}{Y^{DSS}} = \frac{K^{uDSS}}{Y^{DSS}}\left(1 - \frac{1}{G_Z}(1-\delta_k)\right), \tag{150}$$

Using market clearing, the consumption-to-output ratio satisfies:

$$\frac{C^{DSS}}{Y^{DSS}} = \frac{1}{g^{DSS}} - \frac{I^{DSS}}{Y^{DSS}} - + \frac{\phi_p}{2}\left((\Pi^{DSS})^{1-\iota_p} - 1\right)^2 \tag{151}$$

Given C^{DSS} from the consumption block, we can solve for Y^{DSS}, I^{DSS}, K^{uDSS}, and $K^{DSS} = \frac{K^{uDSS}}{G_Z}$.

Labor block. We then solve for L^{DSS} using the production function $Y_t = K_t^{\alpha} L_t^{1-\alpha}$, which yields W^{DSS} using the labor FOC. Meanwhile, wage indexation implies:

$$(\tilde{\Pi}^w)^{DSS} = G_Z (\Pi^{DSS})^{\iota_w}, \tag{152}$$

$$(\Pi^w)^{DSS} = \Pi^{DSS} G_Z, \tag{153}$$

$$\frac{(\Pi^w)^{DSS}}{(\tilde{\Pi}^w)^{DSS}} = (\Pi^{DSS})^{1-\iota_w} \tag{154}$$

Substituting into the wage Phillips curve:

$$\begin{aligned}\omega (L^{DSS})^{1+\frac{1}{\eta}} &= v_w \phi_w (1 - \beta p^{DSS})\left((\Pi^{DSS})^{1-\iota_w} - 1\right)(\Pi^{DSS})^{1-\iota_w} \\ &\quad + (1 - v_w)\lambda^{DSS} W^{DSS} L^{DSS}\end{aligned} \tag{155}$$

Only one unique p^{DSS} can satisfies this equality, which in turn pins down p^{DSS}.

G.3 r^*-Driven Deflationary Steady State

Following CBS, we assume that r^* falls from 0% to -1.1% per annum. First, we find δ^{rDSS} that is consistent with $r^* = -1.1\%$ in the flexible-price steady state.

$$\lambda^{FSS} = \beta p \exp\left(\frac{r^*}{4}\right)\frac{\lambda^{FSS}}{G_Z} + \delta^{rDSS} \tag{156}$$

Note that $p = 1$ since we are in the flexible-price SS.

Lastly, repeat the solution for the self-fulfilling DSS using δ^{rDSS}.

G.4 Log Linearization

We follow Cuba-Borda and Singh (2024) and log linearize the model around the deflationary steady state separately for each case.

G.5 Estimates

Table 3: Decline in r^* Liquidity Trap

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
e_w	invg	0.005	Inf	0.005	0.0004	0.0043	0.0057
e_p	invg	0.005	Inf	0.005	0.0004	0.0041	0.0053
e_g	invg	0.005	Inf	0.012	0.0009	0.0103	0.0133
e_μ	invg	0.005	Inf	0.145	0.0125	0.1253	0.1654
e_z	invg	0.005	Inf	0.014	0.0012	0.0126	0.0163
e_b	invg	0.005	Inf	0.056	0.0096	0.0400	0.0714
ρ_w	beta	0.500	0.1500	0.129	0.0451	0.0607	0.1990
ρ_p	beta	0.500	0.1500	0.084	0.0367	0.0256	0.1371
ρ_g	beta	0.500	0.1500	0.834	0.0379	0.7723	0.8978
ρ_μ	beta	0.500	0.1500	0.180	0.0632	0.0779	0.2757
ρ_b	beta	0.500	0.1500	0.754	0.0361	0.6918	0.8104
ρ_z	beta	0.500	0.1500	0.156	0.0471	0.0764	0.2317

Table 4: Self-fulfilling Liquidity Trap

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
e_w	invg	0.005	Inf	0.008	0.0008	0.0068	0.0093
e_p	invg	0.005	Inf	0.009	0.0008	0.0080	0.0104
e_g	invg	0.005	Inf	0.007	0.0005	0.0062	0.0078
e_μ	invg	0.005	Inf	0.182	0.0150	0.1567	0.2061
e_z	invg	0.005	Inf	0.015	0.0010	0.0133	0.0166
e_b	invg	0.005	Inf	0.004	0.0039	0.0012	0.0082
e_{sunspot}	invg	0.005	Inf	0.011	0.0008	0.0093	0.0119
ρ_w	beta	0.500	0.1500	0.248	0.0657	0.1455	0.3608
ρ_p	beta	0.500	0.1500	0.119	0.0375	0.0573	0.1755
ρ_g	beta	0.500	0.1500	0.941	0.0231	0.9051	0.9777
ρ_μ	beta	0.500	0.1500	0.160	0.0549	0.0700	0.2456
ρ_b	beta	0.500	0.1500	0.542	0.0936	0.4115	0.6955
ρ_z	beta	0.500	0.1500	0.309	0.0716	0.1927	0.4231

References

F. O. Bilbiie. Optimal forward guidance. *American Economic Journal: Macroeconomics*, 11(4):310–45, October 2019. .

O. J. Blanchard and C. M. Kahn. The Solution of Linear Difference Models under Rational Expectations. *Econometrica*, 48(5):1305–1311, July 1980.

D. Capelle and Y. Liu. Optimal Taxation of Inflation. *American Economic Journal: Macroeconomics*, forthcoming.

P. Cuba-Borda and S. R. Singh. Understanding persistent zlb: Theory and assessment. *American Economic Journal: Macroeconomics*, 16(3):389–416, July 2024. .

T. Davig and E. M. Leeper. Generalizing the taylor principle. *American Economic Review*, 97(3):607–635, June 2007. .

R. E. Farmer, D. F. Waggoner, and T. Zha. Understanding markov-switching rational expectations models. *Journal of Economic Theory*, 144(5):1849–1867, September 2009. .

R. E. A. Farmer, D. F. Waggoner, and T. Zha. Generalizing the taylor principle: Comment. *American Economic Review*, 100(1):608–617, March 2010. .